Find all values of x for which the graph of f lies above the graph of g. (Enter your answer using interval notation.)

20. $f(x) = x^2 + x; \quad g(x) = \frac{12}{x}$

want $f(x)$ above $g(x)$ $\Rightarrow$

$f(x) > g(x)$ $\Rightarrow$

$h(x) = f(x) - g(x) > 0$ $\Rightarrow$

$x^2 + x - \frac{12}{x} = \frac{x^2}{1} \cdot \frac{x}{x} + \frac{x}{1} \cdot \frac{x}{x} - \frac{12}{x} = \frac{x^3 + x^2 - 12}{x} > 0$

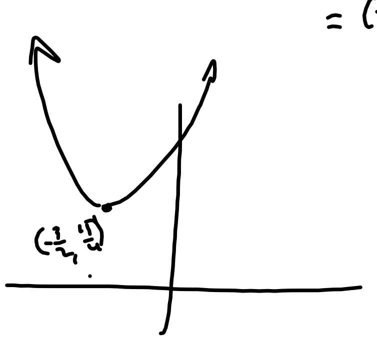
$\pm 1, \pm 2, \vdots$ $x = 2$ makes it zero!

$$\begin{array}{r|rrrr} 2 & 1 & 1 & 0 & -12 \\ & & 2 & 6 & 12 \\ \hline & 1 & 3 & 6 & 0 \end{array}$$

This says $h(x) = (x-2)(x^2+3x+6)$

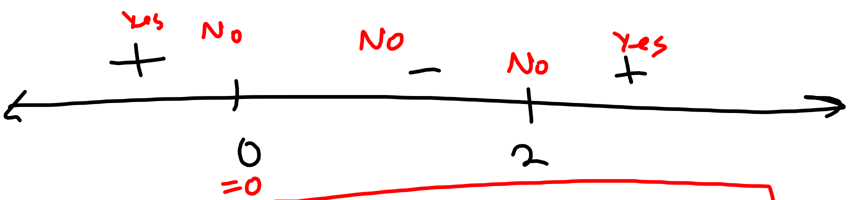
& $x^2 + 3x + 6 = x^2 + 3x + \left(\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{24}{4}$

$= \left(x + \frac{3}{2}\right)^2 + \frac{15}{4}$ SET 0



$\left(x + \frac{3}{2}\right)^2 = -\frac{15}{4}$ $\Rightarrow$ x ain't real & we're keepin' it real for graphs & such.

$h(x) = \frac{(x-2)(x^2+3x+6)}{x} > 0$



want > 0 "+"

$x \in \boxed{(-\infty, 0) \cup (2, \infty) = \mathcal{D}(f)}$

is better than $x \in$, because they want the Domain.

Find the domain of the given function. (Enter your answer using interval notation.)

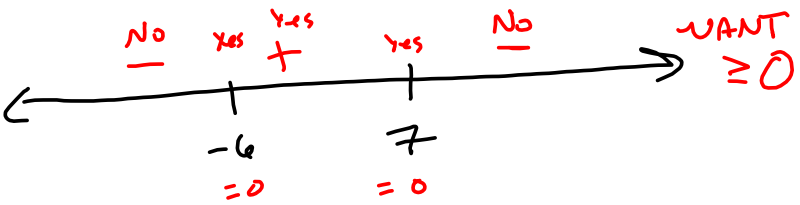
21. $f(x) = \sqrt{42 + x - x^2}$ we need $42 + x - x^2 \geq 0$, b/c $\sqrt{\text{Negative}}$ is bad.

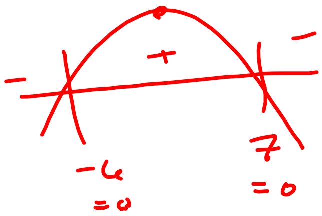
$= \sqrt{g(x)}$

Need $g(x) \geq 0$

$g(x) = 42 + x - x^2$

$= -(x^2 - x - 42) = -(x-7)(x+6)$





$\boxed{x \in [-6, 7]}$

Find the domain of the given function. (Enter your answer using interval notation.)

22. $f(x) = \dfrac{1}{\sqrt{x^4 - 37x^2 + 36}}$

Domain: $\dfrac{\text{stuff}}{0}$ Bad Need $\sqrt{x^4-37x^2+36} \neq 0 \Rightarrow$ $x^4-37x^2+36 \neq 0$

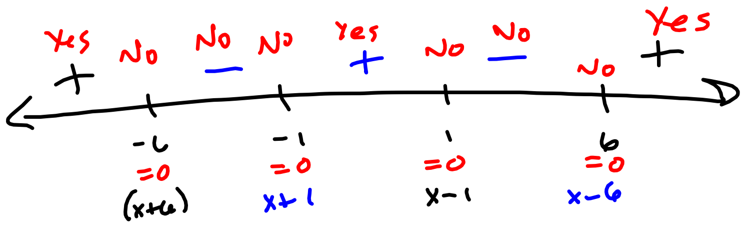
$\sqrt[2n]{\text{negative}}$ Need $x^4-37x^2+36 \geq 0$

$u = x^2 \Rightarrow$

Need $x^4-37x^2+36 > 0$

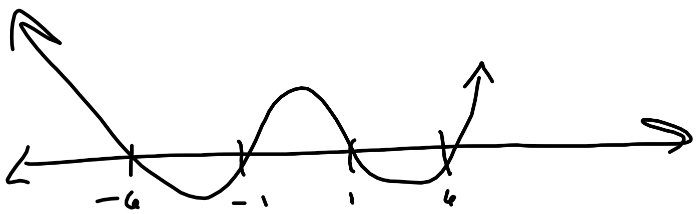
$u^2-37u+36 = (u-36)(u-1)$

$= (x^2-36)(x^2-1) = (x-6)(x+6)(x-1)(x+1)$



WANT > 0 "+"

$\boxed{x \in (-\infty,-6) \cup (1, 1) \cup (6, \infty)}$



Solve the inequality. (This exercise involves expressions that arise in calculus. Enter your answer using interval notation.)

23. $\frac{(1-x)^2}{\sqrt{x}} \geq 3\sqrt{x}(x-1)$

$\mathcal{D}$: Need $x \geq 0$ $\sqrt{x}$ in it
$\therefore \sqrt{x} \neq 0$ $\frac{\text{stuff}}{\sqrt{x}}$ $\Big\}$ $x > 0$

$(-1)^2(x-1)^2 = (1-x)^2$

$$\frac{(1-x)^2}{\sqrt{x}} - \frac{3\sqrt{x}(x-1)}{1} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{(x-1)^2 - 3x(x-1)}{\sqrt{x}} = \frac{(x-1)[(x-1)-3x]}{\sqrt{x}}$$

$$= \frac{(x-1)(-2x-1)}{\sqrt{x}} = \frac{-(x-1)(2x+1)}{\sqrt{x}} \geq 0$$

$x-1=0$ OR $2x+1=0$
$x=1$ $\quad 2x=-1$

$\boxed{x=-\frac{1}{2} \notin \mathcal{D}}$ Ignore

Domain: $(0,\infty)$

$+$ YES $-$

0 Yes 1 No

$=0$

≥ 0: "$+$" OR "$=0$"

$\boxed{x \in (0,1]}$

Solve the inequality. (This exercise involves expressions that arise in calculus. Enter your answer using interval notation.)

24. $\frac{2}{3}x^{-1/3}(x+3)^{1/2}+\frac{1}{2}x^{2/3}(x+3)^{-1/2}<0$

An old trick from Intermediate Algebra:

Factor out the negative powers:

Recall that factoring out is division inside the quantity.

$$x^{-\frac{1}{3}}(x+3)^{-\frac{1}{2}}\left[\frac{2}{3}\left(\frac{x^{-\frac{1}{3}}}{x^{-\frac{1}{3}}}\right)\left(\frac{(x+3)^{\frac{1}{2}}}{(x+3)^{-\frac{1}{2}}}\right)+\frac{1}{2}\left(\frac{x^{2/3}}{x^{-\frac{1}{3}}}\right)\left(\frac{(x+3)^{-\frac{1}{2}}}{(x+3)^{-\frac{1}{2}}}\right)\right]$$

$$=\frac{1}{x^{\frac{1}{3}}\sqrt{x+3}}\left[\frac{2}{3}(1)(x+3)+\frac{1}{2}(x)(1)\right]$$

$$=\frac{1}{x^{\frac{1}{3}}(x+3)^{\frac{1}{2}}}\left[\frac{2}{3}x+2+\frac{1}{2}x\right]=\frac{1}{x^{\frac{1}{3}}(x+3)^{\frac{1}{2}}}\underbrace{\left[\frac{7}{6}x+2\right]}<0 \quad (\text{"}-\text{"})$$

Sign Pattern will be affected by x = 0, -3, -12/7

$\frac{7}{6}x+2=0$

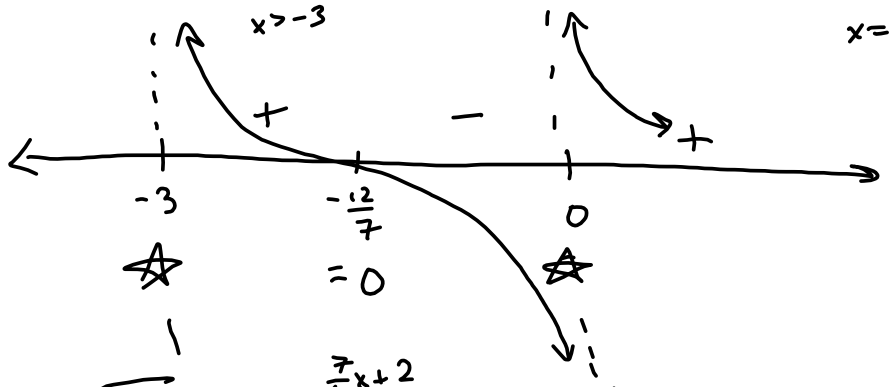
$7x+12=0$

$7x=-12$

$x=-\frac{12}{7}$

$\mathcal{D}$: Need $\underbrace{x+3\geq 0 \;\&\; x+3\neq 0}_{\substack{x+3>0\\ x>-3}} \;\&\; x\neq 0$

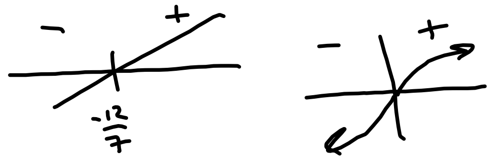
$\mathcal{D}=(-3,0)\cup(0,\infty)$

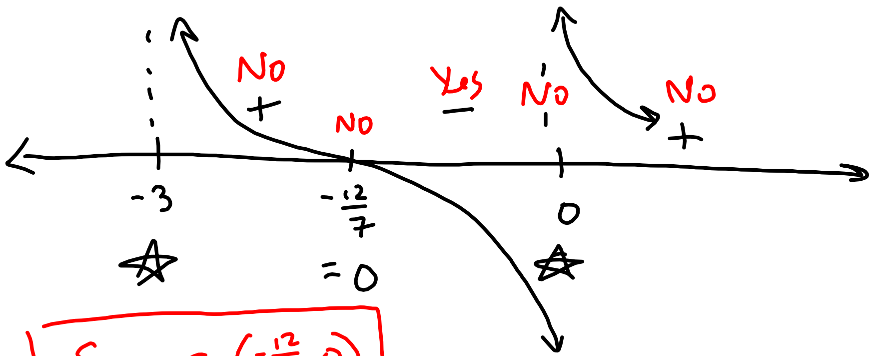


$\sqrt{x+3}$ controls

$\frac{7}{6}x+2$ controls

$x^{\frac{1}{3}}$ controls



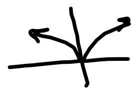


WANT < 0

(" _ ")

$$\boxed{\text{So, } x \in \left(-\tfrac{12}{7}, 0\right)}$$

$x^{\frac{2}{3}}$



Test Value method

$\left(-3, -\frac{12}{7}\right)$ $x=-2$ $\underbrace{\left(\frac{1}{(-2)^{1/3}}\right)}_{-}\underbrace{\left(\frac{1}{(-2+3)^{\frac{1}{2}}}\right)}_{+}\left[\underbrace{\frac{7}{6}(-2)+2}_{-}\right] = +$

$\left(-\frac{12}{7}, 0\right)$ $x=-1$ $\underbrace{\left(\frac{1}{(-1)^{1/3}}\right)}_{-}\underbrace{\left(\frac{1}{(-1+3)^{1/2}}\right)}_{+}\left[\underbrace{\frac{7}{6}(-1)+2}_{+}\right] = -$

$(0, \infty)$ $x=1$ $(+)(+)(+) = +$

$$\frac{1}{x^{\frac{1}{3}}(x+3)^{\frac{1}{2}}}\left[\frac{7}{6}x + 2\right]$$

Solve the inequality. (Enter your answer using interval notation.)

25. $\dfrac{1}{x+1} + \dfrac{1}{x+5} \le \dfrac{1}{x+9}$ LCD : $(x+1)(x+5)(x+9)$

NEVER CLEAR FRACTIONS IN AN INEQUALITY.

PUT EVERYTHING TOGETHER INTO ONE FRACTION ON THE LEFT-HAND-SIDE.

$$\frac{1}{x+1}\left(\frac{(x+5)(x+9)}{(x+5)(x+9)}\right) + \frac{1}{x+5}\left(\frac{(x+1)(x+9)}{(x+1)(x+9)}\right) - \frac{1}{x+9}\left(\frac{(x+1)(x+5)}{(x+1)(x+5)}\right)$$

$$= \frac{x^2+14x+45 + x^2+10x+9 - (x^2+6x+5)}{LCD}$$

$$= \frac{2x^2+24x+54-x^2-6x-5}{LCD} = \frac{x^2+18x+49}{(x+1)(x+5)(x+9)} \stackrel{set}{=} 0 \quad (\text{WANT} \le 0)$$

$\Rightarrow x^2+18x+49 = x^2+18x+9^2-81+49 = (x+9)^2 - 32 = 0$

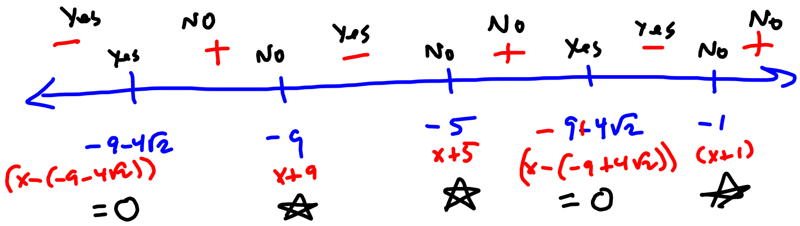
$\Rightarrow (x+9)^2 = 32$

$x+9 = \pm\sqrt{32} = \pm 4\sqrt{2}$

$x = -9 \pm 4\sqrt{2}$

$(x-(-9+4\sqrt{2}))(x-(-9-4\sqrt{2}))$ is factored form

We need a good enough estimate on this to lay out the number line, correctly. Off the top, we have this:



WANT ≤ 0 ("−" OR $=0$)

$-9+4\sqrt{2} \approx -9+4(1.4) = -9+5.6 = -4.4$

$\Rightarrow \boxed{x \in (-\infty, -9-4\sqrt{2}] \cup (-9,-5) \cup [-9+4\sqrt{2}, -1)}$