1 The solutions of the equation $x^2 - 4x - 5 = 0$ are the ---Select--- -intercepts of the graph of $y = x^2 - 4x - 5$.

x-

https://www.desmos.com/calculator

2 The solutions of the inequality $x^2 - 4x - 8 > 0$ are the x-coordinates of the points on the graph of $y = x^2 - 4x - 8$ that lie ---Select--- the x-axis.

Above

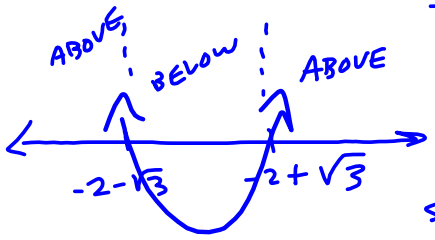
+ABOVE 0

$x^2-4x-8 = x^2-4x+2^2-4-8 = (x-2)^2-12 = 0$

$\Rightarrow (x-2)^2 = 12$

$x-2 = \pm\sqrt{12} = \pm 2\sqrt{3}$

$x = -2 \pm \sqrt{3}$

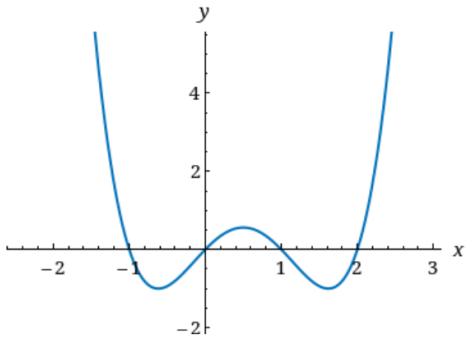


x^2-4x-8

So $x^2-4x-8 > 0$

$\Rightarrow x \in (-\infty, -2-\sqrt{3}) \cup (-2+\sqrt{3}, \infty)$

3 The figure shows a graph of $y = x^4 - 2x^3 - x^2 + 2x$.

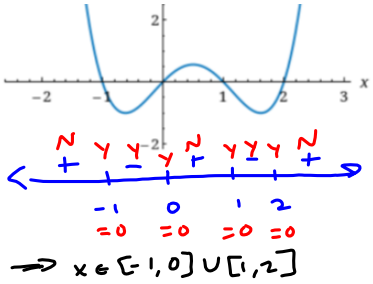


Use the graph to find the solution(s) of each of the following.

(a) The equation $x^4 - 2x^3 - x^2 + 2x = 0$ (Enter your answers as a comma-separated list.)

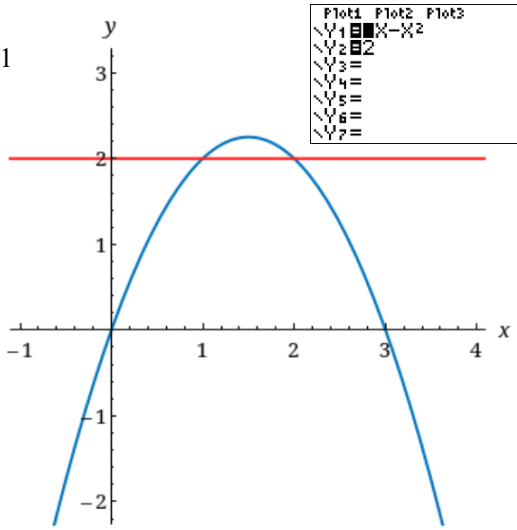
$x = -2, 0, 1, 2$

(b) The inequality $x^4 - 2x^3 - x^2 + 2x \le 0$ (Enter your answer using interval notation.)

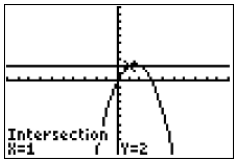


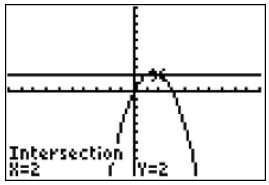
4 The figure shows the graph of $y = 3x - x^2$ and $y = 2$.

Method 1

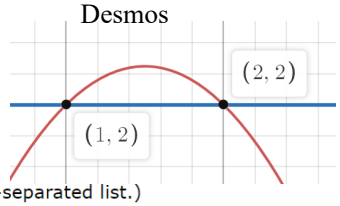


Graphing Calculator





Desmos

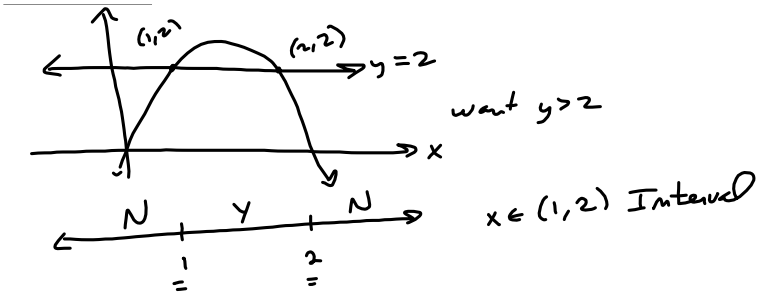


Use the graph to find the solution(s) of each of the following.

(a) The equation $3x - x^2 = 2$ (Enter your answers as a comma-separated list.)

$x = 1, 2$

(b) The inequality $3x - x^2 > 2$ (Enter your answer using interval notation.)



4 The figure shows the graph of $y = 3x - x^2$ and $y = 2$.

Method 2

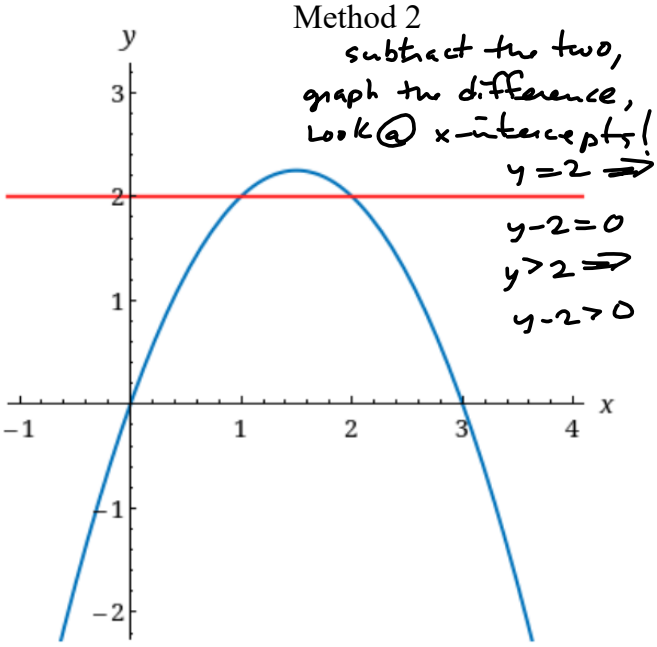
subtract the two, graph the difference, look @ x-intercepts!

$y = 2 \Rightarrow$

$y - 2 = 0$

$y > 2 \Rightarrow$

$y - 2 > 0$



Use the graph to find the solution(s) of each of the following.

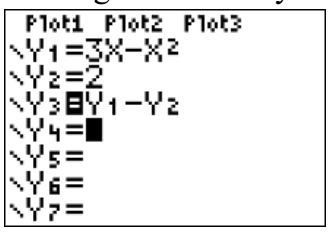
(a) The equation $3x - x^2 = 2$ (Enter your answers as a comma-separated list.)

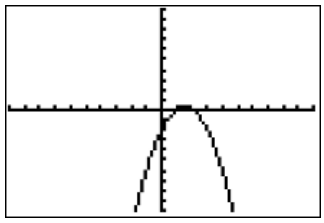
(b) The inequality $3x - x^2 > 2$ (Enter your answer using interval notation.)

Here we used "ZERO" in CALC menu, instead of "INTERSECT."

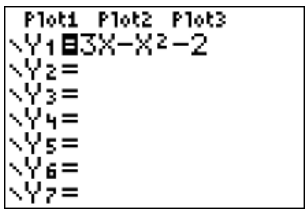
Just find the x-intercepts

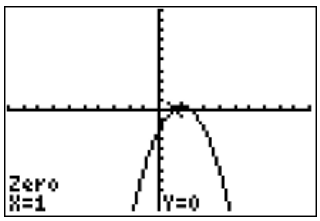
Using "VARS" key.



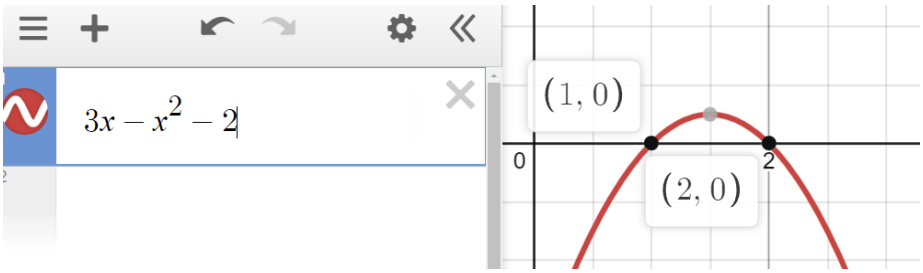


Or just subtract the 2





etc



An equation is given.

5 $y = 7x^3 - 7x^2$; $[-2, 2]$ by $[-1, 1]$ $7x^3-7x^2 = 7x^2(x-1) = 0 \Rightarrow x \in \{0, 1\}$

(a) Use a graphing device to graph the equation in the given viewing rectangle. Find the x- and y-intercepts from the graph and confirm your answers algebraically (from the equation). (If an answer does not exist, enter DNE.)

x-intercept (smaller x-value) $(x, y) =$ (0,0)

x-intercept (larger x-value) $(x, y) =$ (1,0)

y-intercept $(x, y) =$ (0,0)

(b) If the graph appears to be symmetric, confirm that the equation satisfies the corresponding symmetry property. (Select all that apply.)

- ☐ The graph is symmetric with respect to the x-axis.
- ☐ The graph is symmetric with respect to the y-axis.
- ☐ The graph is symmetric with respect to the origin.
- ☐ The graph is not symmetric with respect to the x-axis, y-axis, or the origin.

Try $y = 7x^3 - 7x$

$7(-x)^3 - 7(-x) = -7x^3 + 7x$

$= -(7x^3 - 7x) = -y$

Symmetric thru origin

ODD

Check for symmetry

$y = 7x^3 - 7x^2$

swap x for "$-x$"

$7(-x)^3 - 7(-x)^2 = -7x^3 - 7x^2 \neq y$ or $-y$.

No symmetry

$y = 7x^4 - 7x^2$

$7(-x)^4 - 7(-x)^2$

$= 7x^4 - 7x^2$

$= y$

symmetric w.r.t y-axis

EVEN

6 An equation is given.

$y = x^4 - 2x^3$; $[-2, 3]$ by $[-3, 3]$ $x^3(x-2)$

(a) Use a graphing device to graph the equation in the given viewing rectangle. Find the x- and y-intercepts from the graph and confirm your answers algebraically (from the equation). (If an answer does not exist, enter DNE.)

x-intercept (smaller x-value) $(x, y) =$ (0,0) ✓

x-intercept (larger x-value) $(x, y) =$ (2,0) ✓

y-intercept $(x, y) =$ (0,0) ✓

(b) If the graph appears to be symmetric, confirm that the equation satisfies the corresponding symmetry property. (Select all that apply.)

- ☐ The graph is symmetric with respect to the x-axis.
- ☐ The graph is symmetric with respect to the y-axis.
- ☐ The graph is symmetric with respect to the origin.
- ☑ The graph is not symmetric with respect to the x-axis, y-axis, or the origin. ✓

7 An equation is given.

$$y = -\frac{8}{x^2 + 1}; \quad [-5, 5] \text{ by } [-9, 1]$$

$\frac{8}{x^2+1} = 0 \Rightarrow 8 = 0 \Rightarrow$ ✗ ⇒ No REAL SOL'N!

(a) Use a graphing device to graph the equation in the given viewing rectangle. Find the x- and y-intercepts from the graph and confirm your answers algebraically (from the equation). (If an answer does not exist, enter DNE.)

x-intercept $(x, y) =$ (DNE)

y-intercept $(x, y) =$ (0, −8) → −8!

$x=0: \ -\frac{8}{0^2+1} = \frac{-8}{1} = -8$ → −8!

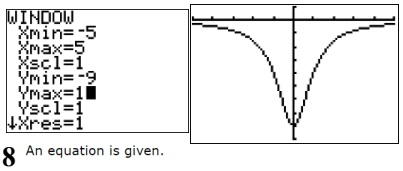
(b) If the graph appears to be symmetric, confirm that the equation satisfies the corresponding symmetry property. (Select all that apply.)

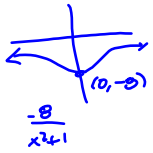
- ☐ The graph is symmetric with respect to the x-axis.
- ☑ The graph is symmetric with respect to the y-axis.
- ☐ The graph is symmetric with respect to the origin.
- ☐ The graph is not symmetric with respect to the x-axis, y-axis, or the origin.

$y = -\frac{8}{x^2+1}$

$\Rightarrow \frac{-8}{(-x)^2+1} = \frac{-8}{x^2+1} = y$

So symmetric wrt y-axis





8 An equation is given.

$$y = \sqrt[3]{1 - x^2}; \quad [-5, 5] \text{ by } [-5, 3]$$

(a) Use a graphing device to graph the equation in the given viewing rectangle. Find the x- and y-intercepts from the graph and confirm your answers algebraically (from the equation). (If an answer does not exist, enter DNE.)

x-intercept (smaller x-value) $(x, y) =$ ( )

x-intercept (larger x-value) $(x, y) =$ ( )

y-intercept $(x, y) =$ ( )

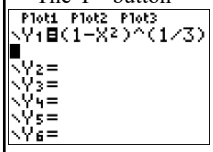
(b) If the graph appears to be symmetric, confirm that the equation satisfies the corresponding symmetry property. (Select all that apply.)

- ☐ The graph is symmetric with respect to the x-axis.
- ☑ The graph is symmetric with respect to the y-axis.
- ☐ The graph is symmetric with respect to the origin.
- ☐ The graph is not symmetric with respect to the x-axis, y-axis, or the origin.

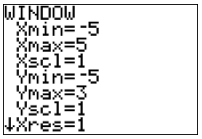
x-axis symmetry is going to be relatively rare, as our focus is on y as a function of x. That means y all by itself and some expression with x in it.

Basically no x-axis symmetry unless you see $|y|$ or y^2 or y^4 etc

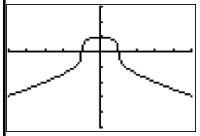
Calculator:
The Y= button



Set the WINDOW



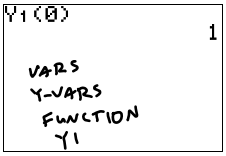
GRAPH



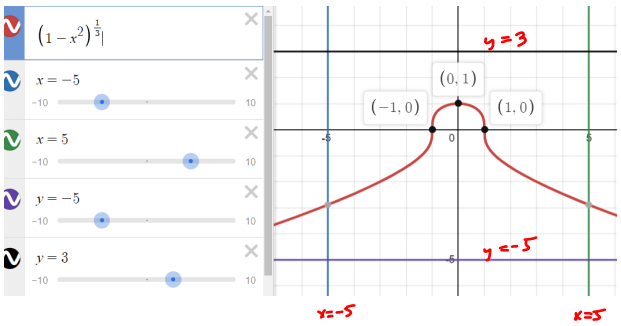
Then calculate ZEROs

and use VARS key to find

Y(0) for the y-intercept.



DESMOS

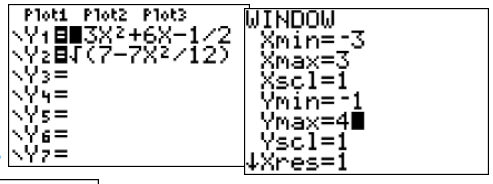


Found x-intercepts by clicking on them!

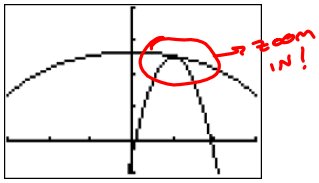
9 Do the given graphs intersect in the indicated viewing rectangle? If so, how many points of intersection are there?

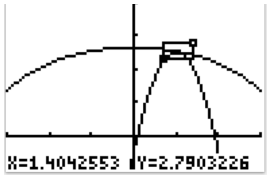
$y = -3x^2 + 6x - \frac{1}{2}$, $y = \sqrt{7 - \frac{7}{12}x^2}$; $[-3, 3]$ by $[-1, 4]$

- ○ Yes, they have one point of intersection.
- ○ Yes, they have two points of intersection.
- ○ Yes, they have three points of intersection.
- ○ No, they do not intersect in the given viewing rectangle.

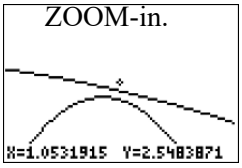


On my calculator, the viewing window doesn't give enough detail to be sure. I zoomed-in on the relevant spot.





Nope. Not after ZOOM-in.



10 Solve the equation both algebraically and graphically. (Round graphical solution(s) to two decimal places. Enter your answers as comma-separated lists. If there is no real solution, enter NO REAL SOLUTION.)

$\frac{4}{x} + \frac{1}{2x} = 7$

M1:

Graph $Y_1 = \frac{4}{x} + \frac{1}{2x} = 4/x + 1/(2x)$

Calculator sees $1/2x$ as $\frac{1}{2}x = \frac{x}{2}$

We want $\frac{1}{2x}$

$Y_2 = 7$

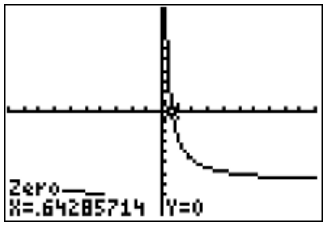
& find intersection

Did we capture ALL SOLNS? Zoom out a few times.

M2

GRAPH $Y_1 = \frac{4}{x} + \frac{1}{2x} - 7$

& find x-intercepts



$x \approx .64285714$

$\approx .64$

$\frac{4}{x} + \frac{1}{2x} = 7$

LCD = 2x

$\frac{4}{x} \cdot \frac{2}{2} + \frac{1}{2x} = \frac{7}{1} \cdot \frac{2x}{2x}$

$\frac{8+1}{2x} = \frac{14x}{2x}$

$\frac{9}{2x} - \frac{14x}{2x} = 0$

$\frac{-14x+9}{2x} = 0$

$\Rightarrow -14x + 9 = 0$

$-14x = -9$

$x = \frac{9}{14} \approx .6428571429$

$\approx .64$

Clean Fractions Method

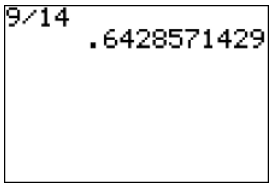
$\left(\frac{4}{x} + \frac{1}{2x} = 7\right) 2x$

$\frac{4 \cdot 2x}{x} + \frac{1 \cdot 2x}{2x} = 7 \cdot 2x$

$8 + 1 = 14x$

$14x = 9$

$x = \frac{9}{14} \approx .64$



11 Solve the equation both algebraically and graphically. (Round graphical solution(s) to two decimal places. Enter your answers as comma-separated lists. If there is no real solution, enter NO REAL SOLUTION.)

$$\frac{7}{x+2} - \frac{9}{2x} = \frac{8}{2x+4}$$

algebraically $x =$

graphically $x =$

LCD $= (x+2)(2)(x) = 2x(x+2)$

$2x+4 = 2(x+2)$

$\left(\frac{7}{x+2}\right)\left(\frac{2x}{2x}\right) - \left(\frac{9}{2x}\right)\left(\frac{x+2}{x+2}\right) = \left(\frac{8}{2(x+2)}\right)\left(\frac{x}{x}\right)$

$\frac{14x-9x-18}{LCD} = \frac{14x-27}{LCD} = \frac{8x}{LCD}$ → $9x$!

$14x-27 = 8x$

$6x = 27$

$x = \frac{27}{6} = \frac{9}{2} = x$

$\frac{7}{\frac{1}{2}+2} - \frac{9}{2(\frac{9}{2})} \stackrel{?}{=} \frac{9}{2(\frac{9}{2})+4}$

$\frac{7}{\frac{9+4}{2}} - \frac{9}{9} = \frac{7}{\frac{13}{2}} - 1$

$= \frac{7\cdot 2}{13} - 1 = \frac{14}{13} - \frac{13}{13} = \frac{1}{13}$

$\stackrel{?}{=} \frac{9}{9+4} = \frac{9}{13}$ No.

$\frac{14x-9x-18}{LCD} = \frac{5x-18}{LCD} = \frac{8x}{LCD}$

$5x-18 = 8x$

$-3x = 18$

$x = -\frac{18}{3} = -6$

$\frac{7}{-6+2} - \frac{9}{2(-6)} \stackrel{?}{=} \frac{8}{2(-6)+4}$

$-\frac{7}{4} + \frac{3}{1} = -1 \stackrel{?}{=} \frac{8}{-12+4} = \frac{8}{-8} = -1$

Yes!

12 Solve the equation both algebraically and graphically. (Round graphical solution(s) to two decimal places. Enter your answers as comma-separated lists. If there is no real solution, enter NO REAL SOLUTION.)

$$x^2 + 4 = 0$$

algebraically $x =$

graphically $x =$

13 Solve the equation both algebraically and graphically. (Round graphical solution(s) to two decimal places. Enter your answers as comma-separated lists. If there is no real solution, enter NO REAL SOLUTION.)

$$x^2 + 5 = 4x$$

algebraically $x =$ ____

graphically $x =$ ____

14 Solve the equation graphically in the given interval. State each answer rounded to two decimals. (Enter your answers as a comma-separated list.)

$$x^2 - 0.5x + 0.04 = 0; \quad [-2, 2]$$

$x =$ ____

15 Solve the equation graphically in the given interval. State each answer rounded to two decimals. (Enter your answers as a comma-separated list.)

$$x^3 - 12x^2 + 47x - 60 = 0; \quad [-1, 6]$$

16 Solve the equation graphically in the given interval. State each answer rounded to two decimals. (Enter your answers as a comma-separated list.)

$$x - \sqrt{x + 3} = 0; \quad [-1, 5]$$

Solve the equation graphically in the given interval. State each answer rounded to two decimals. (Enter your answers as a comma-separated list.)

17 $2 + \sqrt{x} = \sqrt{4 + x^2}$; $[-1, 5]$

$0 = x$ works.

$\sqrt{x^2+4} = \sqrt{x} + 2$

square

$x^2 + 4 = \sqrt{x}^2 + 2(2\sqrt{x}) + 2^2 = x + 4\sqrt{x} + 4$

$x^2 - x = 4\sqrt{x}$

square

$(x^2 - x)^2 = (x(x-1))^2 = x^2(x-1)^2 = x^2(x^2 - 2x + 1) = 16x$

$x^4 - 2x^3 + x^2 = 16x$

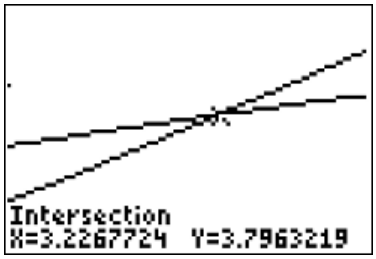
$x^4 - 2x^3 + x^2 - 16x = 0$

$x(x^3 - 2x^2 + x - 16) = 0$

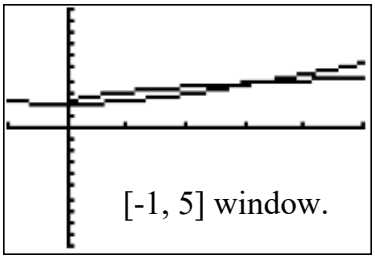
Meh

Messy Cubic Formula Does exist.

$x = 0$ works?



There's also an intersection at (0, 2)



Solve the equation graphically in the given interval. State each answer rounded to two decimals. (Enter your answers as a comma-separated list.)

18 $3x^{1/2} + 3x^{1/3} - 3x = 0$; $[-1, 5]$

$x^{\frac{1}{2}} + x^{\frac{1}{3}} - x = 0$

$\Rightarrow x^{\frac{1}{3}}\left(x^{\frac{1}{2}-\frac{1}{3}} + 1 - x^{\frac{2}{3}}\right) = 0$

$x = 0$ or $x^{\frac{1}{6}} - x^{\frac{2}{3}} + 1 = 0$

$u = x^{\frac{1}{6}}$

$\Rightarrow x^{\frac{1}{6}} = \left(x^{\frac{1}{6}}\right)^4 = u^4$

So $u - u^4 + 1 = 0$

$-u^4 + u + 1 = 0$

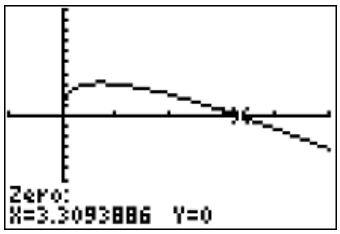
$u^4 - u - 1 = 0$

Not much fun. Need graphs.

This one is out of reach, algebraically.

$D = [0, \infty)$

$D(x^{\frac{1}{3}}) = [0, \infty) = D(\sqrt{x})$



$x \approx 3.31$, $x = 0$

19 Use the graphical method to solve the equation. (Enter your answers as a comma-separated list. Round your answers to two decimal places. If there is no real solution, enter NO REAL SOLUTION.)

$\sqrt{3x + 4} + 2 = x$

$\sqrt{3x+4} = x-2$

$\overset{?}{\Leftrightarrow}$ $3x+4 = (x-2)^2 = x^2 - 4x + 4$

$x^2 - 7x = 0$

$x(x-7) = 0$

$x = 0$ or $x = 7$

$\sqrt{4} + 2 = 2+2 = 4 \neq 7$

$\sqrt{3(7)+4} + 2 =$

$\sqrt{25} + 2 = 5+2 = 7$ ✓

$x = 7$

$x=0$ is extraneous.

$A = B \Rightarrow A^2 = B^2$ but

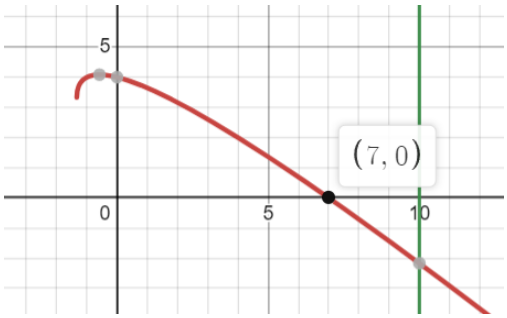
$A^2 = B^2 \Rightarrow A = \pm B$

} More solutions than original

Graphical:

Find x-ints of

$y = \sqrt{3x+4} + 2 - x$



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