3.2 - Polynomial Functions and their Graphs

End Behavior

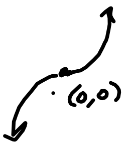
$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$,

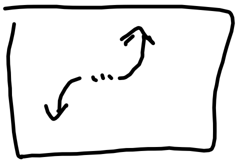
where $a_0, a_1, \dots, a_{n-1}, a_n$ are constant (real)

$\Longrightarrow$ P(x)'s end behavior is that of $a_n x^n$

$P(x) = 7x^5 - 4x^3 + 2x^2 + 7 \longrightarrow$

E.B. is that of $7x^5$

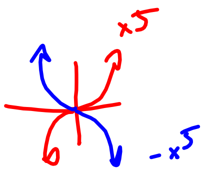


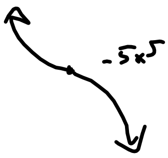


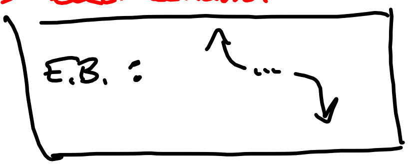
The diagram.

$P(x) = 3x^4 - 5x^5 + 7x^2 - 5x + 1$

$-5x^5$ controls end behavior

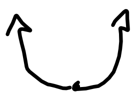


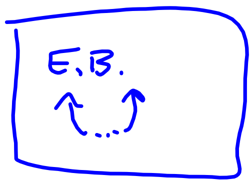




$P(x) = 7x^4 - 5x^3 + 2x^2 + x + 5$

$7x^4$





what I want for #1 is the E.B. diagram on WP#3

Your Book & webassign want you to say it this way:

means as $x \to -\infty$, $P(x) \to \infty$

as $x \to +\infty$, $P(x) \to \infty$

mills Handwritten

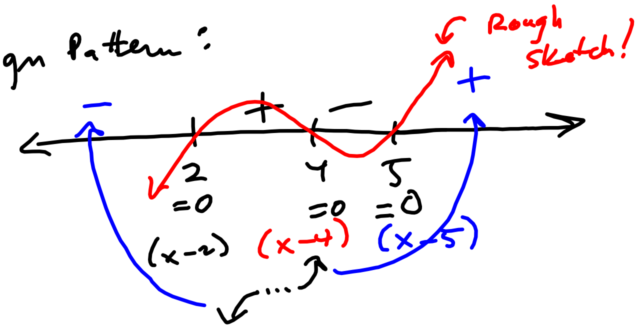
For computer programmers @ Cengage

Zeros of a polynomial function correspond to the *x*-intercepts of its graph.

$P(x) =$ factored form $= (x-2)(x-4)(x-5) = x^3 + \ldots$

Then $x = 2, 4, 5$ are its zeros

Sign Pattern:



I think the book and WebAssign do a good job of test values on the subintervals. That's why I show a more intuitive way that can speed things up, once you see it.

$(+)(+)(+) = +$

to

$(+)(+)(-) = -$ @ $x=5$, moving right to left.

Then it $=0$ @ $x=4$ & $x-4$ is running the show

To the left of 4, $x-4$ & $x-5$ are both negative, so

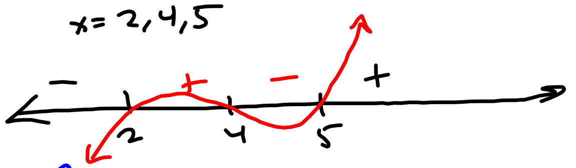
$P(x) = (x-2)(x-4)(x-5)$

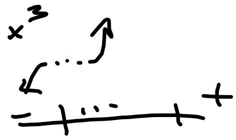
$= (+)(-)(-) = +$ just to the left of $x=4$

Re-Do the sign pattern the Book Way.

$P(x) = (x-2)(x-4)(x-5) \overset{\text{set}}{=} 0 \Rightarrow$

$x = 2, 4, 5$



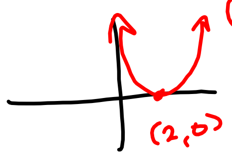


Interval	Test		
$(-\infty, 2)$	1	$(1-2)(1-4)(1-5) = (-1)(-3)(-4) = -12$	−
$(2, 4)$	3	$(3-2)(3-4)(3-5) = (1)(-1)(-2) = 2$	+
$(4, 5)$	4.5	$(4.5-2)(4.5-4)(4.5-5) = (2.5)(.5)(-.5) = -$	
$(5, \infty)$	6	$(6-2)(6-4)(6-5) = (4)(2)(1) = 8$	+

$P(x) = (x-2)^2(x-4)(x-5)$

$(x-2)^2$ means $x=2$ is a zero or root of multiplicity 2.

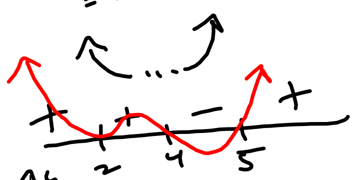
Notice that $(x-2)^2$ just kisses the x-axis at $x=2$

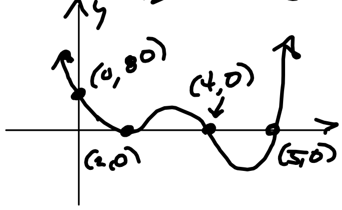


No sign change @ $x=2$, so

$(x-2)^2(x-4)(x-5) = x^2 \cdot x \cdot x + \dots$

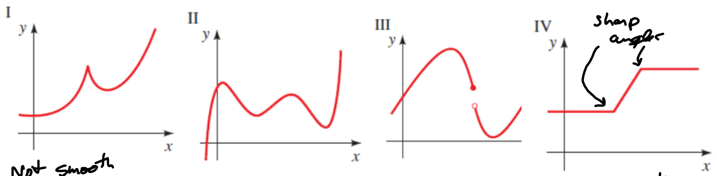
$= x^4 + \dots$





1 Only one of the following four graphs could be the graph of a polynomial function. Which one? Why are the others not graphs of polynomials? (Select all that apply.)

Polynomials are continuous and smooth.



I: Not smooth. You could get a puncture wound.

II: Yes

III: Not continuous

IV: Not smooth

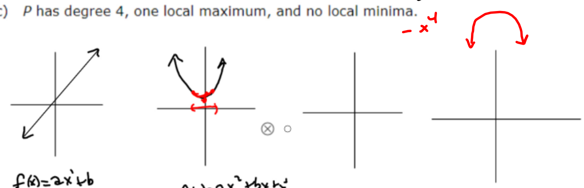
#2 Click Here to See Video

3 Which of the following statements couldn't possibly be true about the polynomial function P?

(a) P has degree 3, two local maxima, and two local minima. No. too many extrema. At most $n-1$ $=3-1=2$

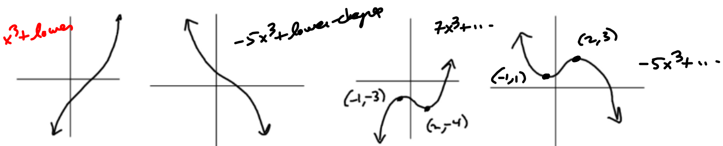
(b) P has degree 3 and no local maxima or minima. x^3

(c) P has degree 4, one local maximum, and no local minima. $-x^4$



$f(x)=2x^1+6$
$n=1$
0 max/min

$f(x)=2x^2+5x+2$
$n=2$
1 local min. - Littlest on the block
1 absolute min - Littlest in the universe.



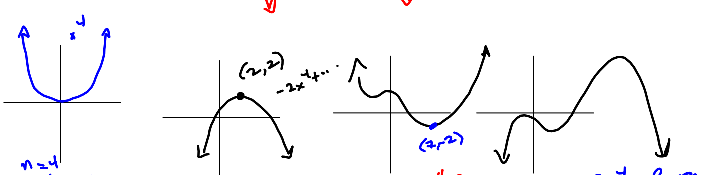
$n=3$
0 maxima
0 minima
extrema → Extremes

$n=3$

$n=3$
local max. (-1,-3)
local min (2,-4)
2 extrema

$n=3$
local max @ (2,3)
Local min @ (-1,1)
2 extrema

$-5x^3+\dots$ End behavior $5x^3+\dots$



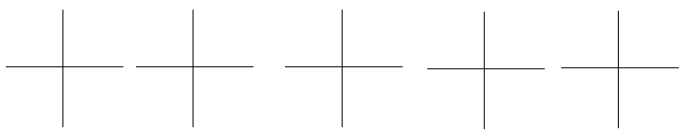
$n=4$
1 extreme
Local & absolute min @ (0,0)

1 extreme
1 local & absolute max.

$11x^4$ + smaller degree
$11x^4+17x^3-5x^2+11x+2$
3 extrema
2 local minima,
1 local maximum
Absolute min @ (3,-2)

$-13x^4$ + lower degree
3 extrema
2 local max
1 local min
1 absolute min.

$n=1$ 0 extrema
$n=2$ 1 extrema
$n=3$ 0 or 2 extrema
$n=4$ 1 or 3 ..
$n=5$ 0 or 2 or 4
$n=6$ 1 or 3 or 5

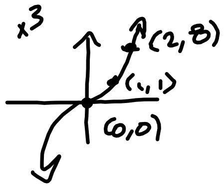


Sketch the graph of each function by transforming the graph of an appropriate function of the form $y = x^n$ from the graphs below. (Select Update Graph to see your response plotted on the screen. Select the Submit button to grade your response.)

4 (a) $P(x) = x^3 - 8$

Start with the graph of a standard function $y = f(x)$:

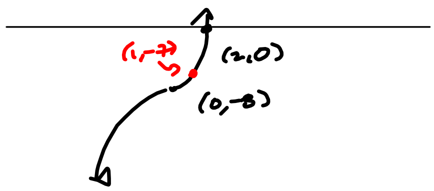
x^3



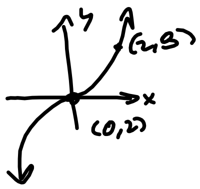
The point A at (0, 0) moves to A' at (x, y) = $(0,-8)$

The point B at (2, 8) moves to B' at (x, y) = $(2,0)$

$x^3 - 8$: $(x,y) \longmapsto (x, y-8)$



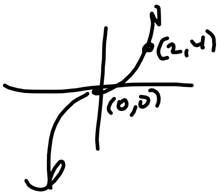
(d) $S(x) = \frac{1}{2}(x - 1)^3 + 2$



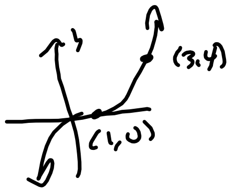
The point A at (0, 0) moves to A' at (x, y) = $(1,2)$

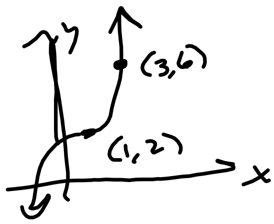
The point B at (2, 8) moves to B' at (x, y) = $(3,6)$

$\frac{1}{2}x^3$



$\frac{1}{2}(x-1)^3$





A polynomial function is given.

5 $P(x) = x(x^2 - 9)$

SET $= 0 \Rightarrow x = 0$ or $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

(a) Describe the end behavior of the polynomial function.

(b) Match the polynomial function with one of the following graphs.

$x(x^2-9) = x^3 - 9x$

Ignore

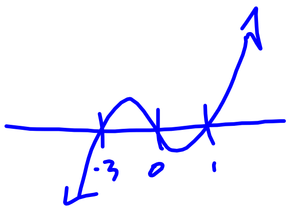
$- \quad -3 \quad 0 \quad 3 \quad +$

$=0 \quad =0 \quad =0$

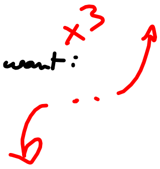
$x(x-3)(x+3)$

$+\infty$

$-\infty$



I want: x^3



webAssign wants

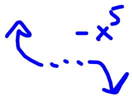
$y \to \infty$ as $x \to \infty$ RIGHT

$y \to -\infty$ as $x \to -\infty$ LEFT

A polynomial function is given.

6 $R(x) = -x^5 + 4x^3 - 9x$

$-x^5$



(a) Describe the end behavior of the polynomial function.

(b) Match the polynomial function with one of the following graphs.

$-x^5 + 4x^3 - 9x = -x(x^4 - 4x^2 + 9)$

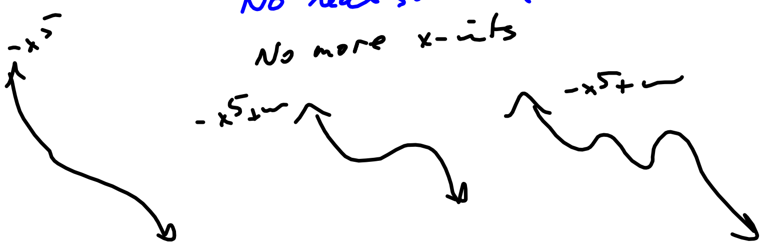
$= -x(u^2 - 4u + 9)$ where $u = x^2$

$u^2 - 4u + 2^2 - 4 + 9 =$

$(u-2)^2 + 5 = 0$

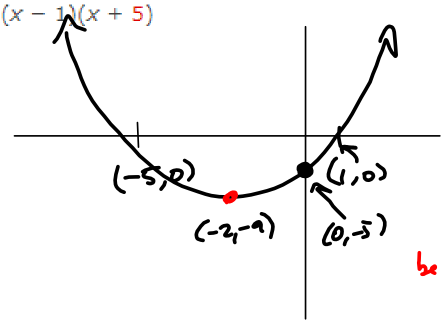
No real solution!

No more x-ints



7 Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

$P(x) = (x - 1)(x + 5)$



E.B. Mills x^2 ↖...↗

webAssign
$y \to \infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$

vertex is $\frac{1}{2}$ way between the x-intercepts

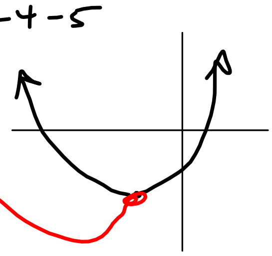
$\frac{-5+1}{2} = -\frac{4}{2} = -2$

$P(-2) = (-2-1)(-2+5)$
$= -3(3) = -9$

$P(0) = (0-1)(0+5) = -1(+5) = -5$

$P(x) = (x-1)(x+5)$
$= x^2 + 4x - 5 = x^2 + 4x + 2^2 - 4 - 5$
$= (x+2)^2 - 9$
$(-2,-9)$

$P(0) = -5$

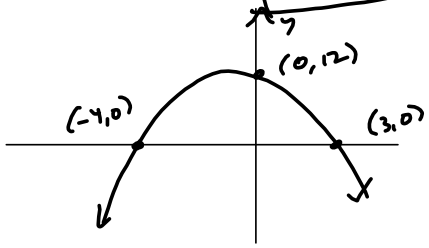


Range: $[-9, \infty)$

8 Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

$P(x) = (3 - x)(x + 4)$ $= -x\cdot x + \ldots = -x^2 + \ldots$ So $-x^2$ End Behavior.

$P(0) = (3)(4) = 12 \rightsquigarrow (0,12)$ y-int



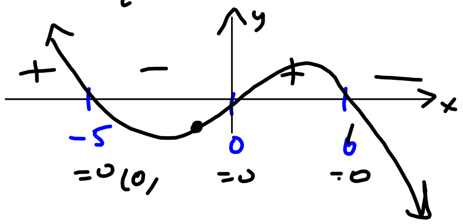
E.B.

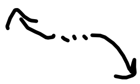
Webassign:
$y \to -\infty$ as $x \to \infty$
$y \to -\infty$ as $x \to -\infty$

9 Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

$P(x) = -x(x - 6)(x + 5)$ $= -x\cdot x\cdot x + \ldots = -x^3 +$ no importance to E.B.

$P(x) = 0 \Rightarrow$
$x = 0, 6, -5$
Better:
$x \in \{-5, 0, 6\}$





$P(0) = 0$

Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

12

$P(x) = x(x+1)(x-1)(3-x) = x \cdot x \cdot x \cdot (-x) + \cdots = -x^4 + \cdots$

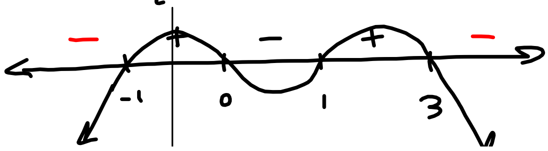
$\longrightarrow$ E.B. :

$P(x) = 0 \Longrightarrow$

$x = 0, -1, 1, 3$

$-1, 0, 1, 3$

$x \in \{-1, 0, 1, 3\}$



Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

13

$P(x) = \frac{1}{2}x(x-2)^2 = \frac{1}{2}x \cdot x^2 + \cdots = \frac{1}{2}x^3 + \cdots$

E.B. :

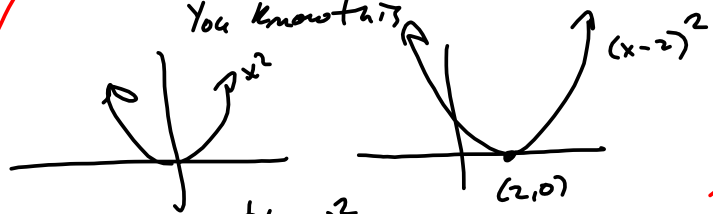
$P(x) = 0 \Longrightarrow$

$x = 0, 2$

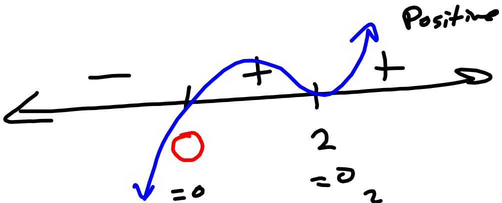
$x=2$ is a zero of multiplicity $m=2$ (because of the square)

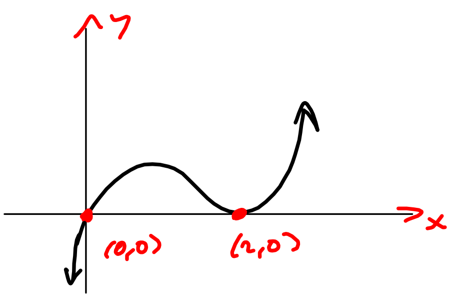
$(x-2)^2$ does not change sign at $x=2$, b/c of the square

You know this



& $(x-2)^2$ controls the sign change at $x=2$ & it says "No sign sign change." Positive to its right and left



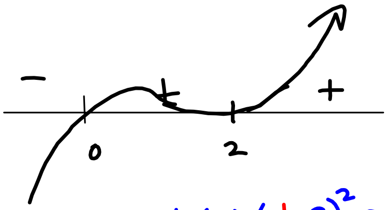


Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

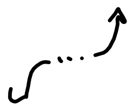
13

$P(x) = \frac{1}{2}x(x-2)^2$

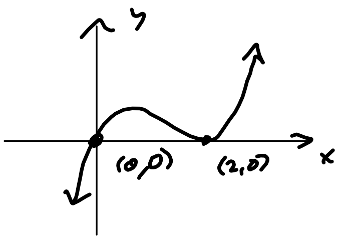
Test-Value Version



x^3



$(-\infty, 0)$	-1	$\frac{1}{2}(-1)(-1-2)^2 = (-)(-)^2 = (-)(+) = -$
$(0, 2)$	1	$\frac{1}{2}(1)(1-2)^2 = (+)(+) = +$
$(2, \infty)$	3	$\frac{1}{2}(3)(3-2)^2 = (+)(+) = +$



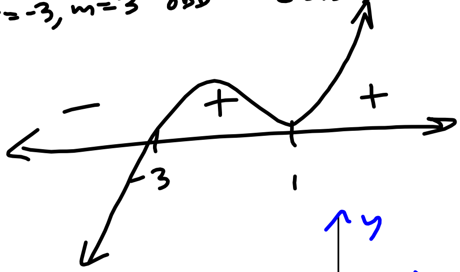
Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

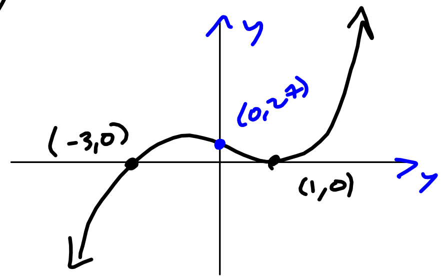
16

$P(x) = (x - 1)^2(x + 3)^3 = x^2 \cdot x^3 + \dots = x^5 + \dots$

$x = 1, m = 2$ Even kisses

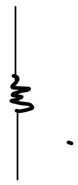
$x = -3, m = 3$ ODD crosses





$P(0) = (-1)^2(3)^3$

$= 1(27)$



Factor the polynomial and use the factored form to find the zeros. (Enter your answers as a comma-separated list. Enter all answers using the appropriate multiplicities.)

22

$P(x) = x^4 - 8x^2 - 9$

Let $u = x^2$, then

$P(-x) = (-x)^4 - 8(-x)^2 - 9 = x^4 - 8x^2 - 9 = P(x)$ Even!

$u^2 - 8u - 9 \overset{\text{SET}}{=} 0 \Rightarrow$

Find the y-intercept of the graph of $P(x)$.

Determine the end behavior of the polynomial $P(x)$.

$(u - 9)(u + 1) = 0 \Rightarrow$

E.B.

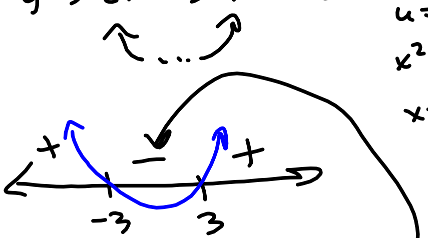
$y \to \infty$ as $x \to \pm\infty$

$u - 9 = 0$ OR $u + 1 = 0$

$u = 9$ $\qquad u = -1$

$x^2 = 9$ $\qquad x^2 = -1$

$x = \pm 3$ $\qquad x = \pm i$ No effect on intercepts in the real plane.



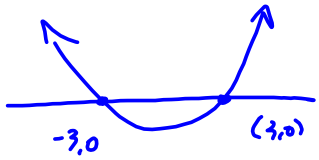
Even $\Rightarrow$ Symmetry $\Rightarrow$ Low point is $(0, P(0))$

Not necessarily!

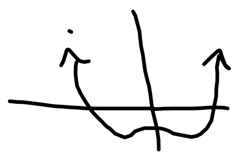
That's a reach

y-int: $P(0) = -9$

$(0, -9)$



-3,0 (3,0)



P(0) might be a local maximum.

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A graphing device is recommended.

Graph the polynomial in the given viewing rectangle.

$y = x^3 - 9x^2$, $[-2, 11]$ by $[-130, 130]$

Find the coordinates of all local extrema, rounded to two decimal places. (If an answer does not exist, enter DNE.)

State the domain and range. (Enter your answers using interval notation. Round your answers to two decimal places.)

Graphing Calculator will clobber this.

Desmos can not be relied upon for 2-digit precision. It rounds to 3 places (afaik), which can be a bad number to try to round to 2 places, sometimes.

Wolfram Alpha will clobber these with "extrema of"

https://www.wolframalpha.com/input?i=extrema+of+x%5E3-9x%5E2