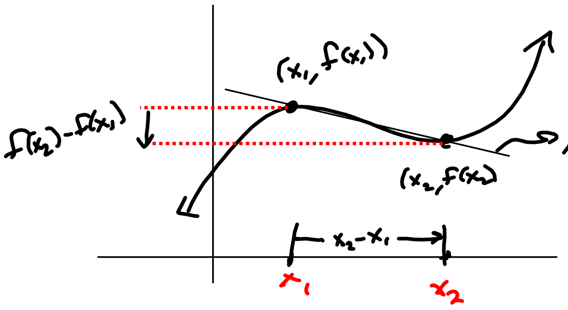
Average Rate of Change of a Function

$$\frac{f(x_2)-f(x_1)}{x_2-x_1}=\frac{y_2-y_1}{x_2-x_1}=\frac{f(b)-f(a)}{b-a}=\frac{\Delta y}{\Delta x}$$



Average Rate of Change is slope of the line segment from $(x_1, f(x_1))$ to $(x_2, f(x_2))$

The average rate of change of a function *f* between *x* = *a* and *x* = *g* is

2 average rate of change = $\frac{f(\boxed{g}) - f(\boxed{a})}{g-a}$.

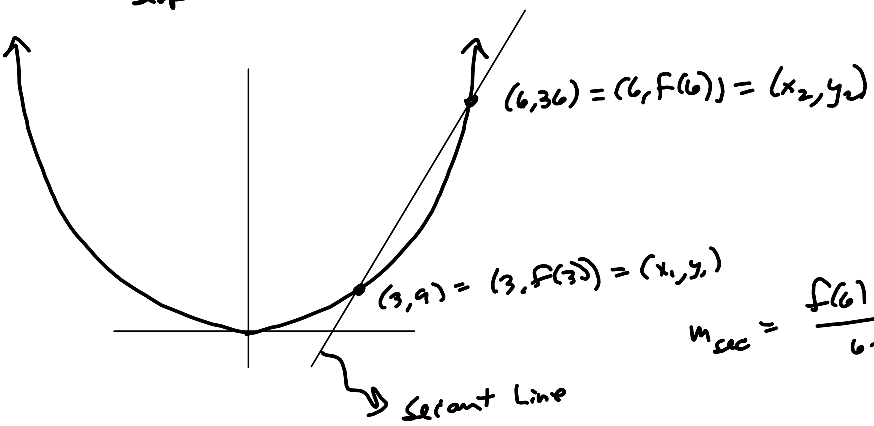
The average rate of change of a function *f*(*x*) = x^2 between *x* = 3 and *x* = 6 is

3 average rate of change = $\frac{\boxed{}}{6-3} = \boxed{}$.

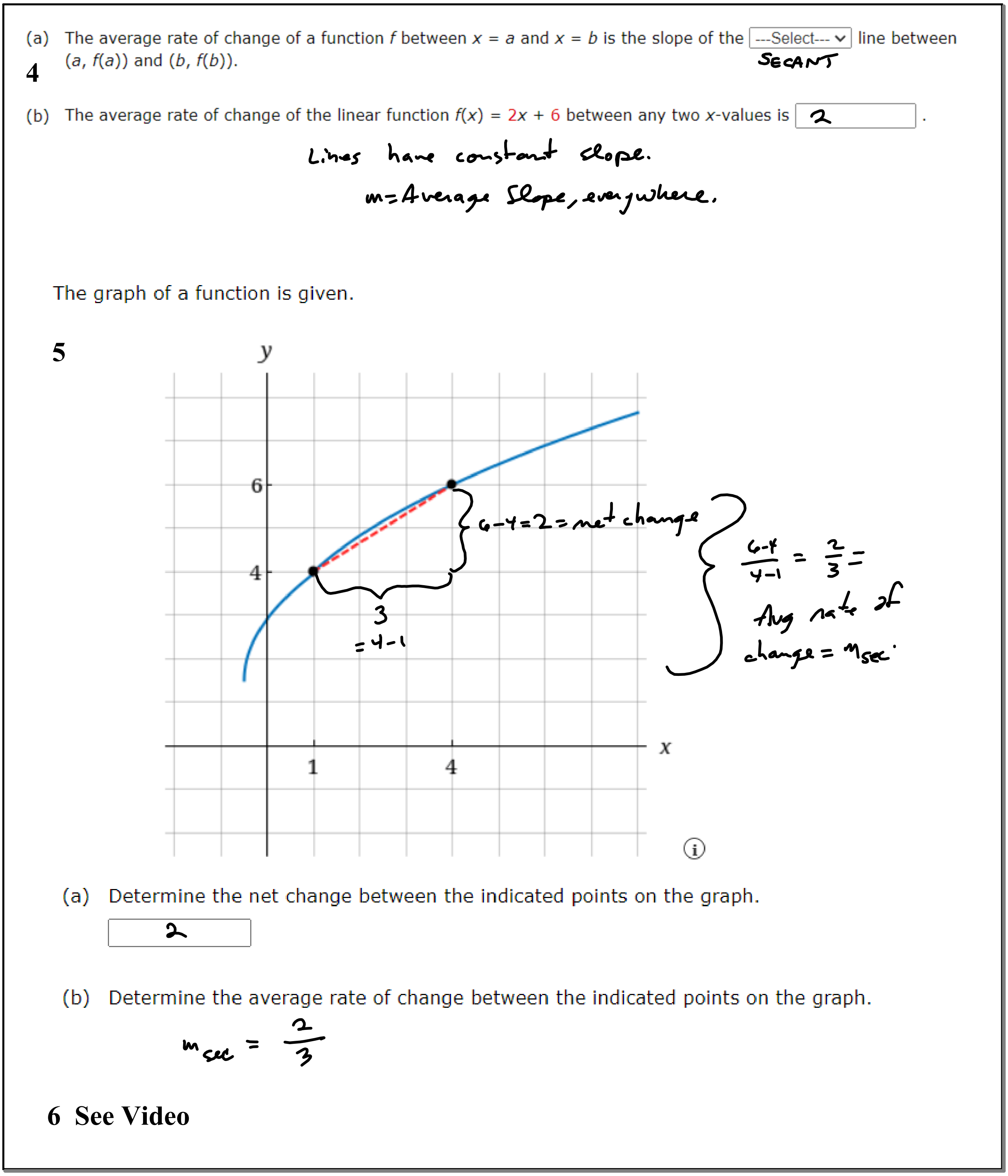
Avg rate of change between $x=3$ & $x=6$ of $f(x)=x^2$ is

$$m_{sec} = \frac{f(6)-f(3)}{6-3} = \frac{6^2-3^2}{3} = \frac{36-9}{3} = \frac{27}{3} = 9$$

slope of the secant line



$$m_{sec} = \frac{f(6)-f(3)}{6-3} = \frac{y_2-y_1}{x_2-x_1} \text{ etc}$$



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(a) The average rate of change of a function *f* between $x = a$ and $x = b$ is the slope of the ---Select--- line between $(a, f(a))$ and $(b, f(b))$.

SECANT

(b) The average rate of change of the linear function $f(x) = 2x + 6$ between any two *x*-values is 2 .

Lines have constant slope.

m=Average Slope, everywhere.

The graph of a function is given.

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$6-4=2=$ net change

3
$=4-1$

$\frac{6-4}{4-1} = \frac{2}{3} =$

Avg rate of change $= m_{sec}$.

(a) Determine the net change between the indicated points on the graph.

2

(b) Determine the average rate of change between the indicated points on the graph.

$m_{sec} = \frac{2}{3}$

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A function is given.

$f(x) = x^3 - 6x^2; \quad x = 0, x = 10$

(a) Determine the net change between the given values of the variable.

$$\frac{f(10)-f(0)}{10-0} = \frac{10^3-6(10)^2-0}{10} = \frac{1000-600}{10} = \frac{400}{10} = 40$$

(b) Determine the average rate of change between the given values of the variable.

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A function is given.

$g(t) = t^4 - t^3 + t^2; \quad t = -3, t = 3$

(a) Determine the net change between the given values of the variable.

(b) Determine the average rate of change between the given values of the variable.

This one's messy enough to want to break it into more steps.

$g(3) = 3^4 - 3^3 + 3^2 = 81 - 27 + 9 = 81 - 18 = 63$

$g(-3) = (-3)^4 - (-3)^3 + (-3)^2 = 81 + 27 + 9 = 117$

$g(3) - g(-3) = 63 - 117 = \boxed{-54 = \text{Net change}}$

(b) $\dfrac{g(3)-g(-3)}{3-(-3)} = \dfrac{-54}{6} = \boxed{-9 = \text{AVG RATE of change} = m_{sec}}$

Evaluating $g(-3)$ & $g(3)$

$t^4 - t^3 + t^2 + 0t + 0$

$$\begin{array}{r|rrrrr} -3 & 1 & -1 & 1 & 0 & 0 \\ & & -3 & 12 & -39 & 127 \\ \hline & 1 & -4 & 13 & -39 & \boxed{117 = g(-3)} \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 1 & -1 & 1 & 0 & 0 \\ & & 3 & 6 & 21 & 63 \\ \hline & 1 & 2 & 7 & 21 & \boxed{63 = g(3)} \end{array}$$

$$\begin{array}{r} {}^{2}39 \\ 3 \\ \hline 117 \end{array}$$

Synthetic Division for evaluating polynomials See C3.

$$\frac{g(3)-g(-3)}{3-(-3)} = \frac{63-117}{6} = \frac{-54}{6} = -9$$

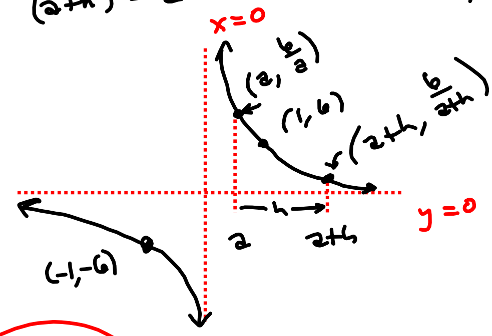
A function is given. Find an expression for the difference quotient (or average rate of change) between $x = a$ and $x = a + h$. Simplify your answer.

9 $g(x) = \frac{6}{x}$

Average Rate of Change between $x=a$ & $x=a+h$:

$$\frac{g(a+h)-g(a)}{(a+h)-a} = \frac{\frac{6}{a+h} - \frac{6}{a}}{h} = \frac{1}{h}\left[\frac{6}{a+h}\cdot\left(\frac{a}{a}\right) - \frac{6}{a}\cdot\left(\frac{a+h}{a+h}\right)\right]$$

$$\boxed{LCD = a(a+h)}$$



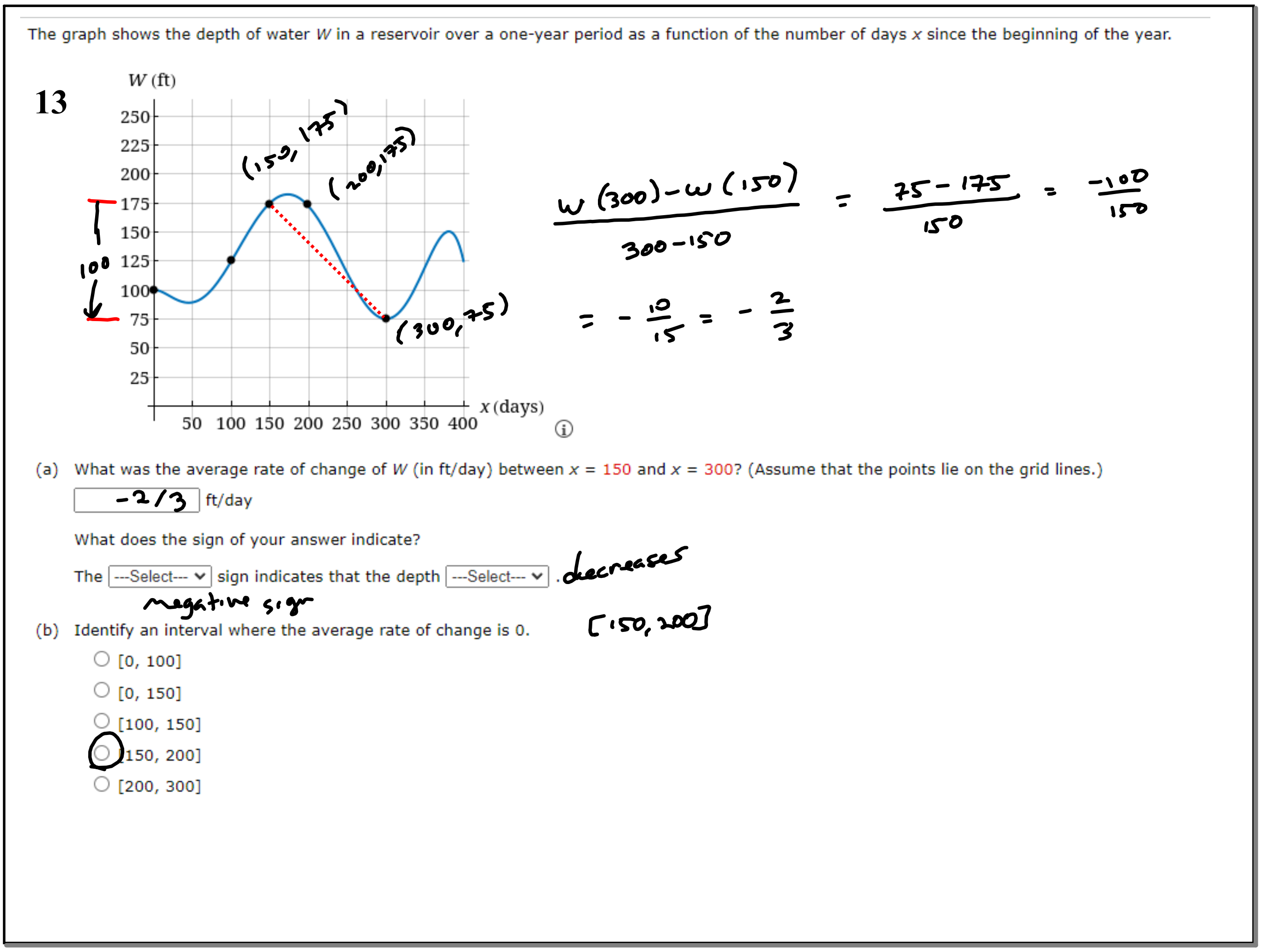
$$= \frac{1}{h}\left[\frac{6a-6(a+h)}{LCD}\right]$$

$$= \frac{1}{h}\left[\frac{6a-6a-6h}{LCD}\right]$$

$$= \frac{1}{h}\left[\frac{-6h}{a(a+h)}\right] = \boxed{\frac{-6}{a(a+h)} = m_{sec}}$$

Click Here!

#12 See Video



The graph shows the depth of water W in a reservoir over a one-year period as a function of the number of days x since the beginning of the year.

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$$\frac{W(300)-W(150)}{300-150} = \frac{75-175}{150} = \frac{-100}{150}$$

$$= -\frac{10}{15} = -\frac{2}{3}$$

(a) What was the average rate of change of W (in ft/day) between x = 150 and x = 300? (Assume that the points lie on the grid lines.)

-2/3 ft/day

What does the sign of your answer indicate?

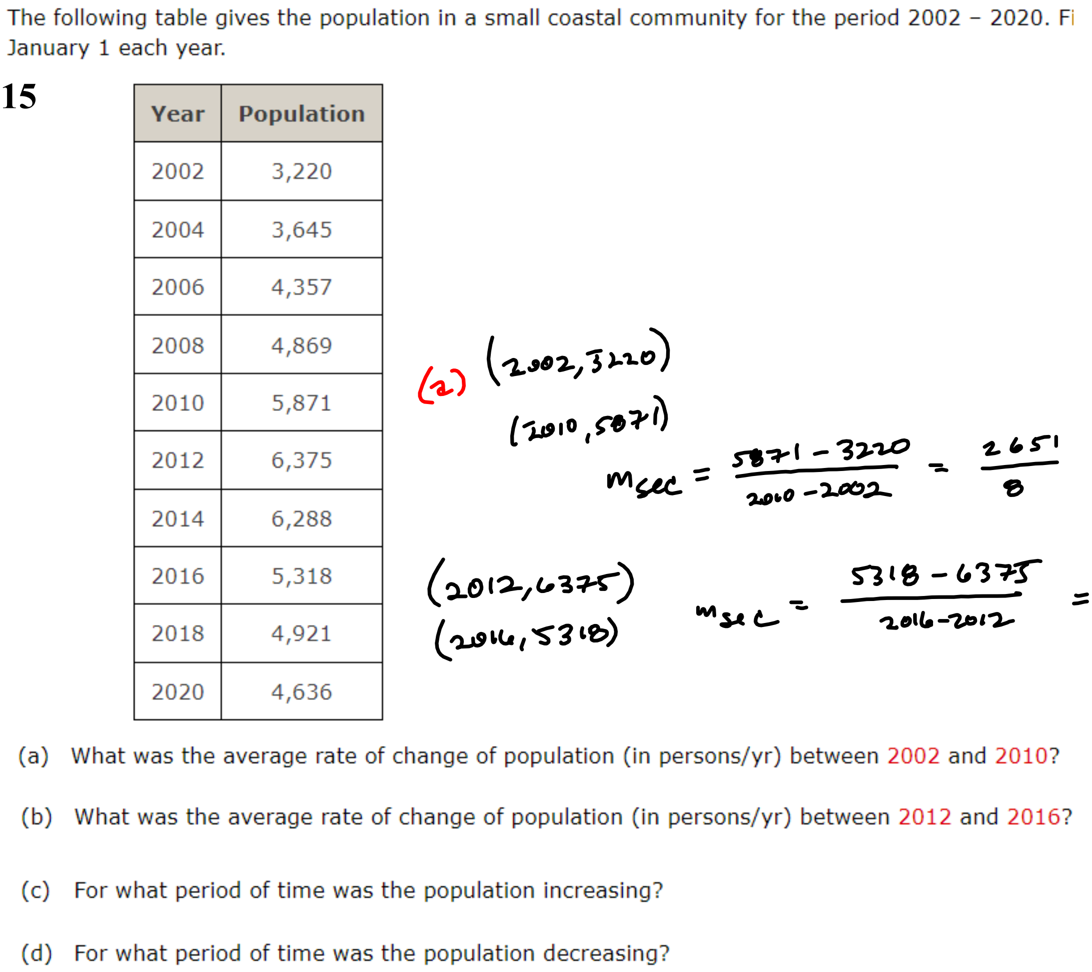
The ---Select--- sign indicates that the depth ---Select--- . decreases

negative sign

(b) Identify an interval where the average rate of change is 0. [150, 200]

○ [0, 100]
○ [0, 150]
○ [100, 150]
● [150, 200]
○ [200, 300]

14 See Video



The following table gives the population in a small coastal community for the period 2002 – 2020. Fi
January 1 each year.

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Year	Population
2002	3,220
2004	3,645
2006	4,357
2008	4,869
2010	5,871
2012	6,375
2014	6,288
2016	5,318
2018	4,921
2020	4,636

(a) (2002, 3220)
(2010, 5871)

$$m_{sec} = \frac{5871-3220}{2010-2002} = \frac{2651}{8}$$

(2012, 6375)
(2016, 5318)

$$m_{sec} = \frac{5318-6375}{2016-2012} = \frac{1057}{4}$$

(a) What was the average rate of change of population (in persons/yr) between 2002 and 2010?

(b) What was the average rate of change of population (in persons/yr) between 2012 and 2016?

(c) For what period of time was the population increasing?

(d) For what period of time was the population decreasing?