Section 4.1 - Exponential Functions Kick-Start

Taken from Section P.3:

If a is any real number and n is a positive integer, then the ***n*th power** of a is

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$$

"n of 'em"

The number a is called the **base**, and n is called the **exponent**.

(.. or "power.")

Example: $(-3)^4 = \underbrace{(-3)(-3)(-3)(-3)}_{4 \text{ of 'em}} = 81$

$-3^4 = -3 \cdot 3 \cdot 3 \cdot 3 = -81$

Multiplying two powers of the same base: Add Exponents.

$$a^m a^n = \underbrace{(a \cdot a \cdot \cdots \cdot a)}_{m \text{ factors}}\underbrace{(a \cdot a \cdot \cdots \cdot a)}_{n \text{ factors}} = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{m+n \text{ factors}} = a^{m+n}$$

$$a^m a^n = a^{m+n}.$$

$3^2 \cdot 3^3 = 3^{2+3} = 3^5 = \underbrace{(3 \cdot 3)(3 \cdot 3 \cdot 3)}_{5 \text{ of 'em}}$

Zero and Negative Exponents:

If $a \neq 0$ is a real number and n is a positive integer, then

$$a^0 = 1 \qquad \text{and} \qquad a^{-n} = \frac{1}{a^n}$$

Also, $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$ & $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Pf: $\frac{1}{\left(\frac{a}{b}\right)} = 1 \cdot \frac{b}{a} = \frac{b}{a}$ "Invert & multiply, saith the math gods."

$2^0 = 1$

$3^{-5} = \frac{1}{3^5} = \frac{1}{243}$

$\left(\frac{2}{3}\right)^{-4} = \frac{1}{\left(\frac{2}{3}\right)^4} = \left(\frac{3}{2}\right)^4$. more on this in a mo'...

Laws of Exponents:

Law	Example	Description
1. $a^m a^n = a^{m+n}$	$3^2 \cdot 3^5 = 3^{2+5} = 3^7$	To multiply two powers of the same number, add the exponents.
2. $\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{3^5}{3^2} = 3^{5-2} = 3^3$	To divide two powers of the same number, subtract the exponents.
3. $(a^m)^n = a^{mn}$	$(3^2)^5 = 3^{2 \cdot 5} = 3^{10}$	To raise a power to a new power, multiply the exponents.
4. $(ab)^n = a^n b^n$	$(3 \cdot 4)^2 = 3^2 \cdot 4^2$	To raise a product to a power, raise each factor to the power.
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$\left(\dfrac{3}{4}\right)^2 = \dfrac{3^2}{4^2}$	To raise a quotient to a power, raise both numerator and denominator to the power.
6. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$	$\left(\dfrac{3}{4}\right)^{-2} = \left(\dfrac{4}{3}\right)^2$	To raise a fraction to a negative power, invert the fraction and change the sign of the exponent.
7. $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}$	$\dfrac{3^{-2}}{4^{-5}} = \dfrac{4^5}{3^2}$	To move a number raised to a power from numerator to denominator or from denominator to numerator, change the sign of the exponent.

$$\left(\frac{2y^{-1}z}{z^2}\right)^{-1}\left(\frac{y}{3z^2}\right)^2 = \left(\frac{z^2}{2y^{-1}z}\right)\left(\frac{y^2}{(3z^2)^2}\right) = \left(\frac{yz^{2-1}}{2}\right)\left(\frac{y^2}{3^2 \cdot z^{2(2)}}\right)$$

$$= \frac{y^{1+2}z^1}{2 \cdot 9z^4} = \frac{y^3}{18z^{4-1}} = \frac{y^3}{18z^3}$$

$$= \frac{y^3z^{1-4}}{18} = \frac{y^3z^{-3}}{18} = \frac{y^3}{18z^3}$$

$$\left(\frac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3} = \left(x^{1-2}y^{-2-3}z^{-3-(-4)}\right)^{-3} = \left(x^{-1}y^{-5}z^1\right)^{-3}$$

$$= \left(\frac{z}{xy^5}\right)^{-3} = \left(\frac{xy^5}{z}\right)^3 = \frac{x^3(y^5)^3}{z^3} = \frac{x^3y^{15}}{z^3}$$

Section 4.1 - Exponential Functions Kick-Start

Taken from Section P.4

The Principal Square Root:

$$\sqrt[2]{a} = b \qquad \text{means} \qquad b^2 = a \qquad \text{and} \qquad b \geq 0$$

The Principal n^{th} Root:

If n is any positive integer, then the **principal *n*th root** of a is defined as follows:

$$\sqrt[n]{a} = b \qquad \text{means} \qquad b^n = a$$

If n is even, we must have $a \geq 0$ and $b \geq 0$.

For practical purposes, we think of $\sqrt[n]{a}$ as $a^{\frac{1}{n}}$

For any rational exponent m/n in lowest terms, where m and n are integers and $n > 0$, we define

$$a^{m/n} = (\sqrt[n]{a})^m \qquad \text{or equivalently} \qquad a^{m/n} = \sqrt[n]{a^m}$$

If n is even, then we require that $a \geq 0$.

$\sqrt[4]{x^4}$. $x^4 \geq 0$ no matter what x, so Domain, here is $(-\infty, \infty)$ and since $\sqrt[4]{y}$ is the positive 4th root of y, we have $\sqrt[4]{x^4} = |x|$, just like $\sqrt{x^2} = |x|$

But, whenever there's a $\sqrt[4]{x}$ around, we must have $x \geq 0$ & so $(\sqrt[4]{x})^4 = x$, not $|x|$.

$\sqrt[4]{x^4} = (x^4)^{\frac{1}{4}} = |x|$, still think of it as "x" and restrict the domain as needed.

$\sqrt[3]{x^3y^6} = (x^3y^6)^{\frac{1}{3}} = x^{3(\frac{1}{3})} \cdot y^{6(\frac{1}{3})} = xy^2$

and no $|x|$ nonsense.

Section 4.1 - Exponential Functions Kick-Start

$\sqrt{32} + \sqrt{18}$

$= 4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$

$2\,|\,32$, $2\,|\,16$, $2\,|\,8$, $2\,|\,4$, 2

$2\,|\,18$, $3\,|\,9$, 3

$\sqrt{32} = \sqrt{2^5} = \sqrt{2^{4+1}} = \sqrt{2^4 \cdot 2^1}$

$= \sqrt{2^4}\sqrt{2^1} = 2^{4(\frac{1}{2})}\sqrt{2} = 2^2\sqrt{2} = 4\sqrt{2}$

$\sqrt{18} = \sqrt{3^2 \cdot 2} = \sqrt{3^2}\sqrt{2} = 3\sqrt{2}$

$\sqrt{36x^2 + 36y^2}$

$= \sqrt{36(x^2+y^2)}$

$= \sqrt{36}\sqrt{x^2+y^2} = \boxed{6\sqrt{x^2+y^2}}$

No. $\sqrt{x^2+y^2}$ is not $\sqrt{x^2} + \sqrt{y^2} = |x| + |y|$

$\sqrt{3^2+4^2} = \sqrt{9+16} \stackrel{?}{=} \sqrt{9} + \sqrt{16} = 3 + 4 = 7?$

No. $\sqrt{9+16} = \sqrt{25} = 5$

$5 \neq 7!$ So $\sqrt{x^2+y^2} \neq \sqrt{x^2} + \sqrt{y^2}$

$\dfrac{\sqrt{xy}}{\sqrt[4]{16xy}} = \dfrac{(xy)^{\frac{1}{2}}}{(16xy)^{1/4}} = \dfrac{x^{\frac{1}{2}}y^{\frac{1}{2}}}{16^{\frac{1}{4}}x^{\frac{1}{4}}y^{\frac{1}{4}}} = \dfrac{x^{\frac{1}{2}-\frac{1}{4}}y^{\frac{1}{2}-\frac{1}{4}}}{(2^4)^{1/4}}$

$= \dfrac{x^{\frac{1}{4}}y^{\frac{1}{4}}}{2} = \dfrac{(xy)^{\frac{1}{4}}}{2} = \dfrac{\sqrt[4]{xy}}{2}$

$2\,|\,16$, $2\,|\,8$, $2\,|\,4$, 2

Have to assume $x > 0$ & $y > 0$.

or assume $x < 0$ when $y < 0$.

$\left(\dfrac{x^{1/2}y^2}{2y^{1/4}}\right)^4 \left(\dfrac{4x^{-2}y^{-4}}{y^2}\right)^{1/2} = \dfrac{x^2y^8}{2^4y^1}\left(\dfrac{4^{\frac{1}{2}}x^{-1}y^{-2}}{y^1}\right)$

$= \dfrac{2x^{2-1}y^{8-2-1-1}}{2^4} = \dfrac{xy^4}{2^3}$

Section 4.1 - Exponential Functions Intro

Section 4.1 Proper:

The **exponential function with base *a*** is defined for all real numbers x by

$$f(x) = a^x$$

where $a > 0$ and $a \neq 1$. Never see $(-3)^x$

While this looks familiar, we haven't actually seen any functions with the variable in the exponent.

Examples $f(x) = 3^x$ or $f(x) = 2^x$

While it may seem of no consequence, extending what we know about integer and rational exponents to ALL real numbers is not a trivial matter, even though your intuition tells you that it *should* work the way we say it does, and that these exponential functions are smooth, positive, and they grow very fast.

$2^{\sqrt{3}} = ?$ $\sqrt{3} = 1.73205\ldots$

whatever $2^{\sqrt{3}}$ is, we expect it must be between $2^{1.732}$ & $2^{1.733}$

$\left(= 2^{\frac{1732}{1000}} \text{ \& } 2^{\frac{1733}{1000}}\right)$

We can approximate $2^{\sqrt{3}}$ as close as we want, using rational exponents,. Just run the square root of 3 out farther and farther.

$\sqrt{3} = 1.732050808\ldots$ (infinite, non-repeating decimal)

by Maple

$\approx 1.7320508075688772935274463415058723669428052538103806280558069794$

Wolfram Alpha

so we surmise that $2^{\sqrt{3}}$ *does* exist, and it is a real number between

$2^{1.7320508075688772935274463415058723669428052538103806280558069793}$

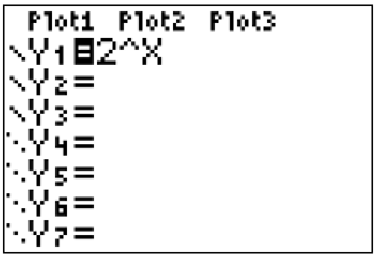
and

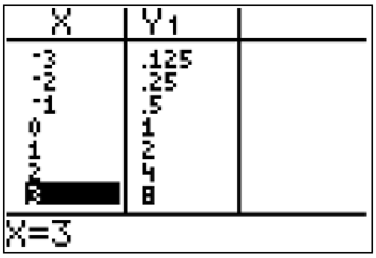
$2^{1.7320508075688772935274463415058723669428052538103806280558069795}$

and *both* numbers are ridiculously close to its actual value.

Section 4.1 - Exponential Functions Intro

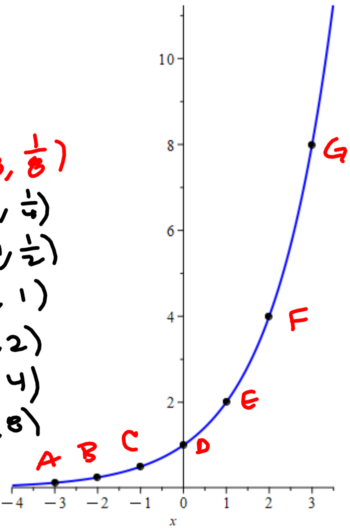
What does $f(x)=2^x$ **look like?**





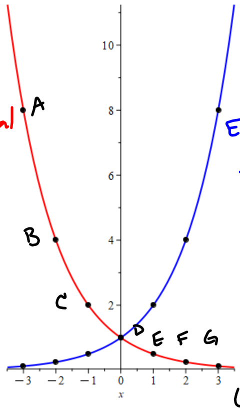
x	$f(x)=2^x$
-3	$2^{-3}=\frac{1}{2^3}=\frac{1}{8}$
-2	$2^{-2}=\frac{1}{2^2}=\frac{1}{4}$
-1	$2^{-1}=\frac{1}{2}$
0	$2^0=1$
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$

$A=(-3,\frac{1}{8})$
$B=(-2,\frac{1}{4})$
$C=(-1,\frac{1}{2})$
$D=(0,1)$
$E=(1,2)$
$F=(2,4)$
$G=(3,8)$



Section 4.1 - Exponential Functions Intro

If $g(x)=(\frac{1}{2})^x$, then $g(x) = (2^{-1})^x = 2^{-x} = f(-x)$ for $f(x)=2^x$!
The reciprocal base results in a reflection about the y-axis!
We can graph $g(x)=(\frac{1}{2})^x = 2^{-x}$ by transforming the

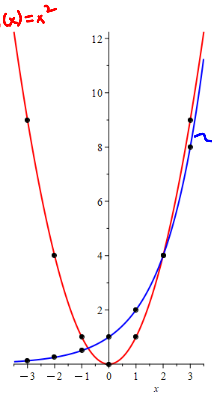


A=(-3, 8)
B=(-2, 4)
C=(-1, 2)
D=(0, 1)
E=(1, ½)
F=(2, ¼)
G=(3, ⅛)

$(\frac{1}{2})^x$: $0<b<1 \Rightarrow b^x$ is exponential decay.

And finally, 2^x : $b>1 \Rightarrow b^x$ " " Growth

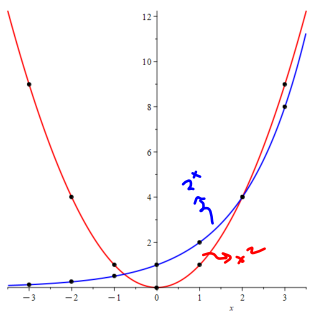
Exponential growth is much greater than polynomial (power-function) growth.



Scaling Constrained.

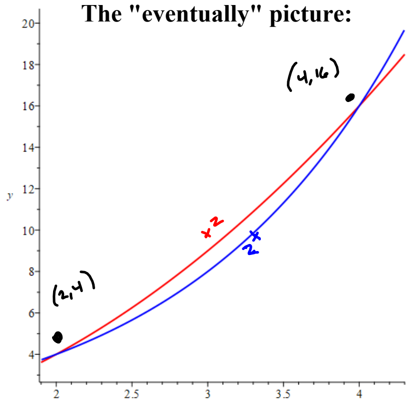
Quantitative correctness

makes things tall and skinny.



Scaling unconstrained for clearer picture, qualitatively.

The "eventually" picture:



Fact: Exponential growth will always catch up to and surpass mere power-function growth.

You'd get the same thing if you compared

x^{500} to 1.001^x
Eventually, 1.001^x will be bigger. It'll just happen way later than x=4!

Exponential Growth:
b^x, $b>1$. $2^x, 1.1^x, 5^x$

Exponential Decay
b^x, $0<b<1$ $(\frac{1}{2})^x, .01^x, .999^x$

$b<0$? Not even real.

Section 4.1 - Exponential Functions Intro

Simple and Compound Interest

Recall: Simple Interest

P= Principal = Starting amount = PRESENT VALUE ($= \$$)

A= Future Value = $A(t)$ as a function of

t = time in years

r = Annual interest rate. (7% means .07)

= Annual proportion of the principal earned each year.

$t = 1$ yr $\quad A(1) = P + Pr = P(1+r) = 100(1.05) = \$105$

$t = \frac{1}{2}$ yr $\quad P + Pr(\frac{1}{2}) = P(1 + \frac{1}{2}r) = 100(1.025) = \$102.50$

$t = 2$ yr $\quad P + Pr(2) = P(1 + 2r) = 100(1.10) - \$110.00$

$A(t) = P + Prt = P(1 + rt)$

\$500 loaned at 5% simple annual interest for 3 years.

It earns \$25 every year, and that's it.

To convert this to compound interest, suppose at the end of the year, that \$25 is added to the principle, so the *next* year, you earn 5% of \$525, and the future value is

$A(2) = \$525 + .05(525) = \$525 + \$26.25 = \$551.25$

This is better than simple interest.

Yr	
1	$500 + .05(500) = 500(1+.05)$
2	$500(1+.05) + .05(500(1+.05)) = 500(1+.05)(1+.05)$
	$= 500(1+.05)^2$
	$= P(1+r)^t$

$500(1.05)^2 = 551.25$!

That's compounded *annually*. For compounding *more* than once per year, it's a little trickier.

Let *m* = the number of times you are going to compound per year.

Define:

$n = mt$ = the total number of *compounding periods* in t years.

$i = r/m$ = the interest rate *per period*.

Here's how it works:

Period	Amount A
1	$P + \frac{r}{m}P = P(1+\frac{r}{m})$
2	$P(1+\frac{r}{m}) + \frac{r}{m}(P(1+\frac{r}{m})) = P(1+\frac{r}{m})[1 + \frac{r}{m}]$
	$= P(1+\frac{r}{m})^2$
⋮	⋮
n	$P(1+\frac{r}{m})^n$

You will typically be given the time t in years. This gives a totally broken-down formula for the future value for compound interest:

$A(t) = P(1+\frac{r}{m})^{mt}$

A more compact version: $A(t) = P(1+i)^n$

See #s 4, 24-end.

We will define *Present Value* and *Effective Annual Rate* as they arise in the exercises.

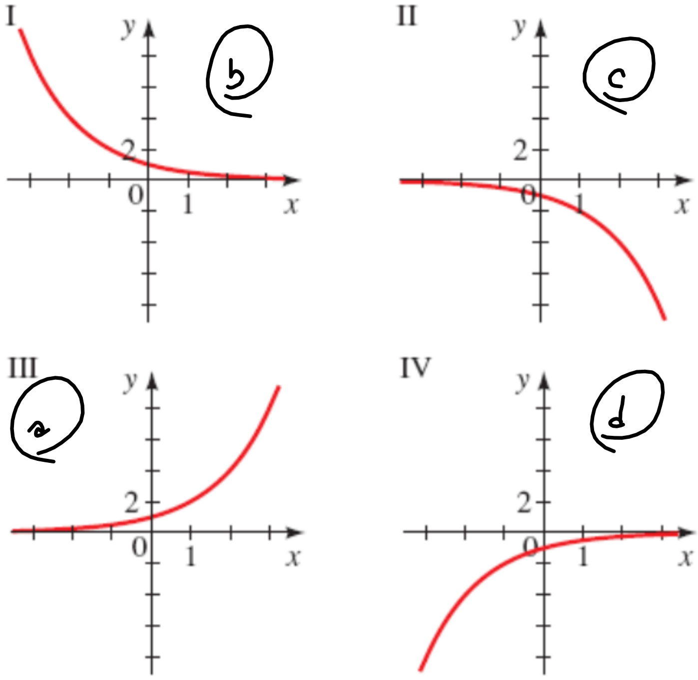
Examples of the above discussion are embedded in the exercises, starting

1 The function $f(x) = 5^x$ is an exponential function with base [5]; $f(-2) =$ [1/25],
$f(0) =$ [1], $f(2) =$ [25], and $f(6) =$ [5^6].

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Match the exponential function with its graph.

2



(a) $f(x) = 2^x$

(b) $f(x) = 2^{-x}$

(c) $f(x) = -2^x$

(d) $f(x) = -2^{-x}$

3

(a) To obtain the graph of $g(x) = 6^x - 1$, we start with the graph of $f(x) = 6^x$ and shift it [---Select--- Down] 1 unit.

$f(x) - 1$

(b) To obtain the graph of $h(x) = 6^{x-1}$, we start with the graph of $f(x) = 6^x$ and shift it to the [---Select--- RIGHT] 1 unit.

$f(x-1)$

4 In the formula $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ for compound interest, the letters P, r, n and t stand for [---Select---], [---Select---], [---Select---] and [---Select---] respectively, and $A(t)$ stands for [---Select---]. So if \$200 is invested at an interest rate of 4% compounded quarterly, then the amount after 4 years is \$ [ ]. (Round your answer to the nearest cent.)

P = Principal

r = annual interest rate

n = # of periods per year.

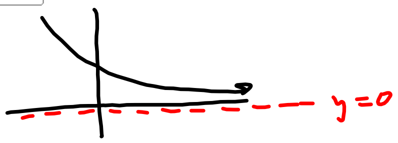
(I use "m" instead of "n", reserving "n" for $n = mt$ = total # of periods.)

t = time in years

5 The exponential function $f(x) = \left(\frac{1}{6}\right)^x$ has the ---Select--- asymptote $y =$ ______ . This means that as $x \to \infty$, we have $\left(\frac{1}{6}\right)^x \to$ 0 .

Horizontal

0



$\left(\frac{1}{6}\right)^x = \frac{1}{6^x} \xrightarrow{x \to \infty} 0$

$\left(\frac{1}{6}\right)^x > 0$ always

6 The exponential function $f(x) = \left(\frac{1}{3}\right)^x + 2$ has the ---Select--- asymptote $y =$ 2 . This means that as $x \to \infty$, we have $\left(\frac{1}{3}\right)^x + 2 \to$ 2 .

$g(x) = \left(\frac{1}{3}\right)^x$

$f(x) = \left(\frac{1}{3}\right)^x + 2 = g(x) + 2$ up 2

Use a calculator to evaluate the function at the indicated values. Round your answers to three decimal places.

7 $f(x) = 9^x$

$f\left(\frac{1}{2}\right) =$ 3

$f(\sqrt{6}) =$ ______

$f(-2) =$ ______

$f(0.4) =$ ______

$9^{\frac{1}{2}} = 3$

$9^{\sqrt{6}} \approx$

$9^{-2} = \frac{1}{9^2} = \frac{1}{81}$

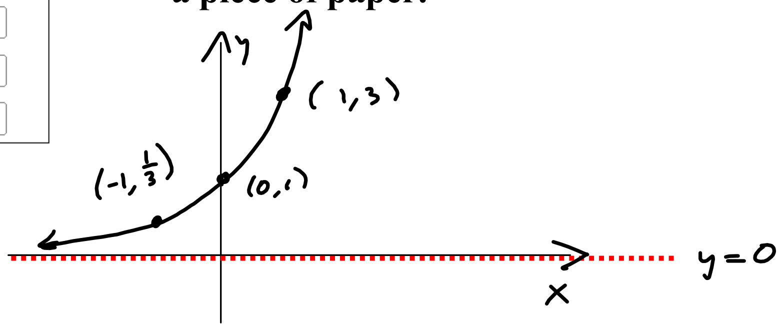
$9^{.4} = 9^{4/10} = 9^{2/5} \approx$

8 Sketch the graph of the function by making a table of values. Use a calculator if necessary. (Simplify your answers completely.)

$f(x) = 3^x$

x	$f(x) = 3^x$
−4	
−2	
0	
2	
4	

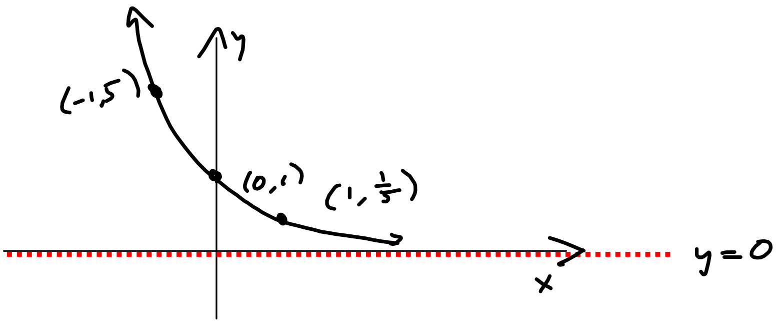
If you want to see me plug points in and turn the crank like a drone, please see the introductory videos. We're to the point where we should be able to throw a quick sketch of this function on a piece of paper.



9 Sketch the graph of the function by making a table of values. Use a calculator if necessary. (Simplify your answers completely.)

$f(x) = \left(\frac{1}{5}\right)^x$

x	$f(x) = \left(\frac{1}{5}\right)^x$
-2	
-1	
0	
1	
2	



10 Graph both functions on one set of axes.

$f(x) = 2^x$ and $g(x) = 2^{-x}$

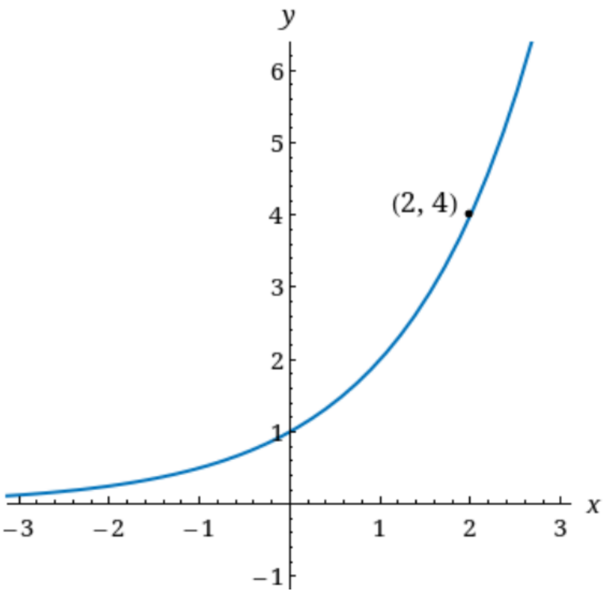
See Intro Video: 00c-Intro-to-4-1-Proper.mp4

11 Graph both functions on one set of axes.

$f(x) = 3^{-x}$ and $g(x) = \left(\frac{1}{3}\right)^x$

See Intro Video: 00c-Intro-to-4-1-Proper.mp4

12 Find the exponential function $f(x) = a^x$ whose graph is given.



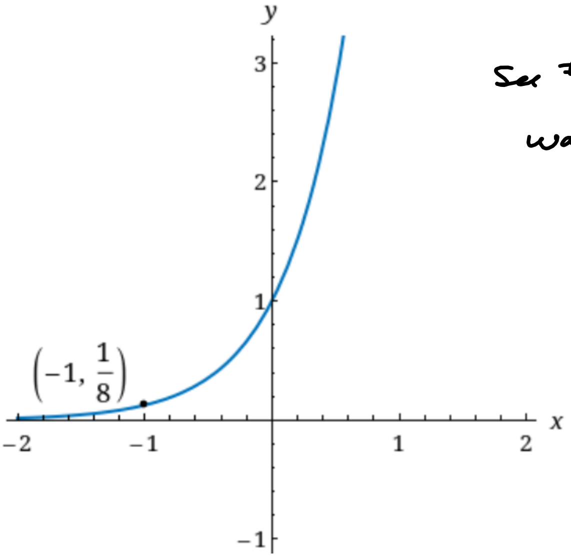
Find a ∋

$a^2 = 4 = 2^2$

$\rightarrow \boxed{a = 2}$

Find the exponential function $f(x) = a^x$ whose graph is given.

13



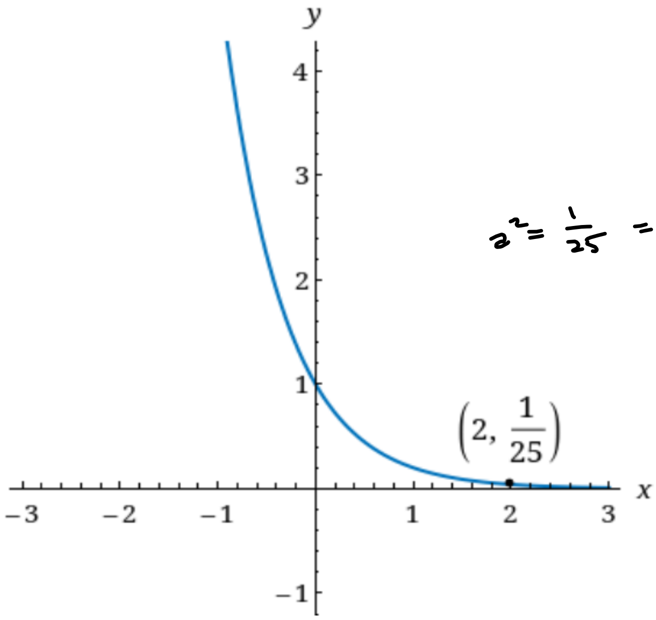
See #12

want

$a^{-1} = \frac{1}{8}$

Find the exponential function $f(x) = a^x$ whose graph is given.

14



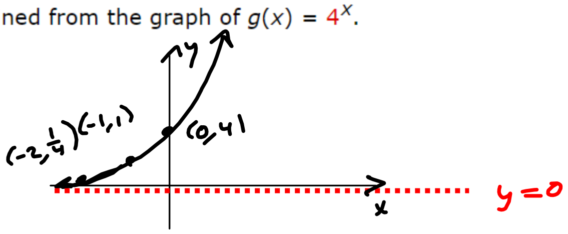
$a^2 = \frac{1}{25} = \frac{1}{5^2} = \frac{1^2}{5^2} = \left(\frac{1}{5}\right)^2$

Consider the following exponential function.

15 $f(x) = 4^{x+1}$ $= g(x+1)$ Left 1 unit

Explain how the graph of f is obtained from the graph of $g(x) = 4^x$.

- ○ shift left 1 unit
- ○ shift downward 1 unit
- ○ shift right 1 unit
- ○ shift upward 1 unit



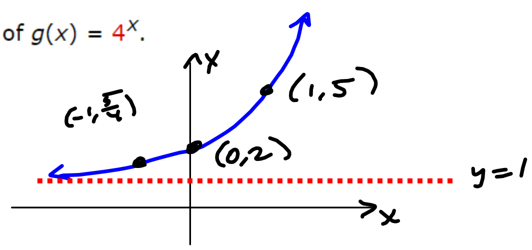
Match the exponential function with one of the graphs labeled I or II.

Consider the following exponential function.

16 $f(x) = 4^x + 1$ $= g(x) + 1$

Explain how the graph of f is obtained from the graph of $g(x) = 4^x$.

- ○ shift left 1 unit
- ○ shift downward 1 unit
- ○ shift right 1 unit
- ○ shift upward 1 unit



Match the exponential function with one of the graphs labeled I or II.

17 In this exercise we compare the graphs of two exponential functions.

Sketch the graphs of $f(x) = 4^{x/2}$ and $g(x) = 2^x$.

$4^{\frac{x}{2}} = 4^{\frac{1}{2}x} = \left(4^{\frac{1}{2}}\right)^x = 2^x$. They're the SAME!

Use the Laws of Exponents to explain the relationship between these graphs.

- ○ $4^{(x/2)} = (2^2)^{(x/2)} = 2^{2(x/2)} = 2^x$
- ○ $4^{(x/2)} = 4^x \cdot 4^x = 2^{2x} \cdot 2^{2x} = (2^x)^4$
- ○ $4^{(x/2)} = (4/2)^x = 2^x$
- ○ $4^{(x/2)} = (2^{x/2} + 2^{x/2}) = 2 \cdot 2^{x/2} = 2^x$
- ○ $4^{(x/2)} = 4^x \cdot 4^{1/2} = 4^x \cdot 2 = 2 \cdot (2^x)^2$

$4^{x+\frac{1}{2}}$?! NO

Compare the graphs of the power function *f* and exponential function *g* by evaluating both of them for $x = 0, 1, 2, 3, 4, 6, 8$, and 10.

18

$f(x) = x^2; \quad g(x) = 2^x$

x	$f(x) = x^2$	$g(x) = 2^x$
0		
1		
2		
3		
4		
6		
8		
10		

Draw the graphs of *f* and *g* on the same set of axes.

See Intro Video: 00c-Intro-to-4-1-Proper.mp4

Compare the graphs of the power function *f* and exponential function *g* by evaluating both of them for $x = 0, 1, 2, 3, 4, 6, 8$, and 10.

19

$f(x) = x^5; \quad g(x) = 5^x$

x	$f(x) = x^5$	$g(x) = 5^x$
0		
1		
2		
3		
4		
6		
8		
10		

See Intro Video: 00c-Intro-to-4-1-Proper.mp4

Draw the graphs of *f* and *g* on the same set of axes.

20 This exercise involves a difference quotient for an exponential function.

If $f(x) = 2^x$, show that

$$\frac{f(x+h)-f(x)}{h} = 2^x\left(\frac{2^h-1}{h}\right).$$

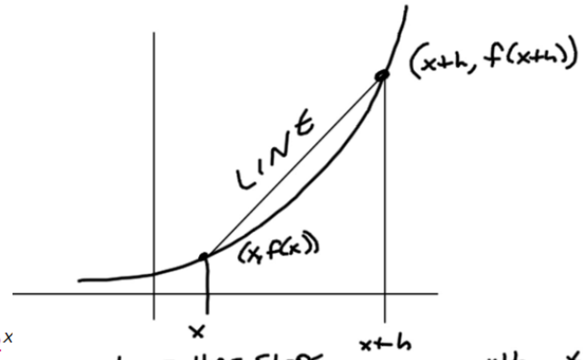
Simplify your answers completely at each step.

$f(x) = 2^x$, so

$$\frac{f(x+h)-f(x)}{h} = \frac{\boxed{} - 2^x}{h}$$

$$= \frac{(2^x)\left(\boxed{}\right) - 2^x}{h}$$

$$= 2^x\left(\frac{2^h-1}{h}\right).$$



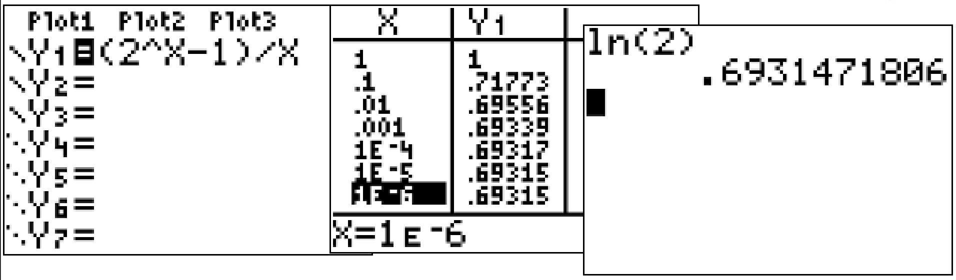
LINE HAS SLOPE

$$\frac{y_2-y_1}{x_2-x_1} = \frac{f(x+h)-f(x)}{x+h-x} = \frac{2^{x+h}-2^x}{h}$$

$$= \frac{2^x2^h-2^x}{h} = \frac{2^x(2^h-1)}{h}$$

$$= 2^x\left(\frac{2^h-1}{h}\right)$$

In calculus, the idea is to let h go to 0.



21 This exercise involves a difference quotient for an exponential function.

If $f(x) = 8^{x-1}$, show that

$$\frac{f(x+h)-f(x)}{h} = 8^{x-1}\left(\frac{8^h-1}{h}\right).$$

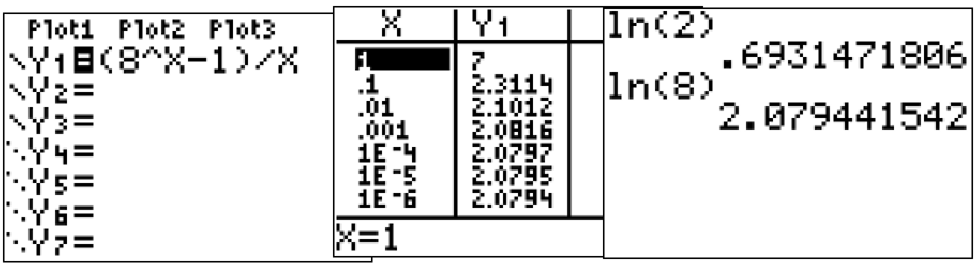
Simplify your answers completely at each step.

$f(x) = 8^{x-1}$ so

$$\frac{f(x+h)-f(x)}{h} = \frac{\boxed{} - 8^{x-1}}{h}$$

$$= \frac{(8^{x-1})\left(\boxed{}\right) - 8^{x-1}}{h}$$

$$= 8^{x-1}\left(\frac{8^h-1}{h}\right).$$



A bacteria culture contains 1,300 bacteria initially and doubles every hour.

22 (a) Find a function N that models the number of bacteria after t hours.

$N(t) =$ ______

(b) Find the number of bacteria after 24 hours.

______ bacteria

Hour		
1	$1300 \cdot 2$	
2	$1300 \cdot 2 \cdot 2$	$= 1300 \cdot 2^2$
3	$1300 \cdot 2^2 \cdot 2$	$= 1300 \cdot 2^3$

Let $N(t)$ = Number of bacteria

Then $N(t) = 1300(2^t)$, where

t = time in hours

In general $N(t) = N_0 \cdot 2^t$, where $N_0 = N(0) =$ Initial Pop.

A certain breed of mouse was introduced onto a small island with an initial population of 320 mice, and scientists estimate that the mouse population is doubling every year.

23 (a) Find a function N that models the number of mice after t years.

$N(t) =$ ______

(b) Estimate the mouse population after 8 years.

______ mice

An investment of $6,000 is deposited into an account in which interest is compounded monthly. Complete the table by filling in the amounts the investment grows to at the indicated times. (Round your answers to the nearest cent.)

24

$r = 5\%$

Time (years)	Amount
1	$
2	$
3	$
4	$
5	$
6	$

$A(t) = P(1+\frac{r}{m})^{mt}$

$r = .05$

$m = 12$

$\frac{r}{m} = \frac{.05}{12}$

$mt = 12t$

$A(t) = 6000\left(1+\frac{.05}{12}\right)^{12t}$

```
Y1(1)
        9792.564796
Y1(1
        9792.564796
Y1(2
        15982.38755
```

```
  X    | Y1
 1     | 9792.6
 2     | 15982
 3     | 26085
 4     | 42573
 5     | 69483
 1E-5  | 6000
 1E-6  | 6000
X=1E-5
```

Table is quicker, but it doesn't carry enough significant digits, so it starts chopping off the decimals, which sucks.

```
        15982.38755
Y1(3
        26084.76094
Y1(4
        42572.78529
Y1(5
        69482.79309
```

```
        26084.76094
Y1(4
        42572.78529
Y1(5
        69482.79309
Y1(6
        113402.4589
```

```
Plot1 Plot2 Plot3
\Y1=6000(1+0.5/1
2)^(12X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

25 If $9,000 is invested at an interest rate of 7.75% per year, compounded semiannually, find the value of the investment (in dollars) after the given number of years. (Round your answers to the nearest cent.)

(a) 6 years
$

(b) 12 years
$

(c) 18 years
$

$P=9000$, $m=2$ periods per year

If $4,000 is invested at an interest rate of 6.5% per year, compounded daily, find the value of the investment (in dollars) after the given number of years. (Round your answers to the nearest cent.)

$m = 365$

26

(a) 3 years
$

(b) 4 years
$

(c) 8 years
$

$\left.\begin{matrix} r = .065 \\ m = 365 \end{matrix}\right\}$ $i = \frac{r}{m} = \frac{.065}{365}$. Don't calculate this leave it!

Keep this "compounded daily" exercise handy, to compare it to "compounded continuously" in the sequel.

```
        42572.78529
Y1(5
        69482.79309
Y1(6
        113402.4589
.065/365
     1.780821918E-4
```

.000178082191 8 is messy, but use it w/o rounding if you don't have a graphing calculator that permits these longer/more complicated expressions.

27 If \$18,000 is borrowed at a rate of 4.75% interest per year, compounded quarterly, find the amount due (in dollars) at the end of the given number of years. (Round your answers to the nearest cent.)

(a) 4 years

\$ ______

(b) 6 years

\$ ______

(c) 8 years

\$ ______

28 The **present value** of a sum of money is the amount that must be invested now, at a given rate of interest, to produce the desired sum at a later date.

How much should be invested now (the present value) to have an amount of \$10,000, 4 years from now, if the amount is invested at an interest rate of 6% per year, compounded semiannually. (Round your answer up to the next cent.)

\$ ______

$r = .06$ Know $A(4) = \$10000$

$m = 2$ want P!

$$P\left(1+\frac{r}{m}\right)^{mt} = A(t) \longrightarrow$$

$$P = \frac{A(t)}{\left(1+\frac{r}{m}\right)^{mt}} = A(t)\left(1+\frac{r}{m}\right)^{-mt}$$

$$P(t) = A\left(1+\frac{r}{m}\right)^{-mt}$$

want $P(4) = 10000\left(1+\frac{.06}{2}\right)^{-2(4)} \approx \$5765.31|1223$

$\approx \$5765.31$

```
6000(1+.06/12)^-
(2*4)
          5765.311223
```

10000

silly!

2

```
Ans*(1+.06/2)^8
                 6000
10000(1+.06/2)^-
(2*4)
          7894.092343
Ans*(1+.06/2)^8
                10000
```

No. A = \$10000

m = 2 were

both missed

$\boxed{\$7894.09 \approx P}$

29 The **present value** of a sum of money is the amount that must be invested now, at a given rate of interest, to produce the desired sum at a later date.

How much should be invested now (the present value in dollars) to have an amount of \$275,000, 4 years from now, if the amount is invested at an interest rate of 7% per year, compounded monthly. (Round your answer to the nearest cent.)

\$ ______

See #28

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

30

Tutorial Exercise

Find the annual percentage yield for an investment that earns 4% per year, compounded monthly.

Step 1

Recall that for an investment that earns compound interest, the annual percentage yield (APY) of the investment is the simple interest rate that yields the same amount at the end of one year.

We are asked to find the APY for an investment that earns 4% per year, compounded monthly. So in this case, the investment calculates interest not only on the principal, but also calculates interest on the interest that has accumulated in previous months. We want to find the simple interest rate that yields the same amount of interest over a year.

First, find the value of the investment after one year with the 4% interest compounded monthly. (Round your final answer to four decimal places.)

$$A_{\text{compound}} = P\left(1 + \frac{\qquad}{12}\right)^{12}$$

$$= P\left(\qquad \right)$$

Recall the formula for simple interest on principal P with rate r is as follows.

I called this *effective rate of interest*, which is the term used in some texts.

I'll try to stick with "APY" or *annual percentage yield*.

Here's the idea: We want the *simple* interest rate with the same future value after one year of some given compound interest rate.

$r = .04$

$m = 12$

want r_e so that

$$P(1 + r_e(1)) = P\left(1 + \frac{r}{m}\right)^{12 \cdot 1} = P\left(1 + \frac{.04}{12}\right)^{12}$$

```
(1+.04/12)^12-1
    .0407415429
```

$$1 + r_e = \left(1 + \frac{.04}{12}\right)^{12}$$

$$r_e = \left(1 + \frac{.04}{12}\right)^{12} - 1 = \left(1 + \frac{r}{m}\right)^{m} - 1$$

$\approx .0407415429$ or

$4.07415429\,\%$

31 Find the annual percentage yield for an investment that earns $5\frac{1}{2}$% per year, compounded quarterly. (Round your answer to two decimal places.)

[ ] %