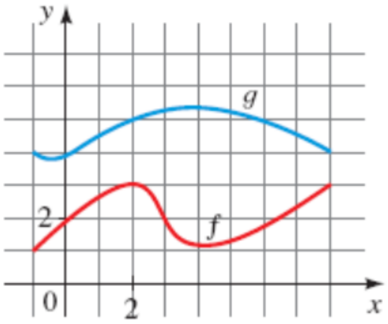
Section 2.7

1



$(f+g)(-1) = f(-1)+g(-1) = 1+4 = \boxed{5 = (f+g)(1)}$

$(f-g)(-1) = f(-1)-g(-1) = 1-4 = \boxed{-3 = (f-g)(-1)}$

$(fg)(-1) = f(-1)g(-1) = (1)(4) = \boxed{4 = (fg)(-1)}$

$\left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \boxed{\frac{1}{4} = \left(\frac{f}{g}\right)(-1)}$

2 By definition, $(f \circ g)(x) =$ --?-- $f(g(x))$. So if $g(5) = 6$ and $f(6) = 24$, then $(f \circ g)(5) = 24$

$= f(g(5)) = f(6) = 24$

3 If the rule of the function f is "add seven" and the rule of the function g is "multiply by 9," then the rule of $f \circ g$ is and the rule of $g \circ f$ is

$(f \circ g)(x) = f(g(x)) = f(9x) = \boxed{9x+7 = (f \circ g)(x)}$

$(g \circ f)(x) = g(f(x)) = g(x+7) = \boxed{9(x+7) = (g \circ f)(x)}$

4 Suppose the rule of the function f is "add one" and the rule of the function g is "multiply by 3."

How can we express these functions algebraically?

$f(x) = x+1$

$g(x) = 3x$

$(f \circ g)(x) = f(g(x)) = f(3x) = \boxed{3x+1 = f(g(x))}$

$(g \circ f)(x) = g(f(x)) = g(x+1) = \boxed{3(x+1) = g(f(x))}$

5 Let f and g be functions.

$= f(x)+g(x)$

(a) The function $(f+g)(x)$ is defined for all values of x that are in the domain(s) of both f & g.

$\mathcal{D}(f \pm g) = \mathcal{D}(f) \cap \mathcal{D}(g) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)\}$

(b) The function $(fg)(x)$ is defined for all values of x that are in the domain(s) of both f & g.

Same

(c) The function $(f/g)(x)$ is defined for all values of x that are in the domain(s) of f and g, and $g(x) \neq 0$.

6 Let f and g be functions.

The composition $(f \circ g)(x)$ $= f(g(x))$ is defined for all values of x for which x is in the domain of $g(x)$ and $g(x)$ is in the domain of $f(x)$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

7 Find $f+g$, $f-g$, fg, and f/g and their domains.

$f(x) = 4 - x, \quad g(x) = x^2 - 3x$

$\mathcal{D}(f) = \mathcal{D}(g) = (-\infty, \infty)$

For f/g, set $g(x) = 0$

$x(x-3) = 0$

$x = 0, 3$

$\mathcal{D}(f+g) = \mathcal{D}(f-g) = \mathcal{D}(fg) = \mathcal{D}(f) \cap \mathcal{D}(g) = \boxed{(-\infty, \infty) = \mathbb{R}}$

$\mathcal{D}(f/g) = \underbrace{\mathcal{D}(f) \cap \mathcal{D}(g)} \cap \{x \mid g(x) \neq 0\}$

$= (-\infty, \infty) \cap (\mathbb{R} \setminus \{0, 3\})$

$= (-\infty, \infty) \cap ((-\infty, 0) \cup (0, 3) \cup (3, \infty))$

$\boxed{\mathcal{D}(\frac{f}{g}) = (-\infty, 0) \cup (0, 3) \cup (3, \infty)}$

$\boxed{f \pm g = 4 - x \pm (x^2 - 3x)} =$

$\boxed{fg = (4-x)(x^2-3x)}$

$\boxed{\frac{f}{g} = \frac{4-x}{x^2-3x}}$

Find $f + g$, $f - g$, fg, and f/g and their domains.

8 $f(x) = \sqrt{16 - x^2}$, $g(x) = \sqrt{x + 1}$

$\mathcal{D}(f) = \{x \mid 16 - x^2 \geq 0\}$

$16 - x^2 = (4-x)(4+x) \geq 0$

N Y Y Y N

-4 $=0$ 4 $=0$

$-x^2$

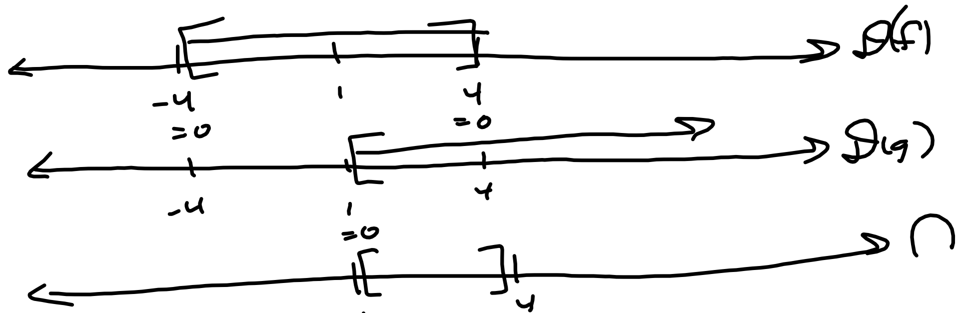
$\mathcal{D}(f) = [-4, 4]$

$\mathcal{D}(g) = \{x \mid x+1 \geq 0\}$

$x + 1 \geq 0$

$x \geq 1$

$= [1, \infty)$



$\boxed{f \pm g = \sqrt{16 - x^2} \pm \sqrt{x+1}}$

$\boxed{(fg) = \sqrt{16 - x^2}\sqrt{x+1}}$

$= \sqrt{(16 - x^2)(x+1)}$

$\boxed{\dfrac{f}{g} = \dfrac{\sqrt{16 - x^2}}{\sqrt{x+1}}} \Rightarrow \mathcal{D}\left(\dfrac{f}{g}\right) =$

$\dfrac{f}{g}$: $\mathcal{D}(f) \cap \mathcal{D}(g)$ part

N Y Y N

1 4

$g \neq 0$ part

Y N Y

1

$\cap$

$= \boxed{(1, 4] = \mathcal{D}\left(\dfrac{f}{g}\right)}$

9 Find $f + g$, $f - g$, fg, and f/g and their domains.

$$f(x) = \sqrt{25 - x^2}, \quad g(x) = \sqrt{x^2 - 4}$$

$\mathcal{D}(f) = \{x \mid 25 - x^2 \geq 0\}$, $\mathcal{D}(g) = \{x \mid x^2 - 4 \geq 0\} = (-\infty, -2] \cup [2, \infty)$

$-$ $+$ $-$

-5 5

$=0$ $=0$

-2 2

$=0$ $=0$

$f \pm g = \sqrt{25 - x^2} \pm \sqrt{x^2 - 4}$

$fg = \sqrt{25 - x^2}\sqrt{x^2 - 4}$

$\frac{f}{g} = \frac{\sqrt{25 - x^2}}{\sqrt{x^2 - 4}}$

$\mathcal{D}(f \pm g) = \mathcal{D}(fg) = \mathcal{D}(f) \cap \mathcal{D}(g)$

AND $\cap$

-5 -2 2 5

$= \boxed{[-5, -2] \cup [2, 5] = \mathcal{D}(f) \cap \mathcal{D}(g)}$

$\mathcal{D}\left(\frac{f}{g}\right)$: Need $g(x) = \sqrt{x^2 - 4} \neq 0$, too!

$x^2 - 4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$ Throw out ± 2: $\boxed{\mathcal{D}\left(\frac{f}{g}\right) = [-5, -2) \cup (2, 5]}$

10 Find the domain of the function. (Enter your answer using interval notation.)

$$f(x) = \sqrt{x} + \sqrt{7 - x}$$

Need $x \geq 0$ & $7 - x \geq 0$

AND!

$7 \geq x$

$x \leq 7$

0 7

$=0$ $=0$

$\cap$, AND

$= \boxed{[0, 7] = \mathcal{D}(f)}$

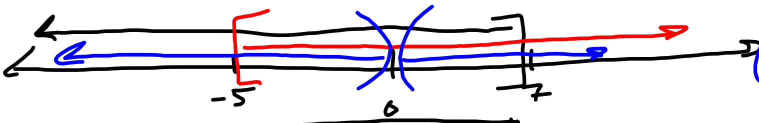
11 Find the domain of the function. (Enter your answer using interval notation.)

$$f(x) = \sqrt{x+5} - \frac{\sqrt{7-x}}{x}$$

Need $x+5 \geq 0$, $7-x \geq$ and $x \neq 0$

$x \geq -5$ $\quad$ $7 \geq x$

$x \leq 7$



AND $\quad$ $\cap$

$= \boxed{[-5,0) \cup (0,7] = \mathcal{D}(f)}$

12 Find the domain of the function. (Enter your answer using interval notation.)

$$h(x) = (x-5)^{-1/4} = \frac{1}{(x-5)^{1/4}} = \frac{1}{\sqrt[4]{x-5}}$$

Need $x-5 \geq 0$

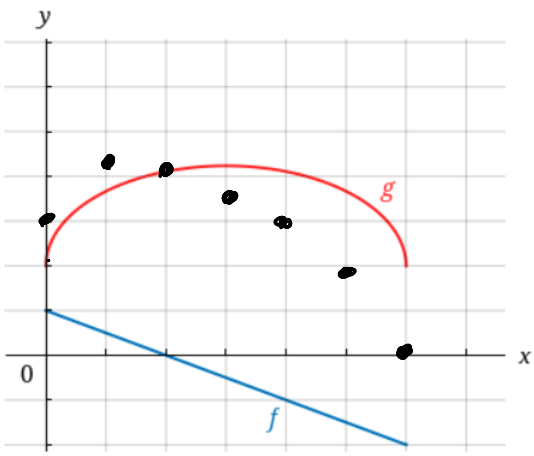
& $\sqrt[4]{x-5} \neq 0 \Leftrightarrow x-5 \neq 0$

$x-5 > 0$

$x > 5$

$= \boxed{(5,\infty) = \mathcal{D}(h)}$

13 Use graphical addition to sketch the graph of $f + g$.

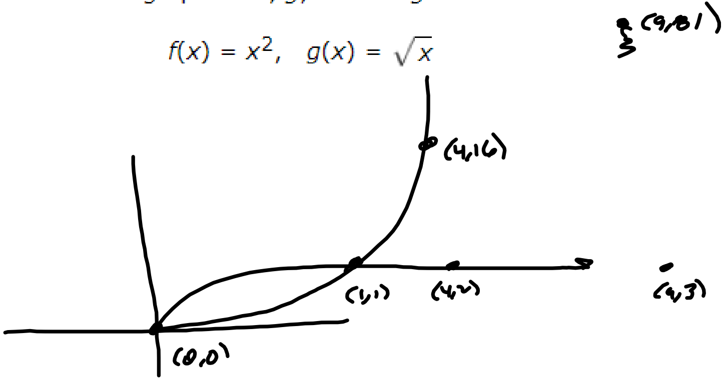


14 See #13.

15 A graphing device is recommended.

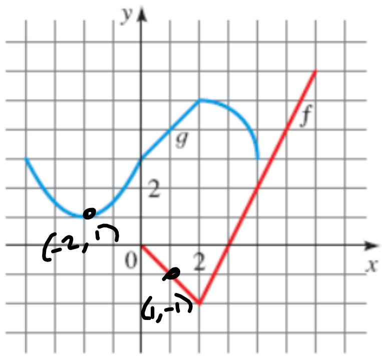
Draw the graphs of f, g, and $f + g$ on a common screen to illustrate graphical addition.

$f(x) = x^2, \quad g(x) = \sqrt{x}$



Use the given graphs of f and g to evaluate the expression. (Assume that each point lies on the gridlines.)

16



$f(g(-2)) = f(1) = -1$

17 See #16

Use the following table to evaluate the expression.

18

x	1	2	3	4	5	6
$f(x)$	2	3	5	1	6	3
$g(x)$	3	4	5	2	1	6

$\boxed{f(g(2)) = f(4) = 1}$

19 See #18

Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

20

$f(x) = 2x + 5, \quad g(x) = 6x - 1$

$\mathcal{D} = (-\infty, \infty)$. Lines (Non-Vertical)

$f \circ g = f(g(x)) = f(g) = 2g + 5 = 2(6x-1) + 5$

$= 12x - 2 + 5 = \boxed{12x + 3 = f \circ g}$

$g \circ f = g(f) = 6f - 1 = 6(2x+5) - 1 = 12x + 30 - 1 \boxed{= 12x + 29 = g(f(x))}$

$f \circ f = f(f) = 2f + 5 = 2(2x+5) + 5$

$= 4x + 20 + 5 = \boxed{4x + 25 = f \circ f}$

$g \circ g = g(g(x)) = (g \circ g)(x) = g(g) = 6g - 1 = 6(6x-1) - 1$

$= 36x - 6 - 1 = \boxed{36x - 7 = g \circ f}$

#21 Click here to see video.

22 Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

$$f(x) = \frac{x}{x+9}, \quad g(x) = 2x - 9$$

$(f \circ g)(x) = f \circ g = f(g(x)) = f(2x-9)$

$f(x) = \frac{x}{x+9}$

$f(g) = \frac{g}{g+9} =$

$= \frac{2x-9}{2x-9+9}$

$= \frac{2x-9}{2x}$

$\mathcal{D}(f \circ g) = \{x \in \mathcal{D}(g) \mid g(x) \in \mathcal{D}(f)\}$

scratch: $\mathcal{D}(f) = \mathbb{R} \setminus \{-9\}$, $\mathcal{D}(g) = (-\infty, \infty)$ (Polynomial)

$g(x) \in \mathcal{D}(f)$ means

$g(x) \neq -9$

$\Rightarrow 2x - 9 \neq -9$

$2x \neq 0$

$x \neq 0 \Rightarrow \{x \mid x \neq 0\} = \mathcal{D}(f \circ g) = \mathbb{R} \setminus \{0\}$

$= \boxed{(-\infty, 0) \cup (0, \infty) = \mathcal{D}(f \circ g)}$

$g \circ f = g(f) = 2f - 9 = 2\left(\frac{x}{x+9}\right) - 9 = \frac{2x - 9(x+9)}{x+9}$

$= \frac{2x - 9x - 81}{x+9} = \boxed{\frac{-7x-81}{x+9} = (g \circ f)(x)}$

$\mathcal{D}(g \circ f) = \{x \in \mathcal{D}(f) \mid \underbrace{f(x) \in \mathcal{D}(g) = \mathbb{R}}_{\text{No Restriction}}\}$

$= \mathcal{D}(f) =$

$\boxed{(-\infty, -9) \cup (-9, \infty) = \mathcal{D}(g \circ f)}$

$f \circ f = f(f) = \frac{f}{f+9} = \frac{\frac{x}{x+9}}{\frac{x}{x+9} + 9} = \frac{\frac{x}{x+9}}{\frac{x + 9(x+9)}{x+9}} = \frac{x}{x+9} \cdot \frac{x+9}{x+9x+81}$

$= \boxed{\frac{x}{10x+81} = f \circ f}$

$\mathcal{D}(f \circ f) = \{x \in \mathcal{D}(f) \mid \underline{f(x) \in \mathcal{D}(f)}\}$

$f(x) \neq -9$

$$\Rightarrow \quad \frac{x}{x+9} \neq -9$$

$$\frac{x}{x+9} + \frac{9(x+9)}{x+9} \neq 0$$

$$\frac{10x+81}{x+9} \neq 0$$

$$\Rightarrow \quad 10x + 81 \neq 0$$

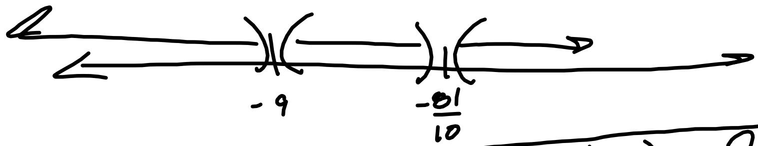
$$10x \neq -81$$

$$\boxed{x \neq -\frac{81}{10}}$$

combine: $x \neq -9$ & $x \neq -\frac{81}{10} \Rightarrow$

$$\mathcal{D}(f \circ f) = \mathbb{R} \setminus \left\{-9, -\frac{81}{10}\right\} =$$

=



$$= \boxed{(-\infty, -9) \cup \left(-9, -\frac{81}{10}\right) \cup \left(-\frac{81}{10}, \infty\right) = \mathcal{D}(f \circ f)}$$

Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

23

$$f(x) = \frac{1}{\sqrt{x}}, \quad g(x) = x^2 - 3x$$

$\mathcal{D}(f) = \{ x \mid \underbrace{x \geq 0}_{\sqrt{}} \text{ and } \underbrace{x \neq 0}_{\overline{0}} \} = (0, \infty) = \{ x \mid x > 0 \}$

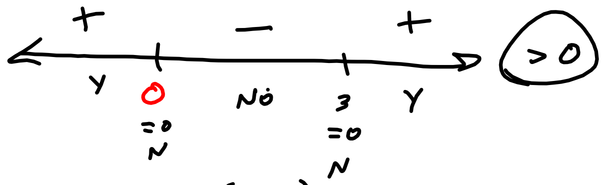
$\mathcal{D}(g) = (-\infty, \infty)$

$f \circ g = f(g) = \frac{1}{\sqrt{g}} = \frac{1}{\sqrt{x^2 - 3x}}$

$\mathcal{D}(f \circ g) = \{ x \mid x \in \underbrace{\mathcal{D}(g)}_{\mathbb{R} = (-\infty, \infty)} \text{ and } \underbrace{g(x) \in \mathcal{D}(f)}_{x^2 - 3x > 0} \}$

Need $x^2 - 3x > 0$

$x(x-3) > 0$



$= (-\infty, 0) \cup (3, \infty)$

$g \circ f = \left(\frac{1}{\sqrt{x}}\right)^2 - 3\left(\frac{1}{\sqrt{x}}\right)$

$= \frac{1}{(\sqrt{x})^2} - \frac{3}{\sqrt{x}}$

$= \boxed{\frac{1}{x} - \frac{3}{\sqrt{x}} = g \circ f}$

$= \frac{1 - 3\sqrt{x}}{x}$

$f \circ f = \frac{1}{\sqrt{f}} = \frac{1}{\sqrt{\frac{1}{\sqrt{x}}}} = \frac{1}{\frac{\sqrt{1}}{\sqrt{\sqrt{x}}}} = 1 \cdot \frac{\sqrt{\sqrt{x}}}{1} = \left((x)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= \boxed{\sqrt[4]{x} = f \circ f}$

24 Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

$f(x) = x^2, \quad g(x) = \sqrt{x - 7}$

$\boxed{\mathcal{D}(f) = \mathbb{R}}$

$\mathcal{D}(g) = [7, \infty)$

Need:

$x - 7 \geq 0$

$\Rightarrow x \geq 7$

$\Rightarrow \mathcal{D}(g) = \{x \mid x \geq 7\}$

$=$ 

$= \boxed{[7, \infty) = \mathcal{D}(g)}$

$f \circ g = f(g) = f(\sqrt{x-7}) = (\sqrt{x-7})^2 = \boxed{x - 7 = f \circ g}$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

No restriction! $\mathcal{D}(f) = \mathbb{R}$!

$= \{x \mid x \geq 7\}$

$= \boxed{[7, \infty) = \mathcal{D}(f \circ g)}$

$g \circ g$ looks tricky

$g \circ g = g(g) = \sqrt{g - 7} = \sqrt{\sqrt{x-7} - 7}$

Need $x \geq 7$ and we need $\sqrt{x-7} - 7 \geq 0$

$\sqrt{x-7} - 7 \geq 0$

$\sqrt{x-7} \geq 7$

$(\sqrt{x-7})^2 \geq 7^2$

$x - 7 \geq 49$

$x \geq 56$

If $0 < A < B$ $\Rightarrow A^2 < B^2$

$2 < 3$

$2^2 < 3^2$

$x \geq 56$ and $x \geq 7$

$\Rightarrow$ 

$\boxed{[56, \infty) = \mathcal{D}(g \circ g)}$

25 Find $f \circ g \circ h$.

$$f(x) = x - 4, \quad g(x) = \sqrt{x}, \quad h(x) = x - 4$$

$$f(g(h(x))) = f(g(x-4)) = f\left(\sqrt{x-4}\right) = \boxed{\sqrt{x-4} - 4 = f \circ g \circ h}$$

26 See #25

Find $f \circ g \circ h$.

27

$$f(x) = \sqrt{x}, \quad g(x) = \frac{x}{x-4}, \quad h(x) = \sqrt[7]{x}$$

$$f(g(h(x))) = f(g(\sqrt[7]{x})) = f\left(\frac{\sqrt[7]{x}}{\sqrt[7]{x} - 4}\right) = \boxed{\sqrt{\frac{\sqrt[7]{x}}{\sqrt[7]{x} - 4}} = f \circ g \circ h}$$

#28 Click Here to See Video

29 Find the functions f and g such that $F = f \circ g$. (Enter your answers as a comma-separated list. Use non-identity functions for $f(x)$ and $g(x)$.)

$F(x) = \frac{1}{x+9} = f(g(x))$

$f(x) = \frac{1}{x}$

$g(x) = x+9$

$\Big\} f(g(x)) = \frac{1}{g} = \frac{1}{x+9}$ ✓

30 Find the functions f, g, and h such that $F = f \circ g \circ h$. (Enter your answers as a comma-separated list. Use non-identity functions for $f(x)$, $g(x)$ and $h(x)$.)

$F(x) = \frac{6}{x^2+4}$

$h(x) = x^2$

$g(x) = x+4$

$f(x) = \frac{6}{x}$

$f(g(h(x))) = f(g(x^2)) = f(x^2+4) = \frac{6}{x^2+4}$

$h(x) = x^2$

$g(x) = \frac{1}{x+4}$

$f(x) = 6x$

$f(g(h(x))) = f(g(x^2)) = f\left(\frac{1}{x^2+4}\right) = 6\left(\frac{1}{x^2+4}\right)$

Using an identity function to cheat work

$f(x) = x$

$g(x) = x$

$h(x) = \frac{6}{x^2+4}$

$f(g(h(x))) = f(g(\frac{6}{x^2+4})) = f(\frac{6}{x^2+4}) = \frac{6}{x^2+4}$

$h(x) = x$

$g(x) = x$

$f(x) = \frac{6}{x^2+4}$

$f(g(h(x))) = f(g(x)) = f(x) = \frac{6}{x^2+4}$

$h(x) = x$

$g(x) = x^2+4$

$f(x) = \frac{6}{x}$

A stone is dropped in a lake, creating a circular ripple that travels outward at a speed of 90 cm/s.

31 (a) Find a function g that models the radius as a function of time t, in seconds.

(b) Find a function f that models the area of the circle as a function of the radius r, in cm.

(c) Find $f \circ g$.

What does this function represent?

a

$\frac{90 \text{ cm}}{s}$ = rate the diameter is changing wrt time.

$\Rightarrow$ radius is half that

$\frac{1}{2}\left(\frac{90 \text{ cm}}{s}\right) = 45 \frac{\text{cm}}{s}$

r = radius = $r(t)$ — in cm

radius as a function of

t = time (in secs)

$\Rightarrow \boxed{r(t) = 45t \text{ cm}}$

this is my g

b

f = area of circle as a function of t, $f = f(t)$

To start with

f = Area $= \pi r^2$ cm^2

&

$f \circ g = f(g) = f(45t)$

$= \pi (45t)^2$

A spherical balloon is being inflated. The radius of the balloon is increasing at the rate of 9 cm/s.

32 (a) Find a function f that models the radius as a function of time t, in seconds. $r(t) = 9t\ (+ r(0)) = f(t)$

?

(b) Find a function g that models the volume as a function of the radius r, in cm. (Formulas for volume are given on the inside front cover of the textbook.)

$V = \frac{4}{3}\pi r^3 = V(r) = g(r)$

(c) Find $g \circ f$.

$\Rightarrow \boxed{V(r(t))\ \frac{4}{3}\pi (9t)^3 = g \circ f}$

b, c

What does this function represent?

Volume as a function of time, in $\frac{\text{cm}^3}{s}$