

Section 3.6 - Rational Functions

Be sure to go over #s 8-10 on Writing Project #3. I think they make a decent introduction, covering everything pretty well. The rest is repetition and maybe a few (too many un-) helpful tips from me.

Rational Function Videos from Writing Project #3:

#8 - Rational Function with horizontal and vertical asymptotes.

#8 Bonus - Find #8's intersection with its horizontal asymptote.

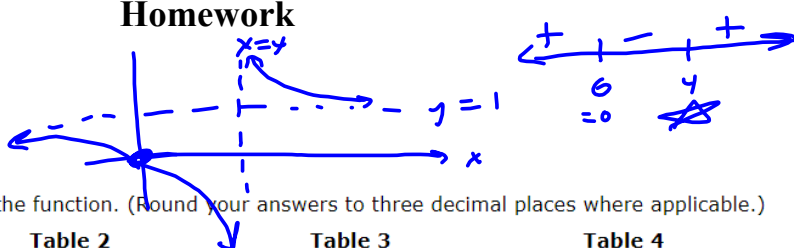
#9 - Rational Function with a hole, built off of #8.

#10 - Rational Function with a slant asymptote, also built off #8.

Homework

A rational function is given.

$$r(x) = \frac{x}{x-4}$$



1

(a) Complete each table for the function. (Round your answers to three decimal places where applicable.)

Table 1

| x | r(x) |
|-------|----------------------|
| 3.5 | <input type="text"/> |
| 3.9 | <input type="text"/> |
| 3.99 | <input type="text"/> |
| 3.999 | <input type="text"/> |

Table 2

| x | r(x) |
|-------|----------------------|
| 4.5 | <input type="text"/> |
| 4.1 | <input type="text"/> |
| 4.01 | <input type="text"/> |
| 4.001 | <input type="text"/> |

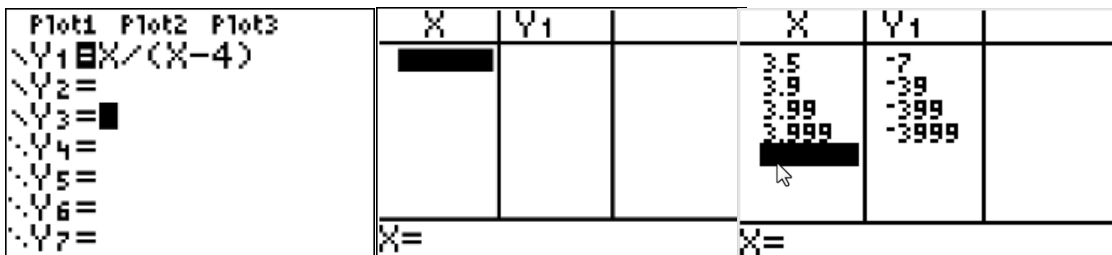
Table 3

| x | r(x) |
|------|----------------------|
| 10 | <input type="text"/> |
| 50 | <input type="text"/> |
| 100 | <input type="text"/> |
| 1000 | <input type="text"/> |

Table 4

| x | r(x) |
|-------|----------------------|
| -10 | <input type="text"/> |
| -50 | <input type="text"/> |
| -100 | <input type="text"/> |
| -1000 | <input type="text"/> |

(b) Describe the behavior of the function near its vertical asymptote, based on Tables 1 and 2.



Using a Computer Algebra System (CAS):

$$f := x \mapsto \frac{x}{x-4}$$

f(3.5)

$$f := x \mapsto \frac{x}{x-4}$$

-7.00000000

f(3.9)

-39.00000000

f(3.99)

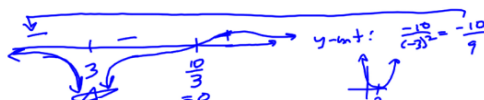
-399.00000000

f(3.999)

-3999.00000000

2

$$r(x) = \frac{x}{x-4}$$



(a) Complete each table for the function. (Round your answers to three decimal places where applicable.)

(b) Describe the behavior of the function near its vertical asymptote, based on Tables 1 and 2.

(c) Determine the horizontal asymptote, based on Tables 3 and 4.

$$\begin{aligned} \text{as } x \rightarrow 3^- \quad r(x) &\rightarrow -\infty \\ \text{as } x \rightarrow 3^+ \quad r(x) &\rightarrow -\infty \end{aligned}$$

$$\frac{3x-10}{(x-3)^2} \quad \text{near } n=3$$

$$\approx \frac{-1}{(\pm \text{small})^2} = \frac{-1}{\text{small}^2} = -\text{BIG}$$

The following question is about the rational function

3

$$r(x) = \frac{(x+1)(x-2)}{(x+2)(x-8)} = \frac{x^2 + \text{smaller}}{x^2 + \text{smaller}} \quad x \rightarrow \pm\infty \rightarrow \frac{x^2}{x^2} = \boxed{1=y}$$

The function r has horizontal asymptote $y =$

Book Way:

$$\frac{x(1+\frac{1}{x})(x(1-\frac{2}{x}))}{x(1+\frac{2}{x})(x(1-\frac{8}{x}))}$$

$$= \frac{(1+\frac{1}{x})(1-\frac{2}{x})}{(1+\frac{2}{x})(1-\frac{8}{x})} \quad x \rightarrow \pm\infty \rightarrow \frac{(1)(1)}{(1)(1)} = \boxed{1=y}$$

$\frac{2}{x}, \frac{1}{x}, \frac{2}{x}, -\frac{8}{x} \quad x \rightarrow \pm\infty \rightarrow 0$

$\frac{x^2}{x^2}$ is a tie in degree \rightarrow
just look at the leading terms

9

$$\text{Let } r(x) = \frac{x^2 + x}{(x+1)(4x-8)} = \frac{x(x+1)}{4(x+1)(x-2)} = \frac{x}{4(x-2)}$$

This has a HOLE
Ⓢ $x = -1$!

- (a) The graph of r has a vertical asymptote $x = 2$. True if $x \neq -1$
- (b) The graph of r has a vertical asymptote $x = -1$. FALSE. It's a hole.
- (c) The graph of r has a horizontal asymptote $y = 1$. FALSE
- (d) The graph of r has a horizontal asymptote $y = \frac{1}{4}$. TRUE.

(c) H.A. Tie of degree 2
check Biggest stuff

$$r(x) = \frac{x^2 + m}{4x^2 + n} \quad x \rightarrow \pm \text{BIG} \rightarrow \frac{x^2}{4x^2}$$

$$= \boxed{\frac{1}{4} = y = \text{H.A.}}$$

Find the x- and y-intercepts of the rational function. (If an answer does not exist, enter DNE.)

14

$$r(x) = \frac{x-5}{x+8} \quad \text{Set } \neq 0 \Rightarrow x=5 \rightsquigarrow (5,0) \text{ x-int}$$

$$y\text{-int} = \frac{-5}{8} \rightsquigarrow (0, -\frac{5}{8})$$

Find all horizontal and vertical asymptotes (if any). (If an answer does not exist, enter DNE. Enter your answers as a comma-separated list of equations.)

17

$$r(x) = \frac{6}{x-3} \quad \text{Proper} \quad \frac{d=0}{d=1} \text{ } \neq 1 > 0 \Rightarrow \boxed{y=0 \text{ is H.A.}}$$

$$\boxed{x=3 \text{ is V.A.}}$$

Find all vertical and horizontal asymptotes (if any). (If an answer does not exist, enter DNE. Enter your answers as a comma-separated list of equations.)

18

$$r(x) = \frac{5x-6}{x^2-16} = \frac{5(x-\frac{6}{5})}{(x-4)(x+4)} \quad \boxed{\text{V.A. } \therefore x=4, x=-4}$$

$$\boxed{\text{H.A. } \therefore 2 > 1 \rightarrow y=0 \text{ is H.A.}}$$

Find all vertical and horizontal asymptotes (if any). (If an answer does not exist, enter DNE. Enter your answers as a comma-separated list of equations.)

19

$$r(x) = \frac{(x+1)(4x-5)}{(x-4)(7x+8)} \Rightarrow \mathcal{D} = \mathbb{R} \setminus \left\{ 4, -\frac{8}{7} \right\}$$

Noting Cancels. No Holes.

$$\boxed{x=4, x=-\frac{8}{7} \text{ V.A.}}$$

= deg 2 / deg 2 \rightarrow H.A.

$$\frac{(x+1)(4x-5)}{(x-4)(7x+8)} \xrightarrow{x \rightarrow \pm\infty} \frac{4x^2}{7x^2} = \boxed{\frac{4}{7} = y \text{ is H.A.}}$$

Book way:

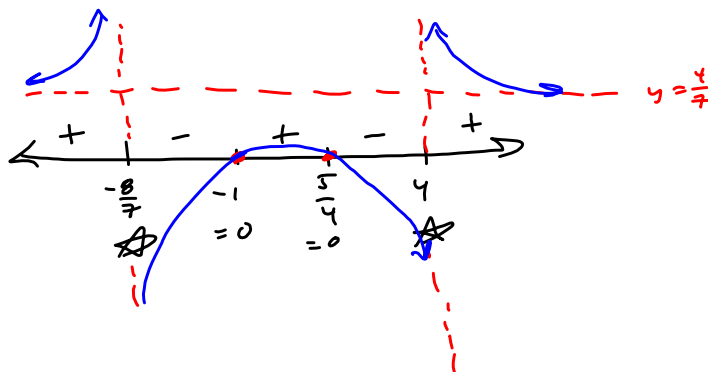
$$\frac{x(1+\frac{1}{x})(x(4-\frac{5}{x}))}{x(1-\frac{4}{x})(x(7+\frac{8}{x}))} = \frac{(1+\frac{1}{x})(4-\frac{5}{x})}{(1-\frac{4}{x})(7+\frac{8}{x})} \xrightarrow{x \rightarrow \pm\infty}$$

$$\frac{(1)(4)}{(1)(7)} = \boxed{\frac{4}{7} = y = \text{H.A.}}$$

x-int: $(-1,0), (5,0)$

y-int: $\frac{(1)(-5)}{(4)(8)} = \frac{-5}{32} = \frac{5}{32} \rightarrow \boxed{(0, \frac{5}{32}) = y\text{-int}}$

$x=4, -\frac{8}{7}$ "V.A."
 $x=-1, \frac{5}{4}$ "=0"



Find all vertical and horizontal asymptotes (if any). (If an answer does not exist, enter DNE. Enter your answers as a comma-separated list of equations.)

21

$$r(x) = \frac{8x^3 - 2}{2x^3 + 5x^2 + 9x}$$

$$y = \frac{8}{2} = 4 = y \text{ is H.A.}$$

$$2x^3 + 5x^2 + 9x = x(2x^2 + 5x + 9)$$

$$b^2 - 4ac = 5^2 - 4(2)(9) < 0$$

so irreducible over \mathbb{R}

$$x=0 \text{ is V.A.}$$

Find all vertical and horizontal asymptotes (if any). (If an answer does not exist, enter DNE. Enter your answers as a comma-separated list of equations.)

22

$$r(x) = \frac{8x^3}{x^3 + 2x^2 + 2x}$$

$$y = 8 \text{ is H.A.}$$

$$x^3 + 2x^2 + 2x = x(x^2 + 2x + 2)$$

$$b^2 - 4ac < 0$$

No real
zeros.

Irreducible Quadratic.

$$r(x) = \frac{8x^3}{x(x^2 + 2x + 2)}$$

$$= \frac{8x^2}{x^2 + 2x + 2} \quad (x \neq 0)$$

$$\rightarrow x=0 \text{ is a HOLE!}$$

Find the intercepts and asymptotes. (If an answer does not exist, enter DNE. Enter your asymptotes as a comma-separated list of equations if necessary.)

24

$$r(x) = \frac{-5x^2 - 50x - 126}{x^2 + 10x + 25}$$

$$y = -5 \text{ is H.A.}$$

$$x^2 + 10x + 25 = (x+5)^2 \stackrel{\text{SET}}{=} 0 \Rightarrow x = -5, m = 2$$

$$D = \mathbb{R} \setminus \{-5\}$$

$$x = -5 \text{ V.A.}$$

$$-5x^2 - 50x - 126 = 0 \Rightarrow$$

$$5x^2 + 50x + 126 = 0 \Rightarrow$$

$$b^2 - 4ac = 50^2 - 4(5)(126)$$

$$= 2500 - 2520 < 0 \Rightarrow$$

No x-ints

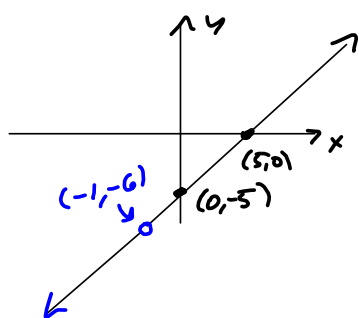
$$y\text{-int } \left(0, -\frac{126}{25}\right)$$

Find the factors that are common in the numerator and the denominator. Then find the intercepts and asymptotes. (If an answer does not exist, enter DNE. Enter your asymptotes as a comma-separated list of equations if necessary.)

$$27 \quad r(x) = \frac{x^2 - 4x - 5}{x + 1} = \frac{(x-5)(x+1)}{x+1} = x-5 \quad (x \neq -1)$$

Hold \textcircled{a} $x = -1$

Find its y -value: $-1 - 5 = -6 \rightarrow (-1, -6)$



$$\begin{array}{r} x \div 7 \\ 0 \overline{) -5} \\ \underline{5} \\ 0 \end{array}$$

$$\begin{array}{l} x-5=0 \\ x=5 \end{array}$$

OBLIQUE

Find the slant asymptote and the vertical asymptote. (If an answer does not exist, enter DNE.)

30

$$r(x) = \frac{x^2}{x-6}$$

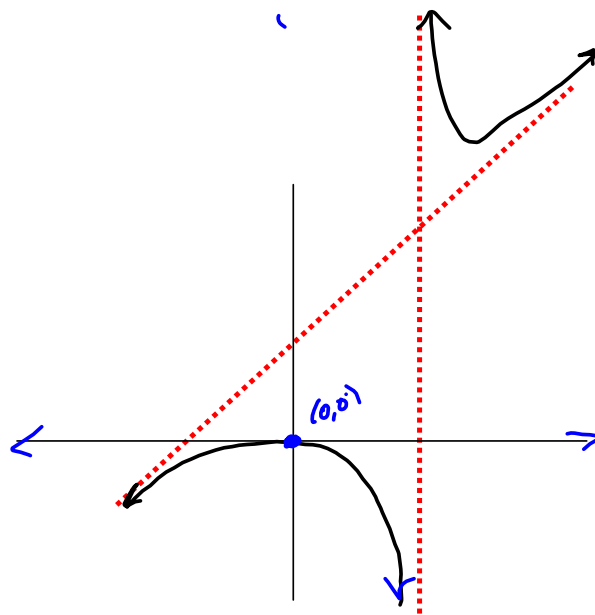
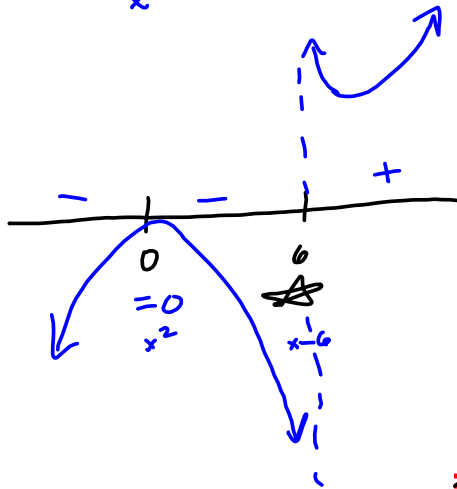
x-int. $x=0$
 $(0,0)$ y-int

$D = \mathbb{R} \setminus \{6\}$
 $x=6$ is v.a.

$\frac{\text{deg}}{\text{deg}} = \frac{2}{1} > 1$ No. H.A.
 Look for SLANT ASYMPTOTE
 By DIVIDING

$$\begin{array}{r} 6 \overline{) 1 \ 0 \ 0} \\ \underline{6 \ 36} \\ 1 \ 6 \ 36 = r \end{array} \rightarrow r(x) = x+6 + \frac{36}{x-6}$$

$y=x+6$ is SA.
 E.B.



Find the slant asymptote and the vertical asymptotes. (If an answer does not exist, enter DNE.)

32

$$r(x) = \frac{x^3 + 9}{2x^2 + x - 1}$$

slant asymptote

$\frac{\text{deg}=3}{\text{deg}=2} \cdot 3 > 2$.
SLANT (oblique)

vertical asymptote (smaller value)

vertical asymptote (larger value)

Sketch a graph of the function.

$$2x^2 + x - 1 = (2x - 1)(x + 1) \stackrel{\text{SET } 0}{\rightarrow}$$

$$x = \frac{1}{2}, -1 = x \text{ V.A.}$$

$$x^3 + 9 = 0$$

$$x^3 = -9$$

$$x = \sqrt[3]{-9}$$

Only has one.

$$y\text{-int: } (0, -9) \text{ y-int}$$

Now Divide:

$$\begin{array}{r} \frac{1}{2}x - \frac{1}{4} \\ 2x^2 + x - 1 \overline{) x^3 + 0x^2 + 0x + 9} \\ \underline{-(x^3 + \frac{1}{2}x^2 - \frac{1}{2}x)} \\ -\frac{1}{2}x^2 \end{array}$$

$$\frac{-\frac{1}{2}x^2}{2x^2} = -\frac{1}{4}$$

$$\frac{1}{2}x - 4 = y = \text{Slant Asymptote}$$

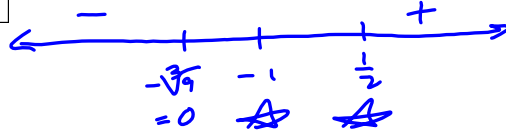
$$x = \frac{1}{2}, -1 \text{ "A"}$$

$$x = \sqrt[3]{-9} = -\sqrt[3]{9} \approx -2.08$$

$$y = \frac{1}{2}x - 4 \dots \uparrow$$

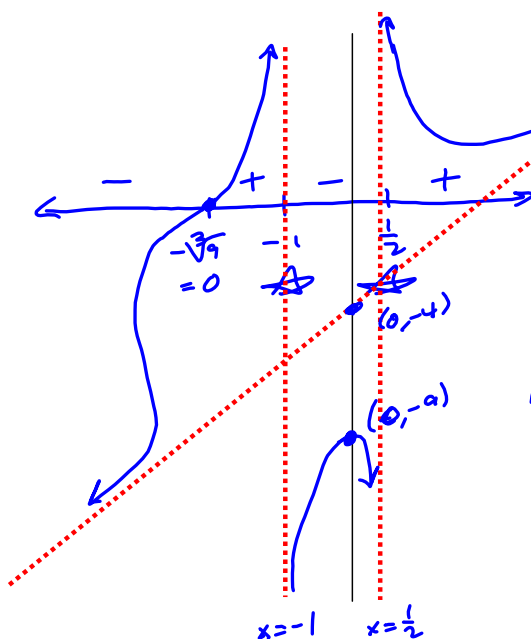
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-9^(1/3)
-2.080083823
    
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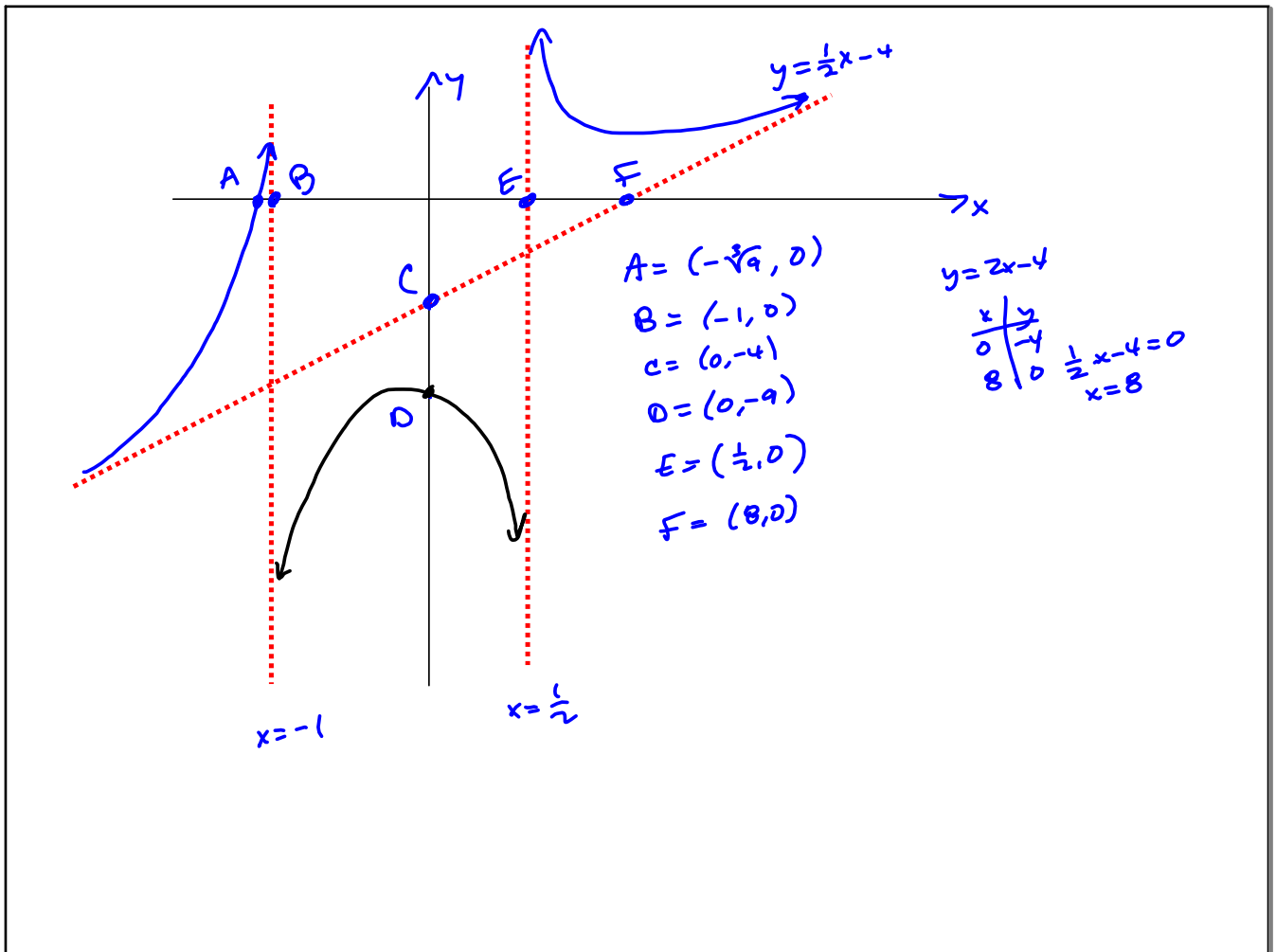


$$r(x) = \frac{(x + \sqrt[3]{9})(x^2 - \sqrt[3]{9}x + 3\sqrt[3]{3})}{(2x - 1)(x + 1)}$$

$$\begin{aligned} (\sqrt[3]{9})^2 &= \\ ((3^2)^{\frac{1}{3}})^2 &= 3^{\frac{4}{3}} \\ &= \sqrt[3]{3^3 \cdot 3} = 3\sqrt[3]{3} \\ 3^{\frac{4}{3}} &= (3^{\frac{1}{3}})^4 = (3^{\frac{1}{3}})^3 \cdot 3^{\frac{1}{3}} \end{aligned}$$



$\frac{1}{2}(0) - 4 = -4$
Needs 1 more draft



33 Let $r(x) = \frac{5x^3 + 5x^2}{x^2 - 4} = \frac{5x^2(x+1)}{(x-2)(x+2)}$

Sketch a graph of the function.

$\frac{\text{deg}=3}{\text{deg}=2}$. $3 > 2$ Slant

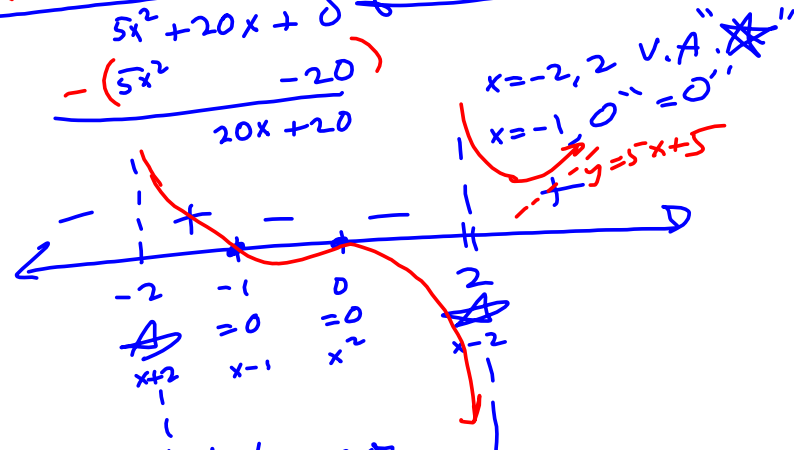
$x = -2, 2 = x$ V.A.

x-int: $(0,0)$ $m=2$ Kiss
 $(-1,0)$ $m=1$

$y = 5x + 5$ is S.A.

$$\begin{array}{r} x^2=4 \quad \sqrt{5x^3 + 5x^2 + 0x + 0} \\ \underline{-(5x^3 \quad -20x)} \\ 5x^2 + 20x + 0 \\ \underline{-(5x^2 \quad -20)} \\ 20x + 20 \end{array}$$

$\frac{5x^3}{x^2} = 5x$
 $\frac{5x^2}{x^2} = 5$



Needs one or two revisions to nail it.

$y = 5x + 5$