

### Section 1.9 Absolute Value Inequalities

Two main concepts.

$$|A| < B \Rightarrow$$

$$-B < A < B$$

Alternate:

$$|A| < B \Rightarrow$$

$$A < B \text{ AND } A > -B$$

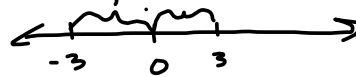
$$|A| > B \Rightarrow$$

$$A > B \text{ OR } A < -B$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$|3| = 3 \quad (|x-0|)$$

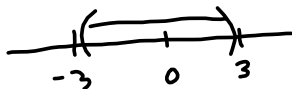
$$|-3| = 3$$



### Introductory concept: Absolute Value Equation

1 The equation  $|x| = 8$  has the two solutions  $x = -8$  (smaller value) and  $x = 8$  (larger value).

$$|x| < 3$$



Everything less than 3 units from 0.

$$-3 < x < 3 \text{ You could}$$

also say

$$-3 < x$$

$$x > -3$$



$$x < 3$$



$$= (-3, \infty) \cap (-\infty, 3)$$

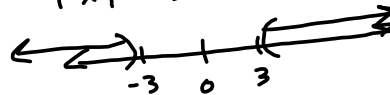
$$|x| < 3 \Rightarrow$$

$$x < 3 \text{ AND } x > -3$$

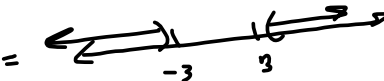
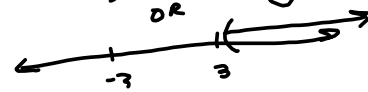
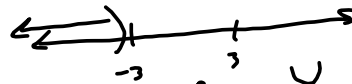
$$\text{etc.}$$

$$(\text{OR } -3 < x < 3)$$

$$|x| > 3$$



$$x < -3 \text{ OR } x > 3$$



$$= (-\infty, -3) \cup (3, \infty)$$

$$|x| > 3 \Rightarrow$$

$$x > 3 \text{ OR } x < -3$$

etc.

WARNING

$-3 > x > 3$  is BAD

This says  $-3 < x$  and  $x > 3$

which is impossible!

Don't write this!

$$|-x| = |(-1)(x)|$$

$$= |-1||x| = |x|$$

$$|3-x| = |x-3|$$

2

(a) The solution of the inequality  $|x| \leq 3$  is the interval  $[-3, 3]$ .

AND

(b) The solution of the inequality  $|x| \geq 3$  is a union of two intervals  $[-\infty, -3] \cup [3, \infty)$ .

OR  $(-\infty, -3] \cup [3, \infty)$ 

3

(a) The set of all points on the real line whose distance from zero is less than 7 can be described by the absolute value inequality  $|x| < 7$ . (Use  $x$  as your variable.)

(b) The set of all points on the real line whose distance from zero is greater than 7 can be described by the absolute value inequality  $|x| > 7$ . (Use  $x$  as your variable.)

4 (a) What is the logical first step in solving the equation  $|6x - 1| = 2$ ?

- Rewrite as the equations  $6x - 1 = 2$  and  $6x - 1 = -2$ .  
 Rewrite as the equations  $6x - 1 = 2$  and  $6x + 1 = 2$ .  
 Rewrite the equation as  $-2 \leq 6x - 1 \leq 2$ .  
 Add 1 to both sides.  
 Divide both sides by 6.

$$6x - 1 = 2 \text{ or } 6x - 1 = -2$$

$$6x - 1 = \pm 2$$

$$6x = 1 \pm 2$$

$$x = \frac{1 \pm 2}{6} \rightarrow \frac{1}{6}$$

$$x \in \left\{ -\frac{1}{6}, \frac{1}{6} \right\}$$

(b) What is the logical first step in solving the inequality  $|4x + 2| \leq 6$ ?

- Rewrite as the equations  $4x + 2 = 6$  and  $4x + 2 = -6$ .  
 Rewrite as the equations  $4x + 2 = 6$  and  $4x - 2 = 6$ .  
 Rewrite the equation as  $-6 \leq 4x + 2 \leq 6$ .  
 Subtract 2 from both sides.  
 Divide both sides by 4.

$\rightarrow$  I prefer  
 $4x + 2 \leq 6$  AND  $4x + 2 \geq -6$

5 Solve the equation. (Enter your answers as a comma-separated list. If there is no solution, enter NO SOLUTION.)

$$|3x| = 18$$

$$|3x| = 18 \rightarrow$$

$$3x = \pm 18$$

$$x = \frac{\pm 18}{3} = \pm 6$$

$$3x = 18 \text{ or } 3x = -18$$

$$x = \frac{18}{3} = 6 \text{ or } x = -\frac{18}{3} = -6$$

6 Solve the equation. (Enter your answers as a comma-separated list. If there is no solution, enter NO SOLUTION.)

$$|-8x| = 15$$

$$\Rightarrow -8x = \pm 15$$

$$x = \frac{\pm 15}{-8} = \mp \frac{15}{8} = \pm \frac{15}{8}$$

$$x \in \left\{ -\frac{15}{8}, \frac{15}{8} \right\}$$

$$\text{OR } |-8x| = |8x| \Rightarrow$$

$$8x = \pm 15 \Rightarrow$$

$$x = \pm \frac{15}{8}$$

7 Solve the equation. (Enter your answers as a comma-separated list. If there is no solution, enter NO SOLUTION.)

$$|x - 9| = 7$$

8 Solve the equation. (Enter your answers as a comma-separated list. If there is no solution, enter NO SOLUTION.)

$$|x - 9| = -3$$

Never!

$$|A| \geq 0$$

$$-3 < 0!$$

- 9 Solve the equation. (Enter your answers as a comma-separated list. If there is no solution, enter NO SOLUTION.)

$$\left| \frac{3}{7}x + 6 \right| - \frac{1}{2} = 8 \implies$$

$$\left| \frac{3}{7}x + 6 \right| = 8 + \frac{1}{2} = \frac{16+1}{2} = \frac{17}{2}$$

$$\implies \left( \frac{3}{7}x + 6 = \pm \frac{17}{2} \right) (14)$$

$$\implies 6x + 84 = \pm \left( \frac{17}{2} \right) (14) = \pm (17)(7) = 119$$

$$6x = -84 \pm 119$$

$$x = \frac{-84 \pm 119}{6}$$

$$\frac{-84+119}{6} = \frac{35}{6}$$

$$\frac{-84-119}{6} = -\frac{203}{6}$$

$$\frac{17}{2}$$

- 10 Solve the equation. (Enter your answers as a comma-separated list. If there is no solution, enter NO SOLUTION.)

$$|x - 7| = |3x + 8|$$

$$|x - 7| = |3x + 8| \implies$$

$$x - 7 = \pm (3x + 8)$$

Messy. Instead, do:

$$x - 7 = 3x + 8 \quad \text{or} \quad x - 7 = -(3x + 8) = -3x - 8$$

$$-2x = +15$$

$$x = \frac{15}{2}$$

$$4x = -1$$

$$x = -\frac{1}{4}$$

$$x \in \left\{ -\frac{1}{4}, \frac{15}{2} \right\}$$



13

Solve the inequality. Express the answer using interval notation.

$$\frac{1}{2}|x| \geq 3$$

14

Solve the inequality. Express the answer using interval notation.

$$|x - 3| \leq 14$$

$$x - 3 \leq 14 \quad \text{AND} \quad x - 3 \geq -14$$

$$x \leq 17 \quad \text{AND} \quad x \geq -11$$

$$\Rightarrow x \in [-11, 17]$$

$$x \leq 17 \quad \text{AND} \quad x \geq -11$$

$$x \in (-\infty, 17] \cap [-11, \infty)$$

$$= [-11, 17]$$

