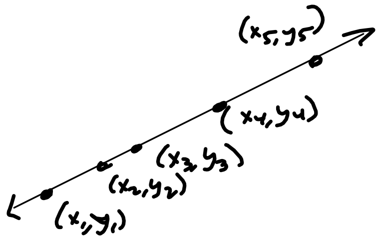
1

Let f be a function with constant rate of change.

(a) Then f is a linear function and f is of the form $f(x) =$ $ax + b$ for some fixed a & b.

(b) The graph of f is a line



$\frac{y_k - y_j}{x_k - x_j} = a =$ same value for any choice of 2 points.

$\frac{f(c) - f(a)}{c - a} = m_{sec} = m_{tan}$

2

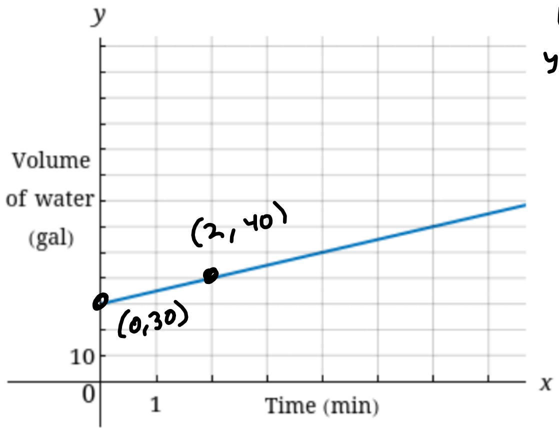
Let f be the linear function $f(x) = -6x + 9$.

(a) The rate of change of f is -6 $(= m)$

(b) The graph of f is a Line with slope -6 and y-intercept $\underline{9}$ b $(0, 9)$

3

A swimming pool is being filled. The graph shows the number of gallons y in the pool after x minutes.



Let $\boxed{\text{Lexicon}}$

$y =$ the # of gallons of water in the pool, as a function of

$x =$ the # of minutes it's being filled

What is the slope of the graph (in gal/min)?

gal/min

$\frac{40 - 30}{2 - 0} = \frac{10}{2} = \boxed{5 \text{ gal/min}}$

If a linear function has positive rate of change, does its graph slope upward or downward?

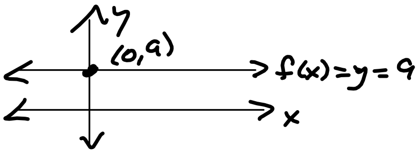
4 ◯ upward **(and to the right)**

◯ downward

Is $f(x) = 9$ a linear function?

5 ◯ Yes, $f(x) = 9$ is a linear function.

◯ No, $f(x) = 9$ is not a linear function.



If so, what are the slope and the rate of change? (If the function is not linear, enter NOT LINEAR.)

slope $= m = 0$

rate of change $= m = 0$

6 Determine whether the given function is linear. If the function is linear, express the function in the form $f(x) = ax + b$. (If the function is not linear, enter NOT LINEAR.)

$f(x) = \frac{8x - 9}{x}$ Looking for $ax+b$

$= \frac{8x}{x} - \frac{9}{x} = 8 - \frac{9}{x}$ is not $ax+b$

Not

For the given linear function, make a table of values and use it to and sketch its graph. (Select Update Graph to see your response plotted on the screen. Select the Submit button to grade your response.)

7 $r(t) = -\frac{2}{3}t + 7$

t	$r(t)$
−6	11
−3	9
0	7
3	5
6	3

$-\frac{2}{3}(-6)+7 = 11$

$-\frac{2}{3}(-3)+7 = 9$

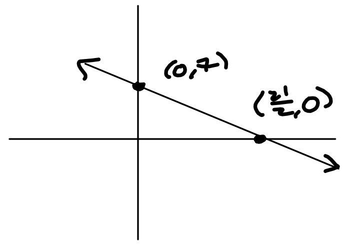
For a hand sketch, the two main points I want to see are the intercepts. In WebAssign, for graphing or measuring slope, always go for the grid corners, for better accuracy, when possible.

t	$r(t)$
0	7
$\frac{21}{2}$	0

$-\frac{2}{3}t + 7 = 0$

$-\frac{2}{3}t = -7$

$t = -\left(\frac{3}{2}\right)(-7) = \frac{21}{2}$



A linear function is given.

8 $f(x) = 3x - 7$

(a) Sketch the graph.

(b) Find the slope of the graph. $m=3$

(c) Find the rate of change of the function.

$m=3$

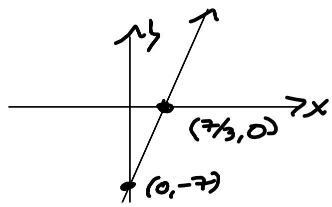
Notice for hand sketch, I plot the intercepts. For the WebAssign, I use the high-school method of using the -7 on the *y*-axis and find my 2nd point using the slope. That's because of how WebAssign tools work, not because I like it.

x	y
0	−7
$\frac{7}{3}$	0

$3x - 7 = 0$

$3x = 7$

$x = \frac{7}{3}$



9 A verbal description of a linear function f is given. Express the function f in the form $f(x) = ax + b$.

The graph of the linear function f has slope $-\frac{5}{6}$ and y-intercept -4.

$f(x) = ax + b = \boxed{-\frac{5}{6}x - 4 = f(x)}$

A table of values for a linear function f is given.

10

x	$f(x)$
−3	11
0	5
2	1
5	−5
7	−9

$= (x_1, y_1) = (x_1, f(x_1))$

$= (x_2, y_2) = (x_2, f(x_2))$

(Avg) Rate of change of f is $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

& Avg rate of change is the rate of change. For lines rate of change = average rate of change.

$= \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{5-11}{0-(-3)} = \dfrac{-6}{3} = -2$

(a) Find the rate of change of f. $= \boxed{m = -2}$

(b) Express f in the form $f(x) = ax + b$.

$f(x) = -2x + 5$

My generic way:

$y = m(x - x_1) + y_1$

$\boxed{y = -2(x - (-3)) + 11}$ → For hand-written work, I'd say STOP! You're done!

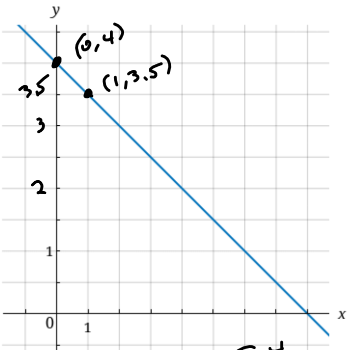
$= -2(x+3) + 11$

$= -2x - 6 + 11$

$= -2x + 5 = y$

The graph of a linear function f is given.

11



(a) Find the rate of change of f. $= \dfrac{3.5-4}{1-0} = \dfrac{-.5}{1} = -.5 = -1/2$

(b) Express f in the form $f(x) = ax + b$. $\boxed{f(x) = -\frac{1}{2}x + 4}$

A large koi pond is filled from a garden hose at the rate of 19 gal/min. Initially, the pond contains 300 gal of water.

12

(a) Find a linear function V that models the volume of water in the pond at any time t.

(b) If the pond has a capacity of 1,896 gal, how long (in min) does it take to completely fill the pond?

Lexicon

Let V = Volume of water in the pond (gal), as a function of t = time spent filling the pond (min)

(a) Given: $(0, V(0)) = (0, 300) = (x_1, y_1) = (t_1, V(t_1)) = (t_1, V_1)$

And, $m = 19 =$ "a" (Textbook)

$y = m(x - x_1) + y_1$

$= 19(t - t_1) + V_1$

$= 19(t - 0) + 300$

$= \boxed{19t + 300 = V(t)}$

(b) Want $V = 1896$ →

$19t + 300 = 1896$ →

$19t = 1596$ →

$t = \dfrac{1596}{19} = \boxed{84 = t}$

$\begin{array}{r} 1896 \\ -\ 300 \\ \hline 1596 \end{array}$

$\boxed{t = 84 \text{ min}}$

13

The manager of a furniture factory finds that it costs \$2,200 to produce 100 chairs in one day and \$4,400 to produce 300 chairs in one day.

(a) Assuming that the relationship between cost and the number of chairs produced is linear, find a linear function C that models the cost (in \$) of producing x chairs in one day.

(b) Draw a graph of C.

Let C = the cost in \$ of producing x = the number of chairs.

(a) we have $(x_1, C_1) = (100, 2200)$ as a function of x.
$(x_2, C_2) = (300, 4400)$

C = the cost in \$ of producing chairs as function of x = the # of chairs produced.

Assume linear relationship:

$m = \frac{C_2 - C_1}{x_2 - x_1} = \frac{4400 - 2200}{300 - 100} = \frac{2200}{200}$

$= \frac{22}{2} = 11$ \$/chair

$y = m(x - x_1) + y_1$

$= m(x - x_1) + C_1$

$= 11(x - 100) + 2200$

$= 11x - 1100 + 2200$

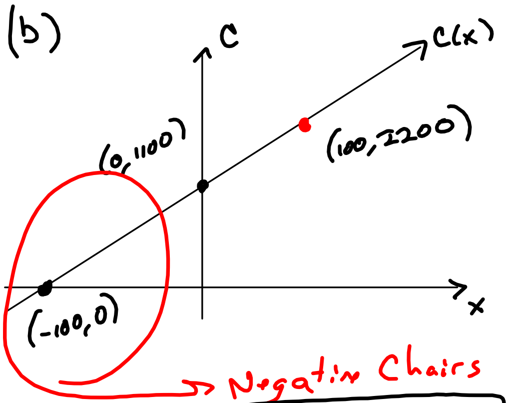
$= \boxed{11x + 1100 = C(x)}$

$11x + 1100 = 0$

$11x = -1100$

$x = \frac{-1100}{11} = -100$

(b)



Negative chairs make no sense.

(c) $\boxed{\text{slope is } 11 \text{ \$/chair}}$

(d) The rate at which the factory's cost increases for every chair produced is the MARGINAL COST, and it is simply the slope $m = \boxed{\$11 \text{ per chair} = 11 \frac{\$}{\text{chair}}}$

Suppose that $f(x) = ax + b$ is a linear function.

14 (a) Use the definition of the average rate of change of a function to calculate the average rate of change of f between any two real numbers x_1 and x_2.

(b) Use your calculation in part (a) to show that the average rate of change of f is the same as the slope a. (Simplify your final answer completely.)

(a) $m_{AVG} = m_{SEC} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{ax_2 + b - (ax_1 + b)}{x_2 - x_1}$

$= \dfrac{ax_2 + b - ax_1 - b}{x_2 - x_1} = \dfrac{a(x_2 - x_1)}{x_2 - x_1} = \boxed{a = m_{SEC}}$

(b) See (a).

Determine whether the given function is linear. If the function is linear, express the function in the form $f(x) = ax + b$. (If the function is not linear, enter NOT LINEAR.)

15 $f(x) = \dfrac{x+1}{2} = \dfrac{x}{2} + \dfrac{1}{2} = \dfrac{1}{2}x + \dfrac{1}{2}$