Section 3.3 - Dividing Polynomials

Recall: Long Division of Integers.

$\frac{37}{5}$

$$\begin{array}{r} 7\text{ r }2 \\ 5\overline{)\,37} \\ -35 \\ \hline 2 \end{array}$$

Divisor = 5
Dividend = 37
Quotient = 7
Remainder = 2

This means $\frac{37}{5} = 7 + \frac{2}{5}$

$$= \frac{7}{1}\cdot\frac{5}{5} + \frac{2}{5} = \frac{35+2}{5} = \frac{37}{5}$$

$\Rightarrow \left(\frac{37}{5} = 7 + \frac{2}{5}\right)\cdot 5 \longrightarrow$

$37 = 5\cdot 7 + 5\cdot\frac{2}{5} = 5\cdot 7 + 2$

Dividend Divisor Quotient Remainder

:

Divide x^2-5x+2

by $x+1$

$$\begin{array}{r l}
 & x - 6 \quad r\,8 \\
x+1 & \overline{)\, x^2 - 5x + 2} \\
 & -(x^2 + x \qquad) \\
 & \overline{\quad 0 \;\; -6x + 2} \\
 & \quad -(-6x - 6) \\
 & \overline{\qquad\qquad\quad 8}
\end{array}$$

$\frac{x^2}{x} = x$

$\frac{-6x}{x} = -6$

x^2-5x+2
$-x^2-x$

This says $f(x) = x^2-5x+2 = (x+1)(x-6) + 8$

$= x^2-5x-6+8$

$= x^2-5x+2 = f(x)$ ✓

Long as we're here:

what's $f(-1)$?

$f(-1) = 8$ = Remainder upon division by $x+1$!

This'll come in *really handy* when trying to evaluate a polynomial of degree *n* = 5!

$f(x) = (x+1)(x-6)+8 \Rightarrow$

$f(-1) = (-1+1)(-1-6)+8$

$= 0+8 = 8!$

$f(x) = x^2-5x+2 \Rightarrow$

$f(-1) = (-1)^2-5(-1)+2$

$= 1+5+2 = 8!$

Division by $x+1 \longrightarrow f(-1)$!

The other way of viewing this division is what the book calls the "Division Algorithm."

Dividend $= f(x) = d(x)q(x) + r(x)$ = divisor · quotient + remainder.

I don't know why it's called an algorithm. It's an interpretation of the result of the division algorithm.

The other interpretation of our work applies to Rational Functions:

$$\frac{x^2+5x-2}{x+1} = x-6 + \frac{8}{x+1}$$

This says the farther you get from $x=-1$ (where it blows up & we see stars), the more $\frac{x^2+5x-2}{x+1}$ is going to look like $x-6$,

because as $x \to \pm\infty$, $\frac{8}{x+1} \longrightarrow 0$

& $x-6+\frac{8}{x+1} \approx x-6$!

$$x-6+\frac{8}{x+1} \longrightarrow x-6 \text{ as } x \to \pm\infty$$

$$x-6+\frac{8}{x+1} \xrightarrow{x\to\pm\infty} x-6$$

Messier Ones

Divide $3x^4 + 2x^2 - 5x + 7$ by $x^2 - x + 2$ to find Quotient and remainder.

$11 \neq 101$

w/o placeholder "0" in the 10s place, 101 is just 11.

$$\begin{array}{r|l} & 3x^2 + 3x - 1 \quad r\ -12x + 9 \\ x^2 - x + 2 & 3x^4 + 0x^3 + 2x^2 - 5x + 7 \\ & -(3x^4 - 3x^3 + 6x^2) \\ & 3x^3 - 4x^2 - 5x + 7 \\ & -(3x^3 - 3x^2 + 6x) \\ & -x^2 - 11x + 7 \\ & -(-x^2 + x - 2) \\ & -12x + 9 = r \end{array}$$

$\frac{3x^4}{x^2} = 3x^2$

$\frac{3x^3}{x^2} = 3x$

$\frac{-x^2}{x^2} = 1$

This says:

$$3x^4 + 2x^2 - 5x + 7 = (x^2 - x + 2)(3x^2 + 3x - 1) - 12x + 9$$

OR

$$\frac{3x^4 + 2x^2 - 5x + 7}{x^2 - x + 2} = 3x^2 + 3x - 1 + \frac{-12x + 9}{x^2 - x + 2}$$

Homework

1

If we divide the polynomial P by the factor $x - c$ and we obtain the equation $P(x) = (x - c)Q(x) + R(x)$, then we say that $x - c$ is the divisor, $Q(x)$ is the [---Select---] , and $R(x)$ is the [---Select---] .

quotient remainder.

Division Algorithm

2

(a) If we divide the polynomial $P(x)$ by the factor $x - c$ and we obtain a remainder of 0, then we know that c is [---Select---] of P.

zero

(b) If we divide the polynomial $P(x)$ by the factor $x - c$ and we obtain a remainder of k, then we know that $P(c) =$ [--?--] .

k Remainder Theorem!

3

Two polynomials P and D are given. Use either synthetic division or long division to divide $P(x)$ by $D(x)$, and express the quotient $P(x)/D(x)$ in the form $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$.

$P(x) = 6x^4 - 3x^3 + 29x^2, \quad D(x) = 3x^2 + 13$

$$
\begin{array}{r}
2x^2 - x + 1 \quad r \quad 13x - 1 \\
3x^2+13 \,\big)\, 6x^4 - 3x^3 + 29x^2 + 0x + 0 \\
-(6x^4 \qquad\quad + 26x^2) \\
\hline
-3x^3 + 3x^2 + 0x + 0 \\
-(-3x^3 \qquad - 13x) \\
\hline
3x^2 + 13x + 0 \\
-(3x^2 \qquad + 1) \\
\hline
\boxed{13x - 1 = r}
\end{array}
$$

$\frac{6x^4}{3x^2} = 2x^2$

$\frac{-3x^3}{3x^2} = -x^1$

$\frac{3x^2}{3x^2} = 1$

$+1 \to 13$

$-1 \to -13$

This says $\frac{6x^4 - 3x^3 + 29x^2}{3x^2+13} = 2x^2 - x + 1 + \frac{13x - 1}{3x^2+13}$

$-1 \to -13$

This looks like $2x^2 - x + 1$ as $x \to \pm\infty$

Two polynomials P and D are given. Use either synthetic division or long division to divide $P(x)$ by $D(x)$, and express P in the form $P(x) = D(x) \cdot Q(x) + R(x)$.

5

$P(x) = 4x^3 - 3x^2 - 4x, \quad D(x) = 4x - 3 = 4(x - \frac{3}{4})$

Long:

$$
\begin{array}{r}
x^2 - 1 \quad r\ -3 \\
4x-3 \overline{\big) 4x^3 - 3x^2 - 4x + 0} \\
-(4x^3 - 3x^2 \qquad\quad) \\
\hline
-4x + 0 \\
-(-4x + 3) \\
\hline
-3
\end{array}
$$

$\frac{4x^3}{4x} = x^2$

$\frac{-4x}{4x} = -1$

This says $P(x) = \underline{(4x-3)(x^2-1) - 3}$

Synthetically

$$
\begin{array}{r|rrrr}
\frac{3}{4} & 4 & -3 & -4 & 0 \\
 & & 3 & 0 & -3 \\
\hline
 & 4 & 0 & -4 & \boxed{-3 = r} \\
 & x^2 & x^1 & c & r
\end{array}
$$

$4(\frac{3}{4}) = 3$

This gives

$4x^3 - 3x^2 - 4x$

$= (x - 3/4)(4x^2 - 4) - 3$

$= (x - \frac{3}{4})(4(x^2 - 1)) - 3$

$= (4x - 3)(x^2 - 1) - 3$

Find the quotient and remainder using long division.

6

$\frac{x^2 - 4x + 10}{x - 3}$

$$
\begin{array}{r|rrr}
3 & 1 & -4 & 10 \\
 & & 3 & -3 \\
\hline
 & 1 & -1 & \boxed{7 = r} \\
 & x^1 & c & r
\end{array}
$$

This says $x^2 - 4x + 10 = \underset{D}{(x-3)}\underset{Q}{(x-1)} + \underset{R}{7}$

Find the quotient and remainder using long division. → only if you need practice

7 $\dfrac{x^3 + 6x^2 - x + 1}{x + 7}$

$$\begin{array}{r|rrrr} -7 & 1 & 6 & -1 & 1 \\ & & -7 & 7 & -42 \\ \hline & 1 & -1 & 6 & -41 = r \end{array}$$

$Q(x) = x^2 - x - 6$

$R(x) = -41$

Find the quotient and remainder using synthetic division.

11 $\dfrac{2x^3 + 7x^2 - 4x + 3}{x - \frac{1}{2}}$

$2(\frac{1}{2}) = 1$

$8(\frac{1}{2}) = 4$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 7 & -4 & 3 \\ & & 1 & 4 & 0 \\ \hline & 2 & 8 & 0 & \boxed{3 = r} \end{array}$$

$Q = 2x^2 + 8x$

$R = 3$

Find the quotient and remainder using synthetic division.

12 $\dfrac{3x^4 + 8x^3 + 5x^2 + x + 7}{x + \frac{2}{3}}$

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 3 & 8 & 5 & 1 & 7 \\ & & -2 & -4 & -\frac{2}{3} & -\frac{2}{9} \\ \hline & 3 & 6 & 1 & \frac{1}{3} & \frac{61}{9} \\ & x^3 & x^2 & x & c & \to R \end{array}$$

$3(-\frac{2}{3}) = -2$

$\cancel{6}\,2(-\frac{2}{3}) = -4$

$\frac{1}{3}(-\frac{2}{3}) = -\frac{2}{9}$

$7 - \frac{2}{9}$

$= \frac{7}{1} \cdot \frac{9}{9} - \frac{2}{9} = \frac{63 - 2}{9} = \frac{61}{9}$

→ $Q = 3x^3 + 6x^2 + x + \frac{1}{3}$

$R = \frac{61}{9}$

Find the quotient and remainder using synthetic division.

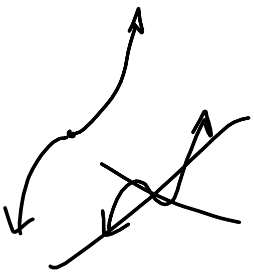
13 $\dfrac{x^3-125}{x-5} = \dfrac{x^3-5^3}{x-5} = (x-5)(x^2+5x+25)$

$R=0,\ Q = x^2+5x+25$

These are always irreducible.

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$



Find the quotient and remainder using synthetic division.

14 $\dfrac{x^4-16}{x+2} = \dfrac{(x^2-4)(x^2+4)}{x+2} = \dfrac{(x-2)\cancel{(x+2)}(x^2+4)}{\cancel{x+2}}$

$= (x-2)(x^2+4) = x^3+4x-2x^2-8$

$= \boxed{x^3-2x^2+4x-8 = Q}$

$\boxed{R=0}$

$x^4-16 = x^4+0x^3+0x^2+0x-16$

-2	1	0	0	0	-16
		-2	4	-8	16
	1	-2	4	-8	0
	x^3	x^2	x^1	x^0	r

$\boxed{Q = x^3+x^2+x}$ $\boxed{R=0}$

Use synthetic division and the Remainder Theorem to evaluate $P(c)$.

15

$$P(x) = 8x^2 + 4x + 5, \quad c = \frac{1}{2}$$

REMAINDER THEOREM

If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

$\frac{1}{2}\,|\;\; 8 \quad 4 \quad 5$

$\quad\quad\; 4 \quad 4$

$\quad 8 \quad 8 \quad \boxed{9 = f(\tfrac{1}{2})}$

$8(\frac{1}{2})^2 + 4(\frac{1}{2}) + 5$

$2 + 2 + 5 = 9$ ✓

$P(x) = 8x^2 + 4x + 5 = (x - \frac{1}{2})(8x^2 + 4x + 5) + 9$

$\Rightarrow P(\frac{1}{2}) = 0(\quad) + 9 = 9$

Use synthetic division and the Remainder Theorem to evaluate $P(c)$.

19

$$P(x) = -x^7 - 3x^2 + 5, \quad c = 1$$

$-x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 - 3x^2 + 0x + 5$

$1 \,|\; -7 \quad 0 \quad 0 \quad 0 \quad 0 \quad -3 \quad 0 \quad 5$

$\quad\quad\quad -7 \quad -7 \quad -7 \quad -7 \quad -7 \quad -10 \quad -10$

$-7 \quad -7 \quad -7 \quad -7 \quad -7 \quad -10 \quad -10 \quad \boxed{-5 = P(1)}$