Section 2.2 Graph of a Function

THE GRAPH OF A FUNCTION

If f is a function with domain A, then the **graph** of f is the set of ordered pairs

$$\{(x, f(x)) \mid x \in A\}$$

plotted in a coordinate plane. In other words, the graph of f is the set of all points (x, y) such that $y = f(x)$; that is, the graph of f is the graph of the equation $y = f(x)$.

The standard way of doing this, when you're ignorant, is to just plotting points and connecting the dots.

Our goal is to give you a MUCH better intuition on what a large family of *basic functions* looks like, and build off from that, by stretching, shrinking, reflecting, and shifting left-right or up-down.

We want you to have an idea of what things look like before you just blindly plug in points and plot them one by one.

For Writing Project #2, I have made videos describing quite a few basic functions. I like what your book does, to an extent. Some nice grahics, there.

Writing Project Videos

Method 1: Plotting Points in Ignorance. The worst way.

We've done the drill and kill.

Now let's build our skill.

Method 2: Graphing Utility.

Zoom Out for major features.

Zoom In for detail.

Desmos is a good website.

Graphing Calculator (I use TI-84 in my demonstrations)

Our Main Method, this chapter:

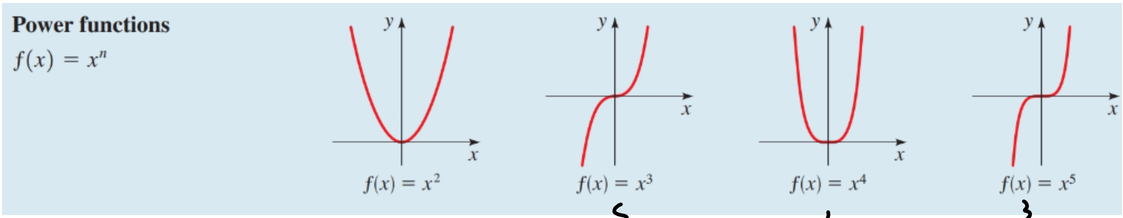
Transformations on Basic functions to build New Functions.

click

Writing Project Videos

Aiming a little too much at WP#2, for as early in the Chapter as we are.

Really, 2.6 is where we put it together, but the more of the rote memory you've got down, the smoother the later stuff will go, so I'm referring you to the basic functions about 4 sections prematurely.



x^{2n} $(x^2, x^4, \dots)$

End Behavior

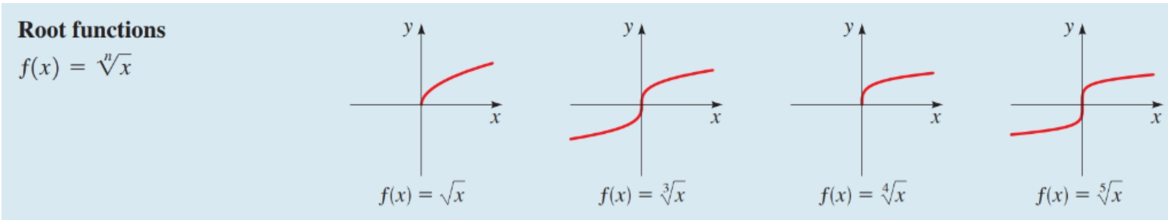
+ +

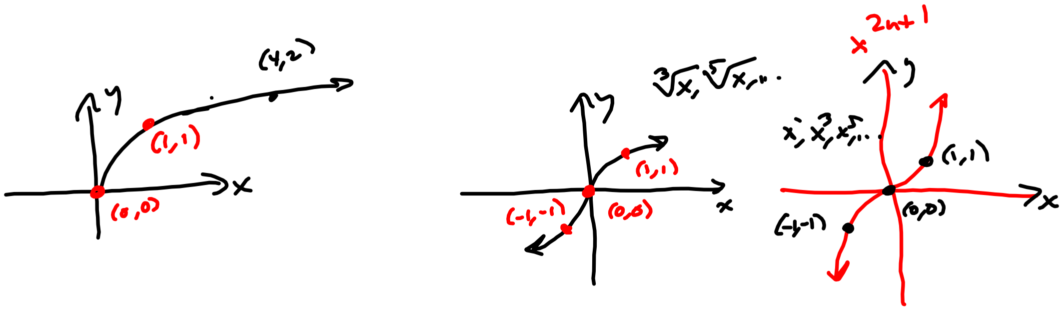
Sign Pattern off @ the edges

x^{2n+1} $(x, x^3, x^5, \dots)$

− +

Sign Pattern off @ the edges





1 $\sqrt[3]{x}$

2 $\sqrt[4]{x}$

3 x^3

To graph the function f, we plot the points $(x,$ --?-- $)$ in a coordinate plane.
$f(x)$

1 To graph $f(x) = x^2 - 5$, we plot the following points.

- ○ $(x, 2x)$
- ○ $(x, x^2 - 5)$
- ○ $(x, 1)$
- ○ $(x, x - 3)$
- ○ $(x, 0)$

$f(4) = 4^2 - 5 = 16 - 5 = 11$

So the point $(4,$ $\boxed{f(4)}$ $)$ is on the graph of f. The height of the graph of f above the x-axis when $x = 4$ is

$4?$

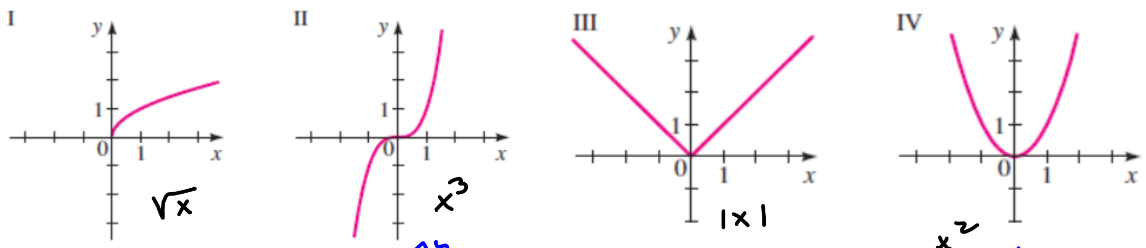
$f(4) = 11$

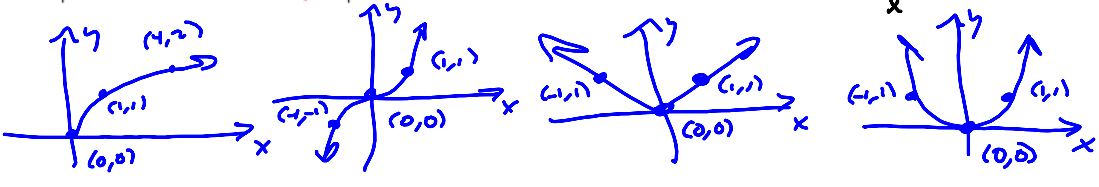
2 If $f(7) = 19$, then the point $(7,$ $\boxed{19}$ $)$ is on the graph of f.

3 If the point $(6, 8)$ is on the graph of f, then $f(6) = 8 \rightsquigarrow (6,8)$ on graph

Match the function with its graph.

4





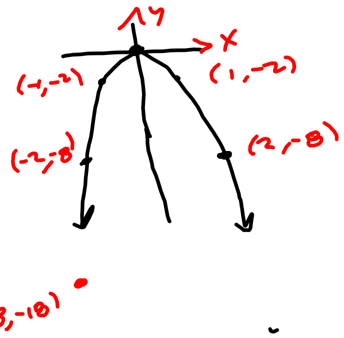
Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

5 $f(x) = -2x^2$

x	$f(x) = -2x^2$
±4	-32
±3	-18
±2	-8
±1	-2
0	0

Twice as tall and upside-down!

Even Function $f(-x) = f(x)$



Don't let the quantitative obscure the qualitative!

Sketch the graph.

Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

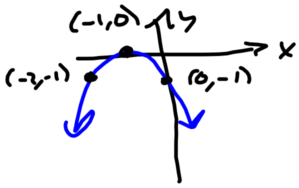
6 $g(x) = -(x + 1)^2$

x	y
-1	0
-2	-1
-3	-4
0	-1
1	-4

$-(-2+1)^2 = -(-1)^2 = -1$

$-(-3+1)^2 = -(-2)^2 = -4$

$-(1+1)^2 = -(2)^2 = -4$



7

Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

$r(x) = 5x^4$

x	$r(x) = 5x^4$
−3	
−2	
−1	
0	
1	
2	
3	

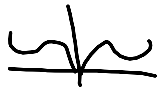
Sketch the graph.

$y = 5x^4$

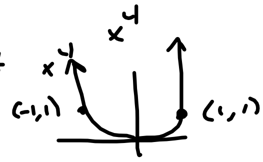
x	$5x^4$
0	0
±1	5
±2	80
±3	405

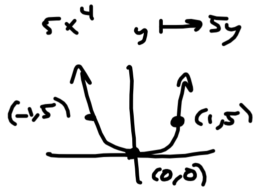
$y = 5(-x)^4 = 5x^4$ symmetric about y-axis. EVEN FUNCTION.

$f(-x) = f(x)$



List

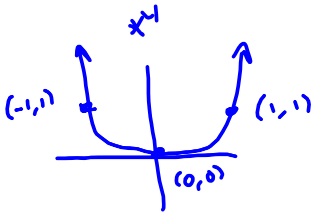


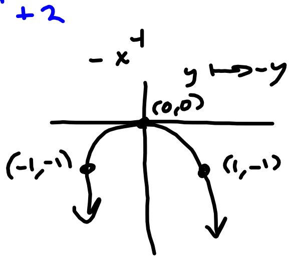


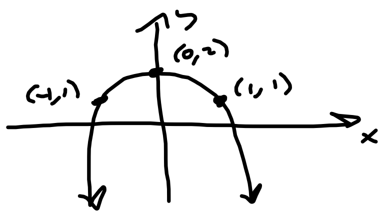
8

Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

$r(x) = 2 - x^4 = -x^4 + 2$

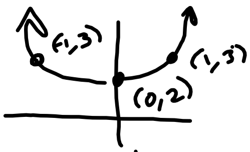






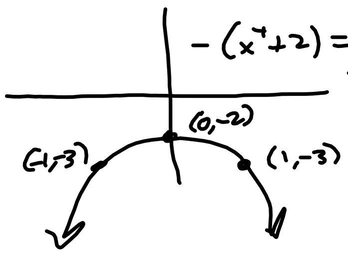
The order of moves matters

Move up 2



Flip:

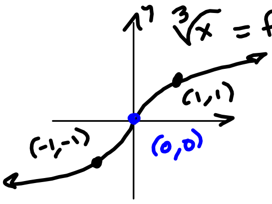
$-(x^4+2) = -x^4 - 2$ wrong!



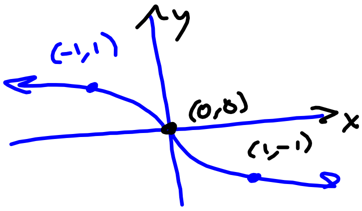
Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

9 $k(x) = \sqrt[3]{-x}$

$\sqrt[3]{x} = f(x) \Longrightarrow K(x) = f(-x)$ = Flipped about y-axis.



$x \longmapsto -x$



$f(x) = \sqrt{x}$ is ODD FUNCTION.

$f(-x) = -f(x)$

$f(-1) = -1$

$f(1) = 1$

Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

10 $k(x) = -\sqrt[3]{x}$

This is identical to #9!

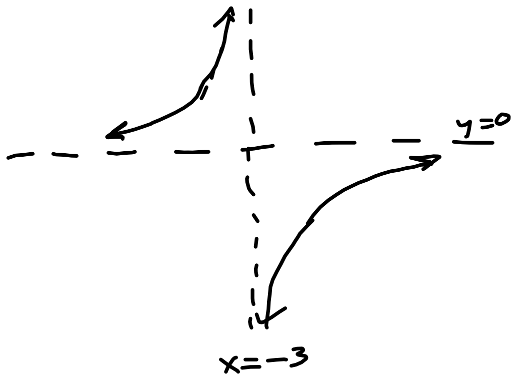
Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

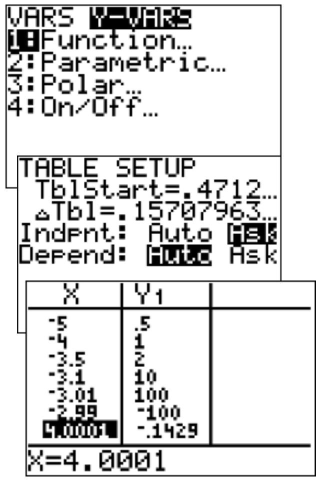
11 $C(t) = -\dfrac{1}{t+3}$

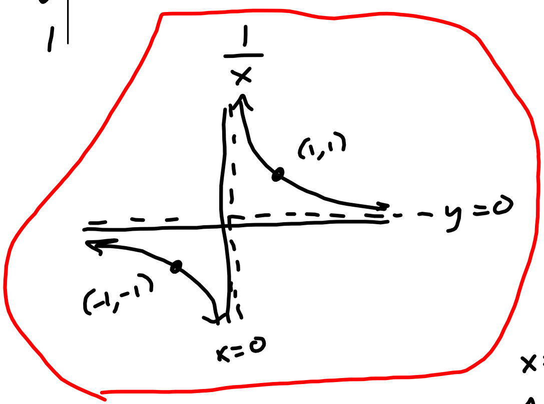
$\mathcal{D} = \mathbb{R} \setminus \{-3\}$

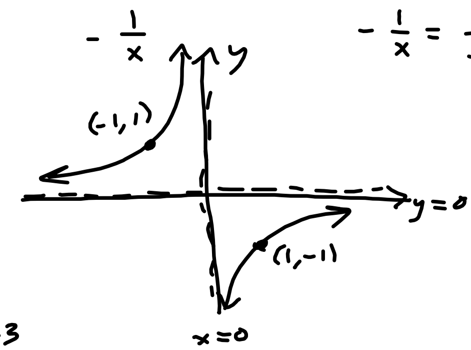
Technique: $x=-3$ has issues

x	y
-5	
-4	
-3.5	
-3.1	
-3.01	
-2.99	
-2.9	
-2.5	
-1	
0	
1	



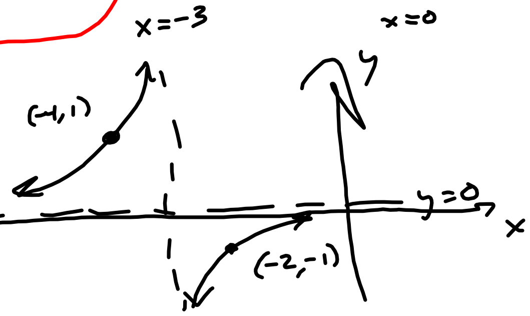






$-\frac{1}{x} = \frac{1}{-x}$

$x \mapsto x-3$ $\quad -\frac{1}{x+3}$

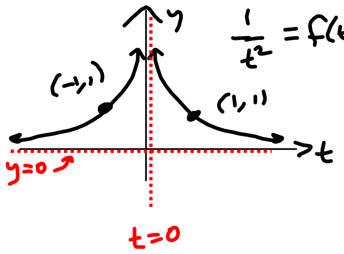


Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

12 $C(t) = \frac{6}{t^2}$

t	$C(t) = \frac{6}{t^2}$
-2	
-1	6
$-\frac{1}{2}$	24
$-\frac{1}{4}$	80
0	UNDEFINED
$\frac{1}{4}$	90
$\frac{1}{2}$	24
1	6
2	

Reciprocal Squared



$\frac{1}{t^2} = f(t)$

$\mathcal{D} = \mathbb{R} \setminus \{0\}$

Need $t^2 \neq 0$

$t = 0$

t	$\frac{1}{t^2}$
-1	1
$-\frac{1}{2}$	4
$-\frac{1}{4}$	16
0	$\nexists$
$\frac{1}{4}$	16
$\frac{1}{2}$	4
1	1

$\frac{1}{(-1)^2} = 1$

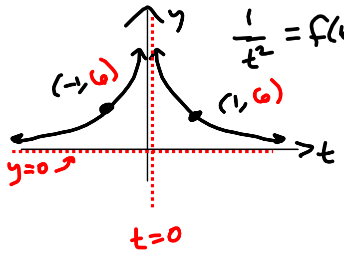
$\frac{1}{(-\frac{1}{2})^2} = 2^2 = 4$

$f(-t) = 16$

6 times the y's from this chart

$c(t) = \frac{6}{t^2} = 6f(t)$

$y \mapsto 6y$



$\frac{1}{t^2} = f(t)$

Sketch the graph.

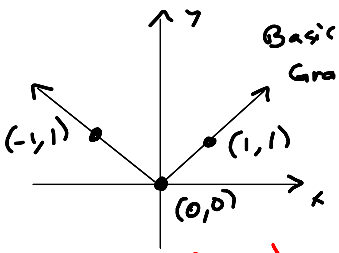
Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

13

$H(x) = |7x|$

$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Basic Function Graph.



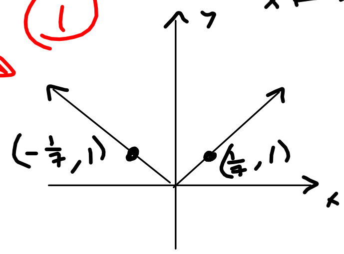
Working off this for both of these

2 ways to view this

$H(x) = |7x| = f(7x)$

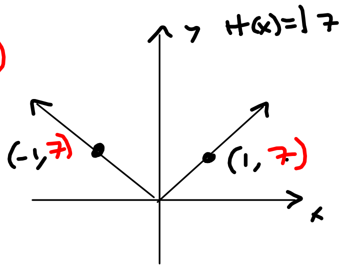
1

$x \longmapsto \frac{1}{7}x$



2

$H(x) = |7x| = 7|x| = 7f(x)$



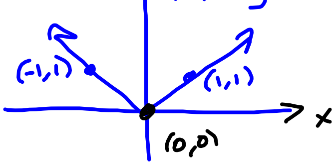
Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

14 $f(x) = |5x - 5| = \underline{5|x-1|}$!

$\to 5|x-1| = f(x) = 5g(x-1)$

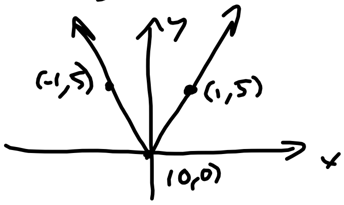
$y \mapsto 5y \qquad x \mapsto x+1$

$|x| = g(x)$



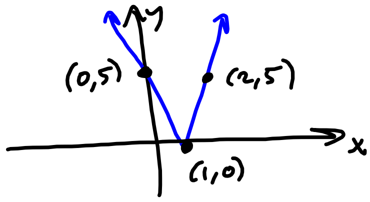
$y \mapsto 5y$

$5|x| = 5g(x)$



$x \to x+1$

$5|x-1| = f(x) = 5g(x-1)$



Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

15 $f(x) = \dfrac{5x}{|5x|}$

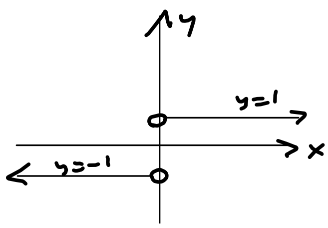
$\mathcal{D}: |5x| \neq 0 \Rightarrow$

$5x \neq 0 \Rightarrow$

$x \neq 0 \Rightarrow \mathcal{D} = \mathbb{R} \setminus \{0\}$

$$= \begin{cases} \frac{5x}{5x} & \text{if } 5x > 0 \\ \frac{5x}{-5x} & \text{if } 5x < 0 \end{cases}$$

$$= \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$



This is not at all like the book, but that's OK. I'm trying to let you see the bigger stuff, and not get bogged in drill and kill.

Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

16 $G(x) = |x| - x$

Break it into its two pieces, like #15, and it's a lot simpler to see.

A graphing device is recommended.

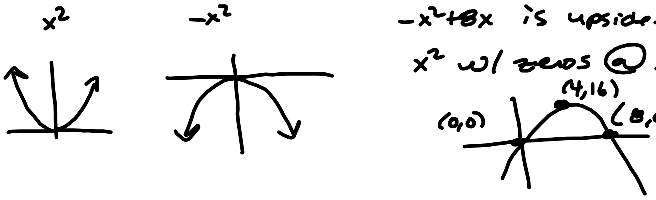
A function f is given.

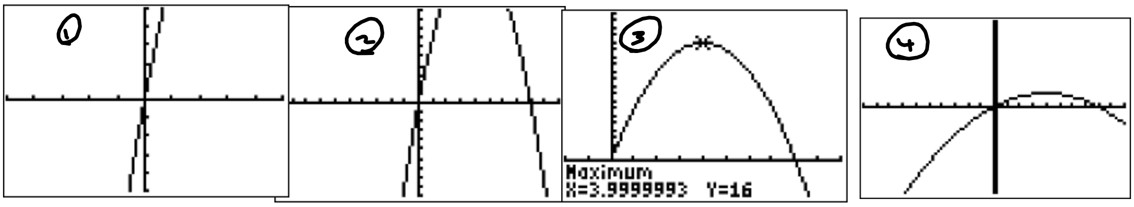
17 $f(x) = 8x - x^2 = -x^2+8x = -x(x-8) \overset{\text{SET}}{=} 0 \Rightarrow x \in \{0, 8\}$

Graph the function in each of the given viewing rectangles. Select the viewing rectangle that produces the most appropriate graph of the function.

- [−5, 5] by [−5, 5]
- [−10, 10] by [−10, 10]
- [−2, 10] by [−5, 20]
- [−10, 10] by [−100, 100]

$-x^2+8x$ is upside-down x^2 w/ zeros @ $x=0,8$.





BEST

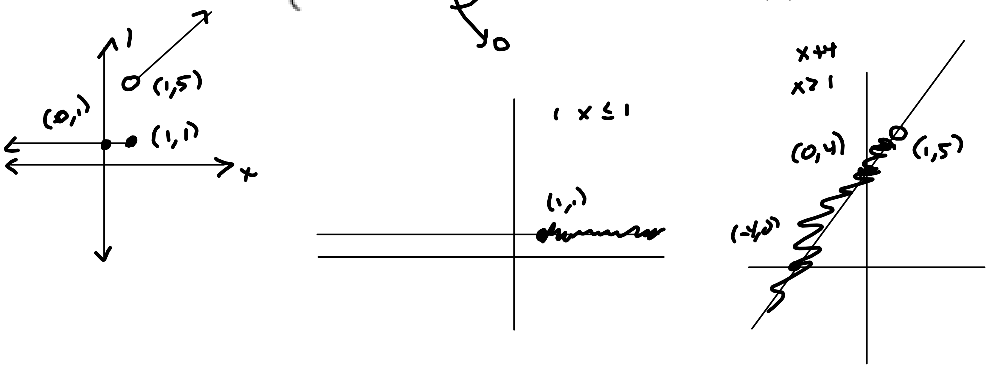
Sketch a graph of the piecewise-defined function.

18 $f(x) = \begin{cases} 1 & \text{if } x \le 1 \\ x + 4 & \text{if } x > 1 \end{cases}$

• (1,1)

1+4=5 o (1,5)

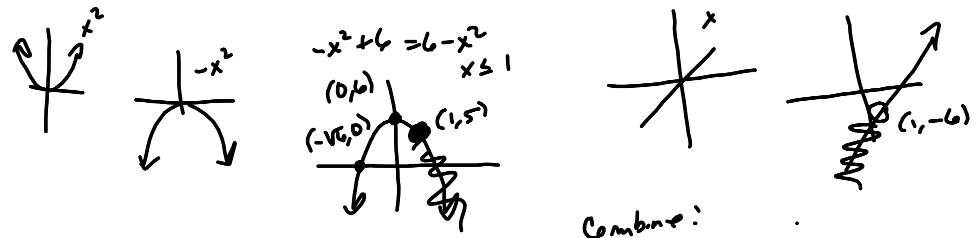
o



Sketch a graph of the piecewise-defined function.

19

$$f(x) = \begin{cases} 6 - x^2 & \text{if } x \le 1 \quad (1,5)\ \bullet \\ x - 7 & \text{if } x > 1 \quad (1,-6)\ \circ \end{cases}$$

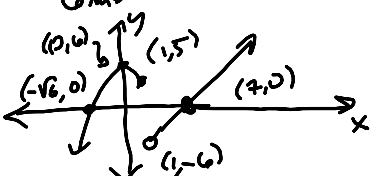


$6 - x^2 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

Combine:



Sketch a graph of the piecewise-defined function.

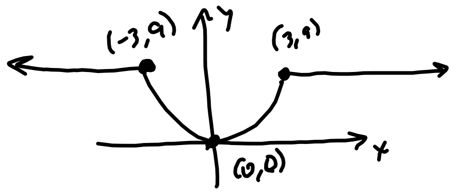
20

$$f(x) = \begin{cases} x^2 & \text{if } |x| \le 3 \\ 9 & \text{if } |x| > 3 \end{cases} = \begin{cases} x^2 & \text{if } -3 \le x \le 3 \\ 9 & \text{if } \boxed{-3 > x > 3 \text{ is BAD!}} \end{cases}$$

means $x \le 3$ AND $x \ge -3$

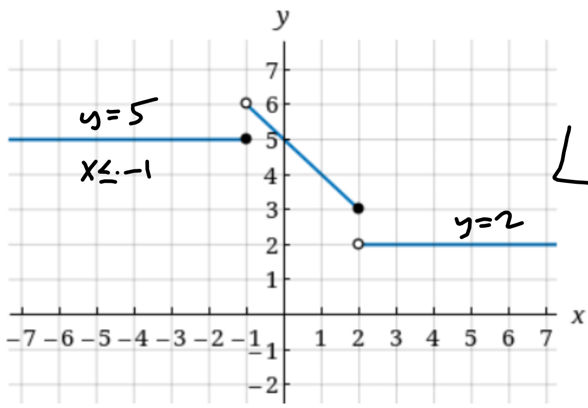
understood to be an "and"

$$= \begin{cases} x^2 & \text{if } -3 \le x \le 3 \quad (\pm 3, 9)\ \bullet \\ 9 & \text{if } x > 3 \text{ OR } x < -3 \quad (\pm 3, 9)\ \circ \end{cases}$$



A graph of a piecewise-defined function is given.

22



Point-Slope:

Book: $y - y_1 = m(x - x_1)$

Me: $y = y_1 + m(x - x_1)$

$\boxed{\text{Me 4 real: } y = m(x - x_1) + y_1}$ Best

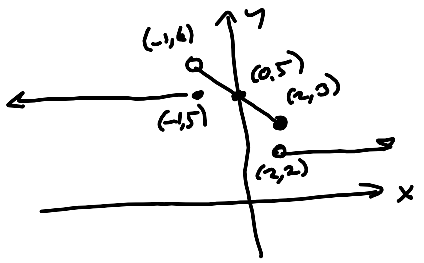
$$f(x) = \begin{cases} 5 & x \le -1 \\ -x+5 & -1 < x \le 2 \\ 2 & x > 2 \end{cases}$$

o $(-1, 6)$ $\quad -1 < x \le 2$

• $(2, 3)$

$$m = \frac{6-3}{-1-2} = \frac{3}{-3} = -1$$

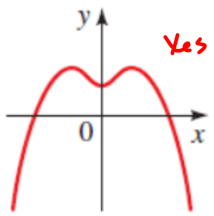
$$\begin{aligned} y &= m(x - x_1) + y_1 \\ &= -1(x - (-1)) + 6 \\ &= -x - 1 + 6 \\ &= -x + 5 \end{aligned}$$



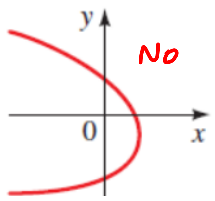
Use the Vertical Line Test to determine whether the curve is the graph of a function of *x*.

23

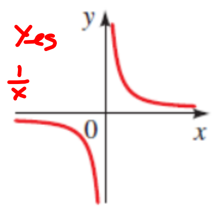
(a)



(b)



(c)



To be a function *f* must map each *x* in its domain to *exactly* one *y* in the range. Hence, the Vertical Line Test.

Consider the following equation.

25

$x = y^8$

Find two distinct values of y that satisfy the equation for the same value of x. (Enter your answers as a comma-separated list. If no such values exist, enter DNE.)

$-1, 1$

Determine whether the equation defines y as a function of x.

$x = y^8$

No. We have $y = \pm 1$ assigned to $x = 1$

$f(1) = 1\,?$ or $-1\,?$ Not function.

y is not a function of x.

$y^8 = x$

$\sqrt[8]{y^8} = \sqrt[8]{x}$

$|y| = \sqrt[8]{x}$

$y = \pm\sqrt[8]{x}$

$x = 1: \quad y = \pm 1$

$x = |y|$ another non-function

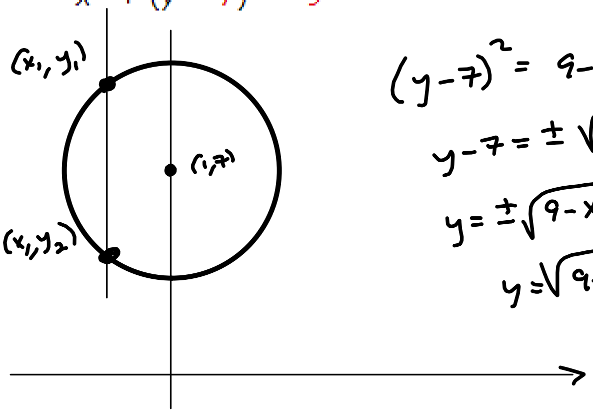
$x = y^2$

$x = y^{2n}$ Not Func.

Determine whether the equation defines y as a function of x.

26

$x^2 + (y-7)^2 = 9$



No. It's 2 functions in a way:

$(y-7)^2 = 9 - x^2$

$y - 7 = \pm\sqrt{9 - x^2}$

$y = \pm\sqrt{9 - x^2} + 7$

$y = \sqrt{9 - x^2} + 7$ Top half of circle.

Domain: Need $9 - x^2 \ge 0$

$(3-x)(3+x) \ge 0$

$(3-4)(3+4) = (-)(+) = -$

N Y Y Y N

$-$ $+$ $-$

-3 3

$=0$ $=0$

$D = [-3, 3]$

Determine whether the equation defines y as a function of x.

27 $\sqrt{y} - x = 6$

$\sqrt{y} = x + 6$

$\sqrt{y}^2 = (x+6)^2$

$y = (x+6)^2$

Yeah!

Determine whether the equation defines y as a function of x.

28 $8|x| + y = 0$

$y = -8|x|$

Yeah!

29 (a) Draw graphs of the functions

$f(x) = x^2 + x - 5$

and $g(x) = |x^2 + x - 5|$.

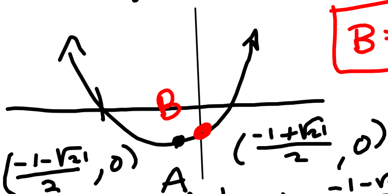
$a=1, b=1, c=-5$

One way:
① Find x-intercepts. Logic
② Complete the square & view as a transformed x^2.
③ Stuff into a grapher

$b^2-4ac = 1^2-4(1)(-5) = 21$

$x = \frac{-1 \pm \sqrt{21}}{2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\boxed{B = (0,-5)}$

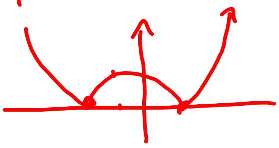


Vertex : $\dfrac{\frac{-1-\sqrt{21}}{2} + \frac{-1+\sqrt{21}}{2}}{2} = \dfrac{\frac{-2}{2}}{2} = \frac{-1}{2}$

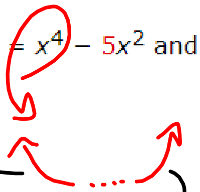
$\left(-\frac{1}{2}\right)^2 + \frac{1}{2} - 5$
$= \frac{1}{4} + \frac{2}{4} - \frac{20}{4}$
$= -\frac{17}{4}$

$\boxed{\left(-\frac{1}{2}, -\frac{17}{4}\right) = A}$

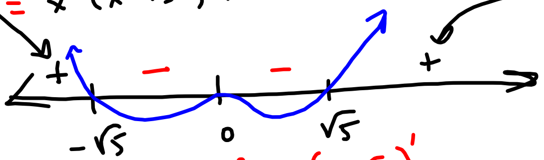
$|x^2+x-5|$



(b) Draw graphs of the functions $f(x) = x^4 - 5x^2$ and $g(x) = |x^4 - 5x^2|$.

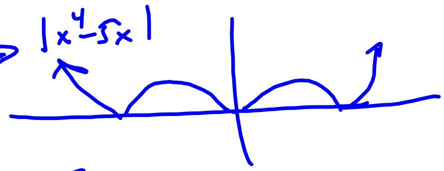


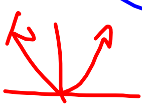
$x^4 - 5x^2$
$= x^2(x^2-5)$
$= x^2(x-\sqrt{5})(x+\sqrt{5})$



x^2 $(x-\sqrt{5})^1$

$\Rightarrow$ $|x^4-5x|$



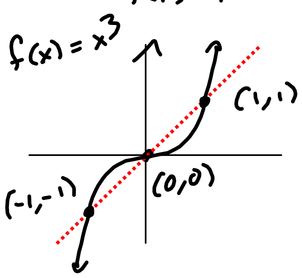


The even power says that the sign doesn't change as we move across $x=0$ boundary

31 Sketch the graph of the function by first making a table of values.

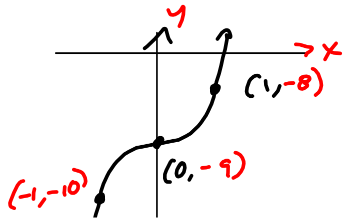
$g(x) = x^3 - 9$

$f(x) = x^3$ = Basic Function



$g(x) = x^3 - 9 = f(x) - 9$

$y \mapsto y - 9$



32 Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

$g(x) = (x - 1)^3$ x^3, right 1 unit

$f(x) = x^3$

$g(x) = f(x-1)$

33 Sketch the graph of the function by first making a table of values.

$f(x) = 4 + \sqrt{x} = \sqrt{x} + 4$

$g(x) = \sqrt{x} \implies f(x) = g(x) + 4$

$\sqrt{x}$ up 4

34 Sketch the graph of the function by first making a table of values. (If an answer is undefined, enter UNDEFINED.)

$f(x) = \sqrt{x - 4}$

$g(x) = \sqrt{x}$ is basic $\longrightarrow f(x) = \sqrt{x-4} = g(x-4)$ Right 4 units

DELAY by 4 units