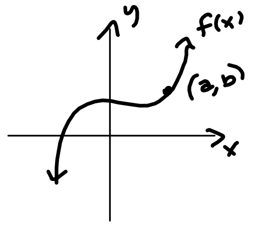
1

Fill in the blank with the appropriate direction (left, right, upward, or downward).

(a) The graph of $y = f(x) + 2$ is obtained from the graph of $y = f(x)$ by shifting upward 2 units.

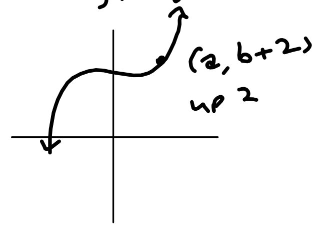
(b) The graph of $y = f(x + 2)$ is obtained from the graph of $y = f(x)$ by shifting leftward 2 units.

$y = f(x)$



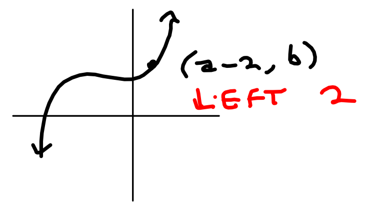
(a) $y = f(x) + 2$

$y \mapsto y+2$



(b) $y = f(x+2)$

$x \mapsto x-2$



2

Fill in the blank with the appropriate direction (left, right, upward, or downward).

(a) The graph of $y = f(x) - 6$ is obtained from the graph of $y = f(x)$ by shifting downward 6 units.

$y \mapsto y-6$

(b) The graph of $y = f(x - 6)$ is obtained from the graph of $y = f(x)$ by shifting right 6 units.

$x \mapsto x+6$

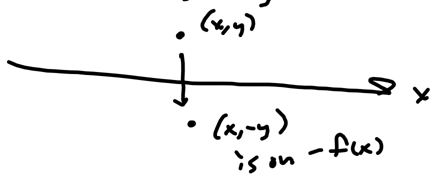
3

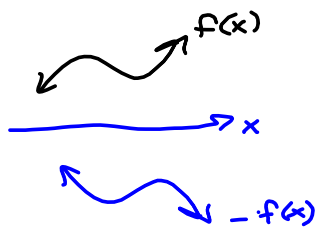
Fill in the blank with the appropriate axis (x-axis or y-axis).

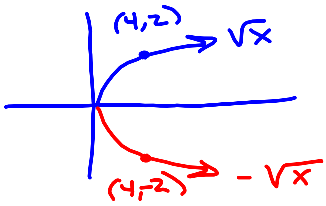
(a) The graph of $y = -f(x)$ is obtained from the graph of $y = f(x)$ by reflecting about the

(b) The graph of $y = f(-x)$ is obtained from the graph of $y = f(x)$ by reflecting about the

$y = -f(x)$ says y is now $-y$.

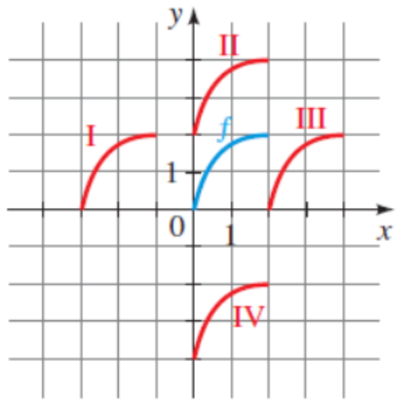






A graph of a function f is given. Match each equation with one of the graphs labeled I-IV.

4



I Left 3 $f(x+3)$ is (b)

II Up 2 $f(x)+2$ is (a)

III RIGHT 2 $f(x-2)$ is (c)

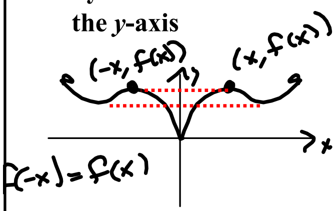
IV DOWN 4 $f(x)-4$ is (d)

5 If a function f is an even function, then what type of symmetry does the graph of f have?

- ○ The function f has symmetry with respect to the y-axis.
- ○ The function f has symmetry with respect to the origin.
- ○ The function f has symmetry with respect to the x-axis.

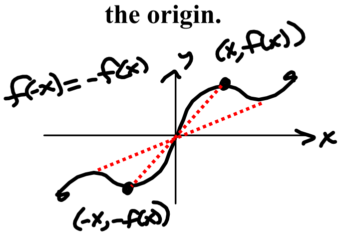
#5 is Even

Symmetric wrt the *y*-axis



#6 is Odd

Symmetric wrt the origin.



6 If a function f is an odd function, then what type of symmetry does the graph of f have?

- ○ The function f has symmetry with respect to the y-axis.
- ○ The function f has symmetry with respect to the origin.
- ○ The function f has symmetry with respect to the x-axis.

Suppose the graph of f is given. Describe how the graph of each function can be obtained from the graph of f.

7 (a) $f(x + 1)$ Left 1

$x \mapsto x - 1$

(b) $f(x) + 8$ up 8

$y \mapsto y + 8$

Explain how the graph of g is obtained from the graph of f.

8 (a) $f(x) = x^2$, $g(x) = (x + 9)^2 = f(x+9)$

(b) $f(x) = x^2$, $g(x) = x^2 + 9 = f(x) + 9$

$y \mapsto y+9$

$f(x)+9 = x^2+9$

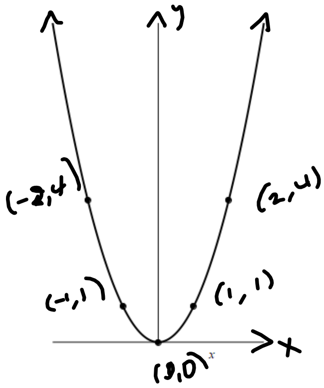
up 9

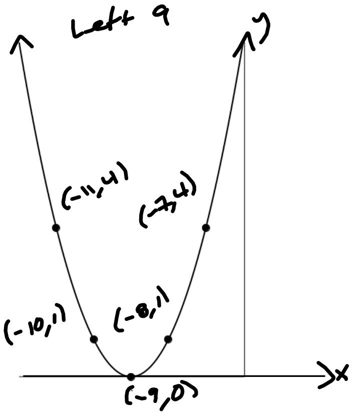
$x \mapsto x-9$

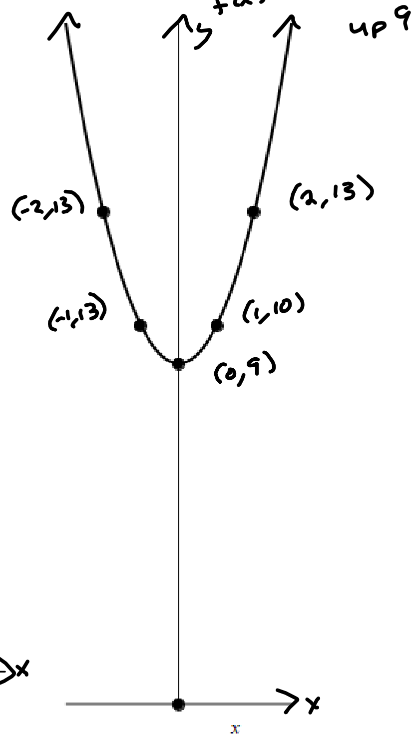
$f(x) = x^2$

$f(x+9) = (x+9)^2$

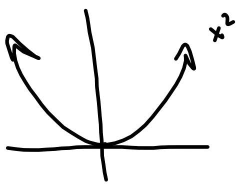
Left 9

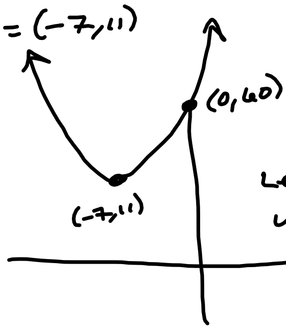






$(x+7)^2 + 11$ $(h,k) = (-7,11)$





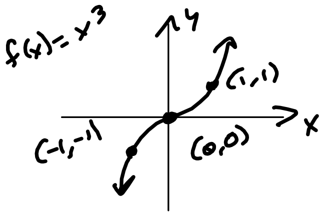
Left 7

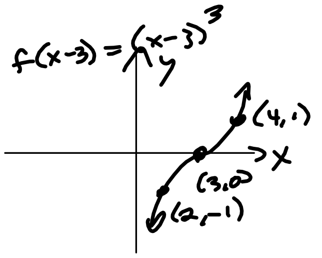
up 11

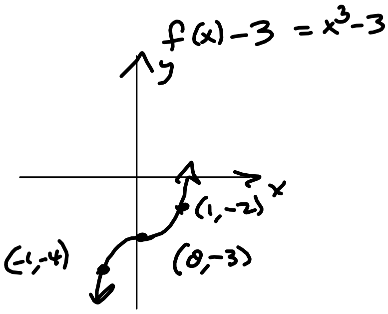
Explain how the graph of g is obtained from the graph of f.

9 (a) $f(x) = x^3$, $g(x) = (x - 3)^3$ $= f(x-3)$ $x \mapsto x+3$ Right 3

(b) $f(x) = x^3$, $g(x) = x^3 - 3$ $= f(x) - 3$ $y \mapsto y-3$ Down 3





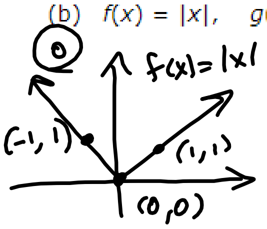


Explain how the graph of g is obtained from the graph of f.

10 (a) $f(x) = |x|$, $g(x) = |x + 4| - 4$ Left 4, Down 4

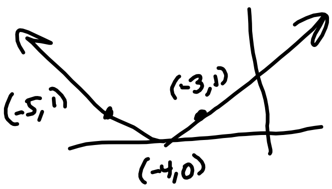
(b) $f(x) = |x|$, $g(x) = |x - 4| + 4$ Right 4, Up 4

$x \mapsto x-4$



2 1 $|x+4| = f(x+4)$

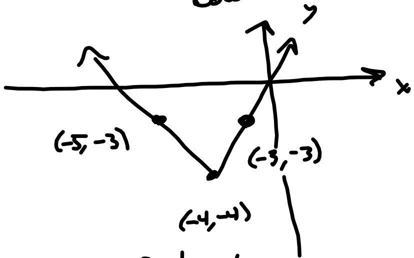
Left 4



2 $g(x) = |x+4| - 4$

$= f(x+4) - 4$

Down 4



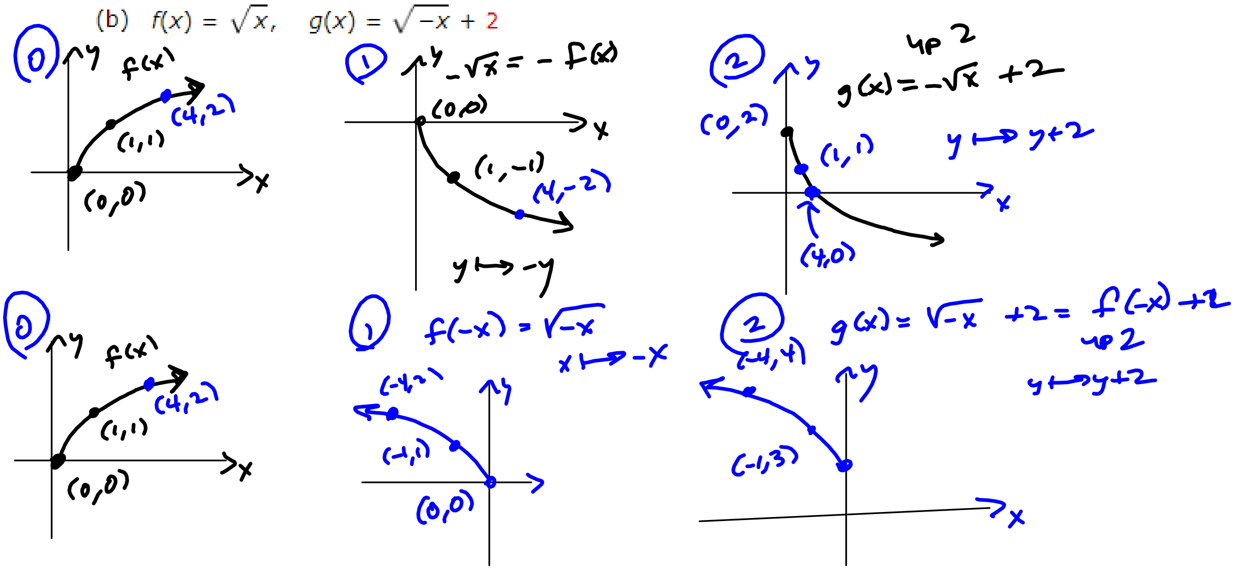
Check Picture!

y-int: $g(0) = 4 - 4 = 0$

Explain how the graph of g is obtained from the graph of f.

11 (a) $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x} + 2$

(b) $f(x) = \sqrt{x}$, $g(x) = \sqrt{-x} + 2$



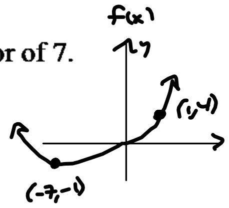
Bridge to Writing Project #2

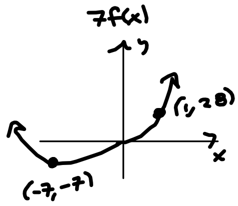
I think by the time you finish #11, you've seen just enough exercises and had enough discussion to bridge from your homework on WebAssign to your Writing Project #2

Recall:

$7f(x)$ multiplies all the y values by 7. It's a vertical stretch by a factor of 7.

(x,y) on the graph of $f(x)$ implies $(x,7y)$ is on the graph of $7f(x)$

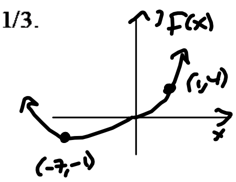


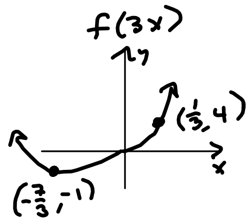


and

$f(3x)$ multiplies all the x values by $\frac{1}{3}$. It's a horizontal shrink by a factor of 1/3.

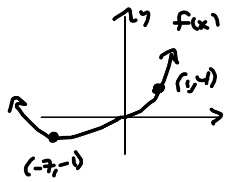
(x,y) on the graph of $f(x)$ implies $\left(\frac{1}{3}x, y\right)$ is on the graph of $f(3x)$.

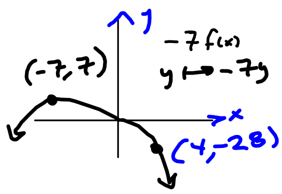




If you can do that, then you can do all your reflections by simply following the rules, above. It's a combo move that's a natural one-step move

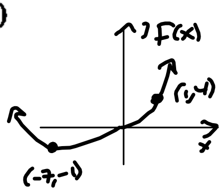
$-7f(x)$ multiplies all the y values by -7. It's a vertical stretch by a factor of 7 AND a reflection about the x-axis. (Up to down)

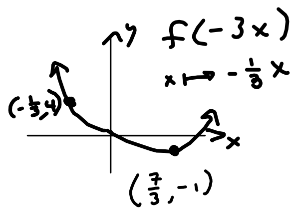




and

$f(-3x)$ multiplies all the x values by $-\frac{1}{3}$. It's a horizontal shrink by a factor of 1/3 AND a reflection about the y-axis. (left-to-right)

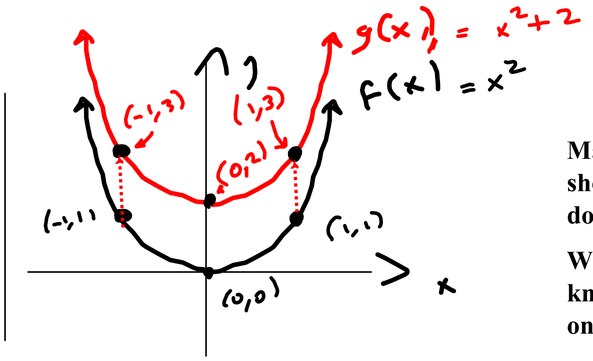




Continuing the Homework...

12 Use the graph of $y = x^2$ (in blue) to graph the following (in red).

(a) $g(x) = x^2 + 2$



$(x,y) \longmapsto (x, y+2)$

$(-1,1) \longmapsto (-1, 1+2) = (-1,3)$

$(0,0) \longmapsto (0, 0+2) = (0,2)$

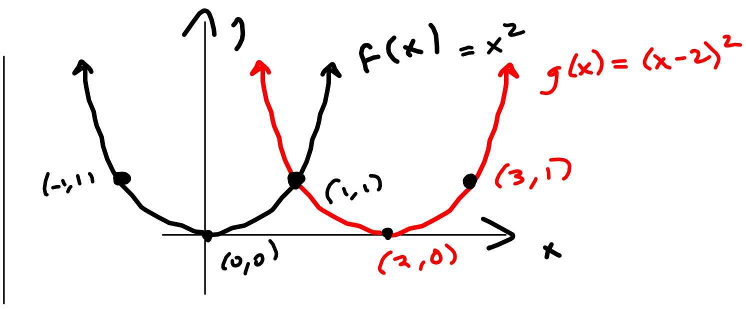
$(1,1) \longmapsto (1, 1+2) = (1,3)$

Matching. Ugh. At least they go fairly quickly. You should be doing a hand sketch for each of these as I am doing.

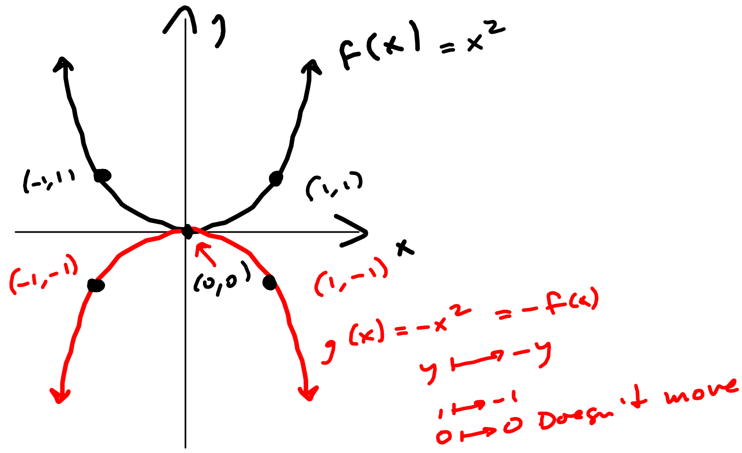
Well, maybe not as many points as I show, for you to know you have the right idea after the 4th or 5th or 10th one, just by following one or two points around.

(b) $g(x) = (x - 2)^2$

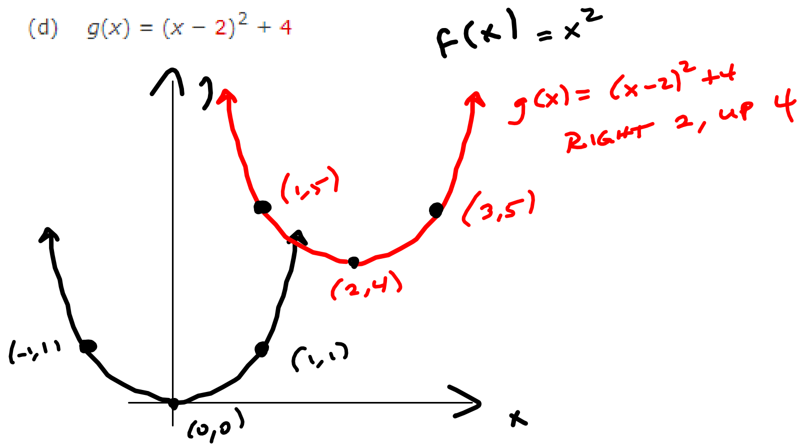
$x \longmapsto x+2$
RIGHT 2



(c) $g(x) = -x^2$



(d) $g(x) = (x - 2)^2 + 4$



Use the graph of $y = \sqrt{x}$ (dashed) to graph the given functions (solid).

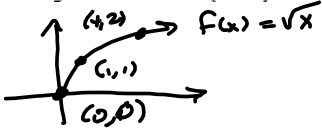
13

(a) $g(x) = \sqrt{x - 1}$

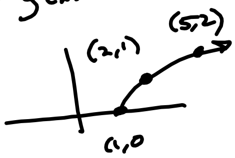
(b) $g(x) = \sqrt{x} + 3$

(c) $g(x) = \sqrt{x + 1} + 1$

(d) $g(x) = -\sqrt{x} + 3$



(a) $g(x) = \sqrt{x-1} = f(x-1)$ RIGHT 1

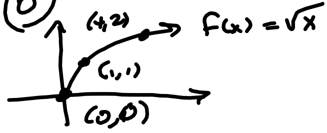


(c) $g(x) = \sqrt{x+1} + 1 = f(x+1) + 1$

Left 1, up 1

(0) $f(x) = \sqrt{x} \rightarrow$ (1) $\sqrt{x+1} \rightarrow$ (2) $\sqrt{x+1} + 1 = g(x)$

(0)

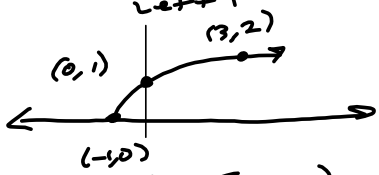


$D = [0, \infty)$

$R = [0, \infty)$

(1) $\sqrt{x+1} = f(x+1)$

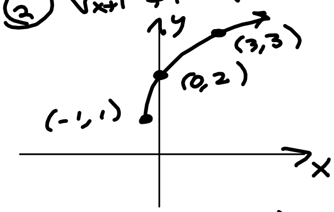
Left 1



$D = [-1, \infty)$

$R = [0, \infty)$

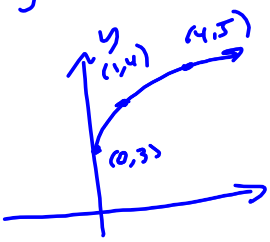
(2) $\sqrt{x+1} + 1 = f(x+1) + 1$



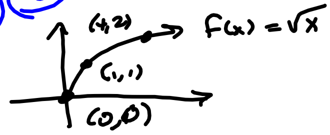
$D = [-1, \infty)$

$R = [1, \infty)$

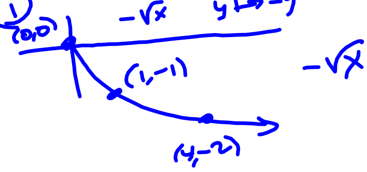
(b) $g(x) = \sqrt{x} + 3$



(d) (0)

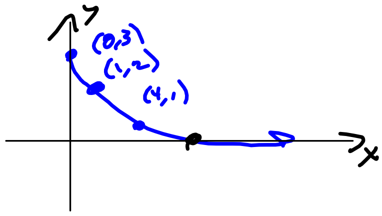


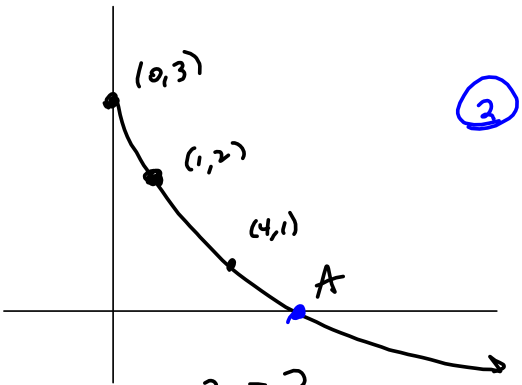
(1) $-\sqrt{x}$ $y \mapsto -y$



$-\sqrt{x}$

(2) $-\sqrt{x} + 3$





$A: -\sqrt{x} + 3 = 0$

$-\sqrt{x} = -3$

$\sqrt{x} = 3$

$x = 3^2 = 9$

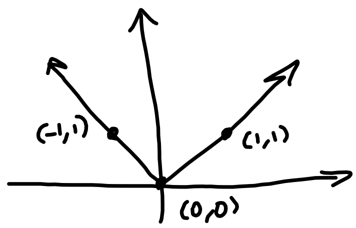
$\boxed{A = (9,0)}$

14 Consider the graph of $y = |x|$.

Match the graph with the function.

$y = |x + 5|$

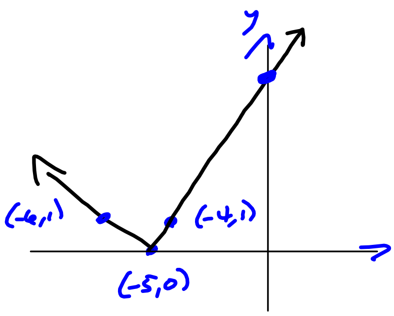
$f(x) = |x|$



$D = (-\infty, \infty)$

$R = [0, \infty)$

$g(x) = |x+5| = f(x+5) \qquad x \longmapsto x-5$



$D = (-\infty, \infty)$

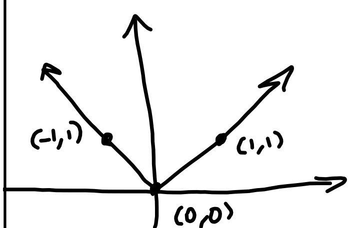
$R = [0, \infty)$

15 Consider the graph of $y = |x|$.

Match the graph with the function.

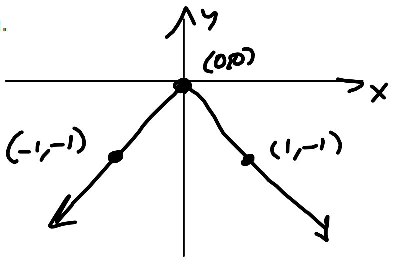
$y = -|x|$

$f(x) = |x|$



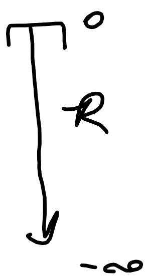
$D = \mathbb{R}$

$R = [0, \infty)$



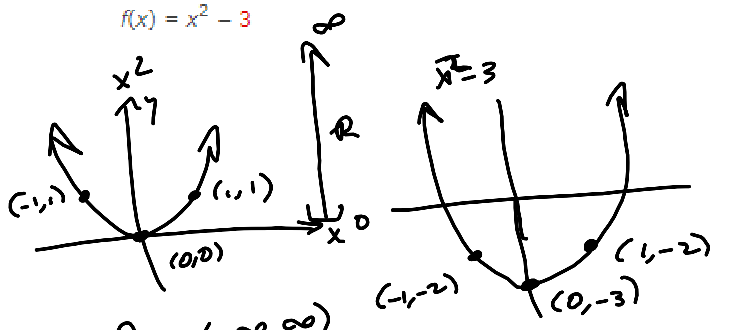
$D = (-\infty, \infty)$

$R = (-\infty, 0]$



16 Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$f(x) = x^2 - 3$



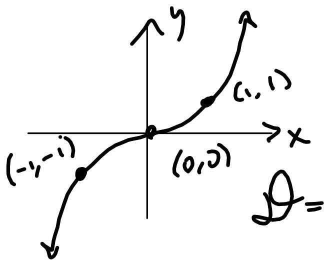
$D = (-\infty, \infty)$
$R = [0, \infty)$

$R = [-3, \infty)$

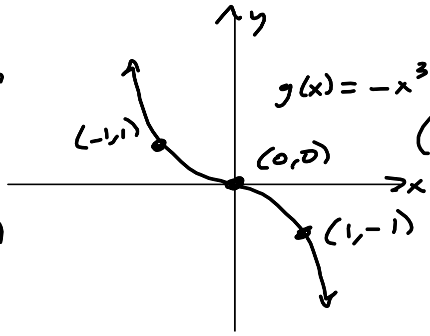
#s 17 - 19 See Video

20 Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$f(x) = -x^3$



Symmetric wrt origin
ODD FUNCTION
$D = R = (-\infty, \infty)$

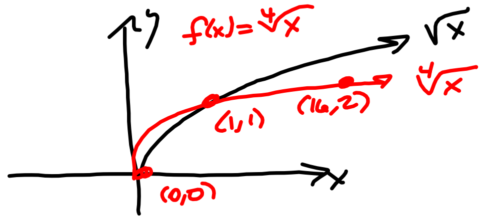


$g(x) = -x^3 = f(x)$
(Same as $(-x)^3 = f(-x)$
b/c
$(-x)^3 = ((-1)x)^3$

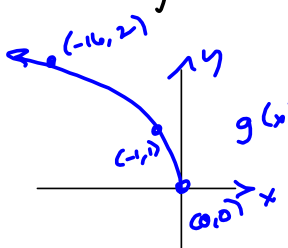
21 Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$g(x)$ ~~$f(x)$~~ $= \sqrt[4]{-x}$

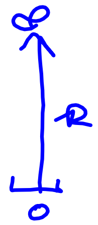
$D = [0, \infty)$
$R = [0, \infty)$

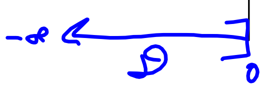


$\Rightarrow g(x)$ is $f(x)$ flipped about the y-axis
$x \mapsto -x$



$g(x) = \sqrt{-x}$





$D = (-\infty, 0]$
$R = [0, \infty)$

22 Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$f(x) = \sqrt[3]{-x}$

Rep/Recap: Natural #s $\mathbb{N} = \{1,2,3,4,5,\dots\}$ = counting numbers

If $n \in \mathbb{N}$, then $2n$ is even an $2n+1$ is odd.

$1, 3, 5, 7, 9, 11, \dots$ are odd

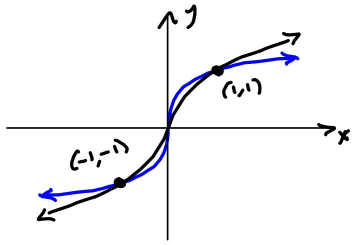
Everything I show and do with $\sqrt[3]{x}$ is going to be the same for $\sqrt[5]{x}, \sqrt[7]{x}, \dots, \sqrt[2n+1]{x}, \dots$

Almost everything

$\sqrt[5]{x} > \sqrt[3]{x}$ for $x \in (0,1)$

$\sqrt[5]{1} = \sqrt[3]{1}$ for $x = 1$

$\sqrt[5]{x} < \sqrt[3]{x}$ for $x \in (1,\infty)$



To summarize, $\sqrt[5]{x}$ is taller when x is small

and $\sqrt[3]{x}$ " " " x " big

22

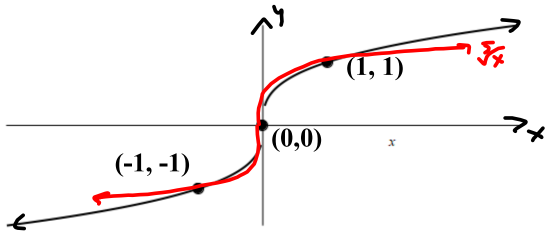
Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

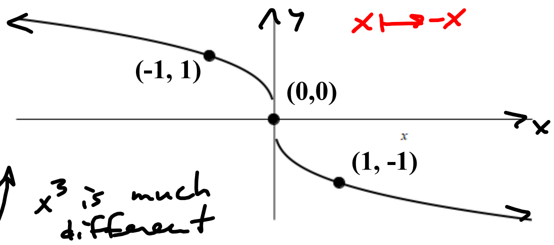
$f(x) = \sqrt[3]{-x}$

$g(x) = \sqrt[3]{x} = x^{1/3}$

Same for $\sqrt[5]{x}, \sqrt[7]{x}, \ldots$, $\sqrt[\text{2n+1}]{x}, \ldots$

$f(x) = \sqrt[3]{-x} = (-x)^{1/3}$



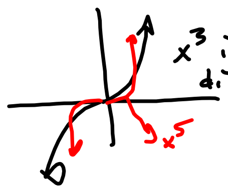


$x \mapsto -x$

ODD

$\mathcal{D} = (-\infty, \infty)$

$\mathcal{R} = (-\infty, \infty)$



x^3 is much different

NOTE!

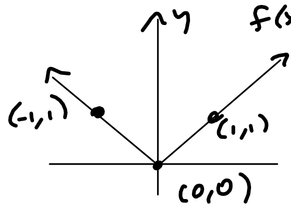
$g(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -g(x)$

23

Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

$y = \frac{1}{6}|x|$

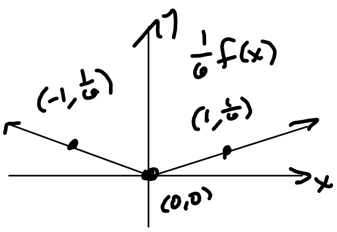
$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



$|\text{STUFF}| = \begin{cases} \text{STUFF} & \text{if stuff} \geq 0 \\ -\text{STUFF} & \text{if stuff} < 0 \end{cases}$

$\frac{1}{6}|x| = \frac{1}{6}f(x)$

$y \mapsto \frac{1}{6}y$

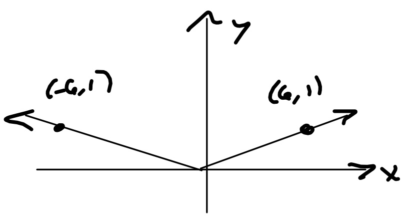


Alternate:

$\frac{1}{6}|x| = |\frac{1}{6}x|$

Horizontal Stretch

$x \longmapsto \frac{1}{\frac{1}{6}}x = 6x$



You can sort of play this game with other functions, but it works best for $y = mx + b$

or $y = m|x| + b$ (Abs-Val)

for $f(x) = x^2$:

$g(x) = 5(x-2)^2 + 5$

$f(x) = x^2 \longrightarrow 5x^2 \longrightarrow 5(x-2)^2 \longrightarrow 5(x-2)^2 + 4$

vertical stretch

$y \longmapsto 5y$

$5(x^2 - 4x + 4) + 5$

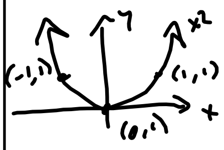
$= 5x^2 - 20x + 20 + 5$

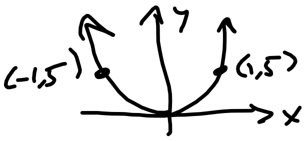
$= 5x^2 - 20x + 25$

$= 5(x^2 - 4x \quad) + 25$

$= 5(x^2 - 4x + 4) - 20 + 25$

$= 5(x-2)^2 + 5$





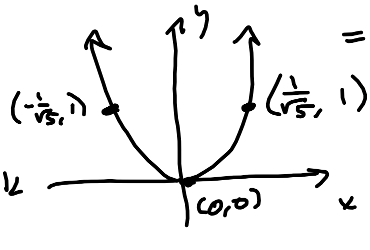
$5(x-2)^2 + 4$

$(\sqrt{5})^2 (x-2)^2 + 4$

$(\sqrt{5}(x-2))^2 + 4$

horizontal shrink

$x \longmapsto \frac{1}{\sqrt{5}} x$



$x^2 \xrightarrow[x \mapsto \frac{1}{\sqrt{5}}x]{} (\sqrt{5}x)^2 \xrightarrow[x \mapsto x+2]{} (\sqrt{5}(x-2))^2 \xrightarrow[y \mapsto y+4]{} (\sqrt{5}(x-2))^2 + 4$

Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

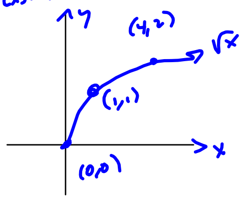
24

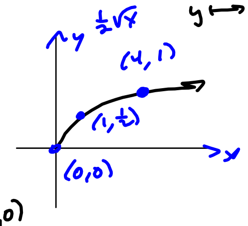
$f(x) = \frac{1}{2}\sqrt{x+2} - 3$

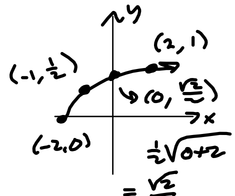
$g(x) = \sqrt{x}$

$\sqrt{x} \longmapsto \frac{1}{2}\sqrt{x} \longrightarrow \frac{1}{2}\sqrt{x+2} \longrightarrow \frac{1}{2}\sqrt{x+2} - 3$

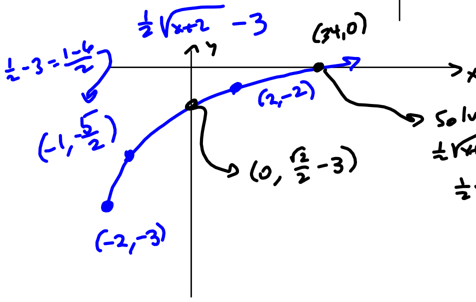
$y \mapsto \frac{1}{2}y \qquad x \mapsto x-2 \qquad y \mapsto y-3$







$\frac{1}{2}\sqrt{0+2} = \frac{\sqrt{2}}{2}$



Solve

$\frac{1}{2}\sqrt{x+2} - 3 = 0$

$\frac{1}{2}\sqrt{x+2} = 3$

$\sqrt{x+2} = 6$

$x+2 = 36$

$x = 34 \rightarrow (34,0)$

25 Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations. (Select Update Graph to see your response plotted on the screen. Select the Submit button to grade your response.)

$f(x) = 4 - 2(x-5)^2 = -2(x-5)^2 + 4$

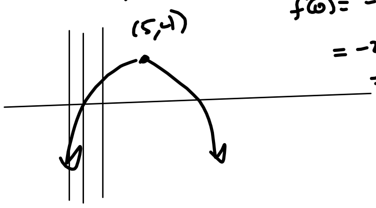
$x^2 \rightarrow -2x^2 \rightarrow -2(x-5)^2 \rightarrow -2(x-5)^2 + 4$

Start with the graph of a standard function $y = g(x) =$ x^2 ✔

The point A at (0, 0) moves to A' at $(x, y) =$ (5,4) ✔

The point B at (1, 1) moves to B' at $(x, y) =$ (6,2) ✔

At a glance:



Not clear where y-axis is w/o investigating

$f(0) = -2(-5)^2 + 4$

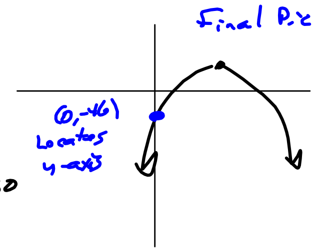
$= -2(25) + 4$

$= -50 + 4$

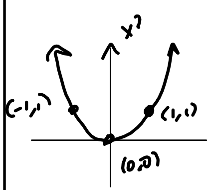
$= -46$

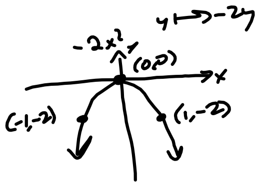
Negative y-int, so

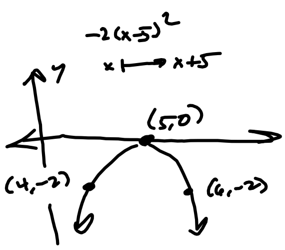
Final Pic

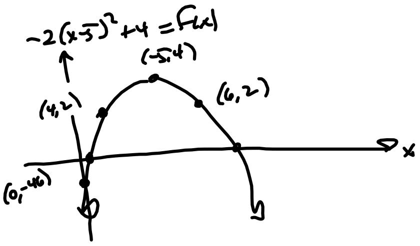


Now, do it like WP#2:









#s 26-28 Click Here for Videos

#29 Click Here for the Video.

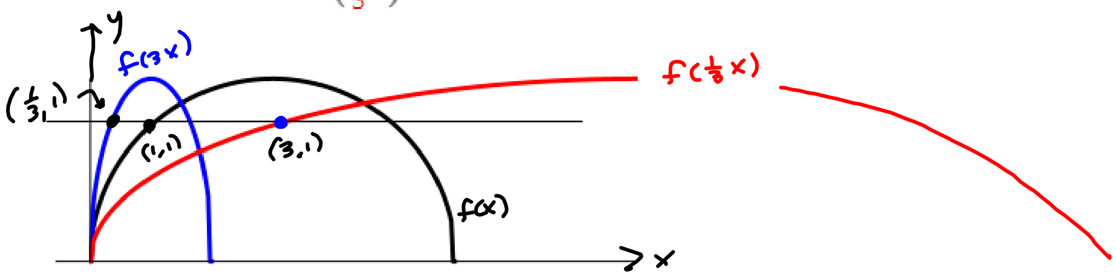
30 A graphing device is recommended.

If $f(x) = \sqrt{2x - x^2}$, graph the following functions in the viewing rectangle [−5, 7] by [−4, 4].

(a) $y = f(x)$

(b) $y = f(3x)$ HOR. SHRINK $x \longmapsto \frac{1}{3}x$

(c) $y = f\left(\frac{1}{3}x\right)$ HOR. STRETCH $x \longmapsto 3x$



Extra Insight/Technique

$f(x) = \sqrt{2x - x^2}$

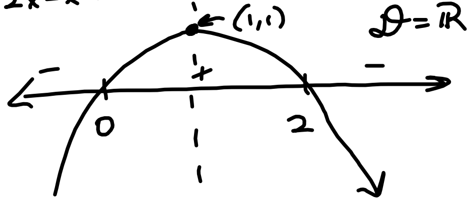
$2x - x^2 = -x^2 + 2x = -x(x-2)$

$-x^2 + 2x$

$= -(x^2 - 2x + 1^2) + 1$

$= -(x-1)^2 + 1$

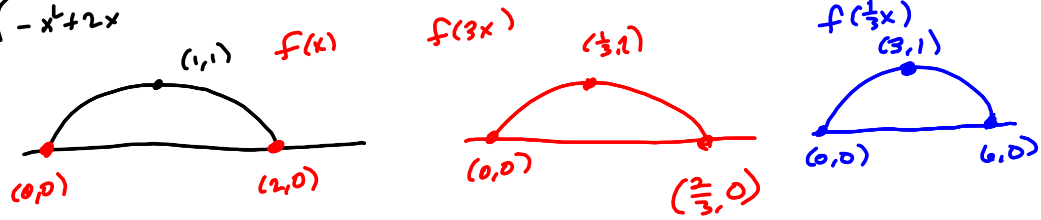
$(h, k) = (1, 1)$



$\mathcal{D} = \mathbb{R}$

$\mathcal{R} = (-\infty, 1]$

$\sqrt{-x^2 + 2x}$

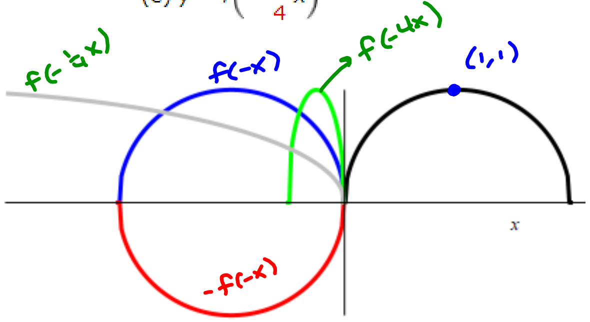


$\sqrt{\frac{1}{4}} = \frac{1}{2}$

A graphing device is recommended.

31 If $f(x) = \sqrt{2x - x^2}$, graph the following functions in the viewing rectangle $[-9, 5]$ by $[-4, 4]$.

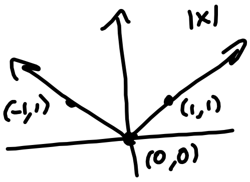
(a) $y = f(x)$
(b) $y = f(-x)$ ↔ flip
(c) $y = -f(-x)$ flip ↕ then ↔
(d) $y = f(-4x)$
(e) $y = f\left(-\frac{1}{4}x\right)$



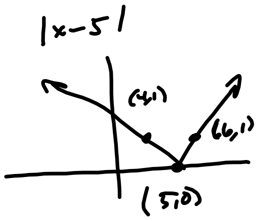
Sometimes you can pull a factor outside of the function and turn it into a vertical stretch/shrink as your first move, rather than a horizontal shrink/stretch, respectively. Stretching *y* can be thought of as shrinking *x* in some situations.

$g(x) = |5x-5|$

$|x|$

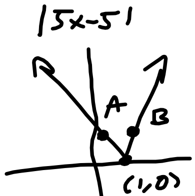


$|x-5|$



$f(x-5)$

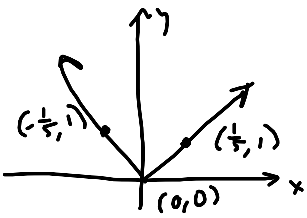
$|5x-5|$



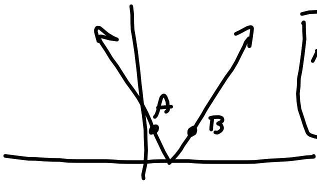
$f(5x-5)$

$$\boxed{\begin{aligned} A &= (\tfrac{4}{5}, 1) \\ B &= (\tfrac{6}{5}, 1) \end{aligned}}$$

$|5x|$



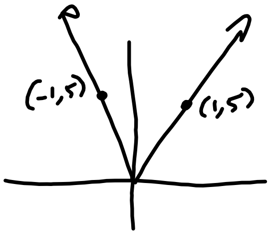
$|5(x-1)|$



$$\boxed{\begin{aligned} A &= (\tfrac{4}{5}, 1) \\ B &= (\tfrac{6}{5}, 1) \end{aligned}}$$

$g(x) = |5x-5| = 5|x-1|$

$5|x|$



$5|x-1| = g(x)$