(a) To solve the equation $x^3 - 8x^2 = 0$, we ---Select--- the left-hand side.

1

(b) Find the solutions of the equation $x^2(x - 8) = 0$. (Enter your answers as a comma-separated list.)

Solve the equation $\sqrt{3x} + x = 0$ by doing the following steps.

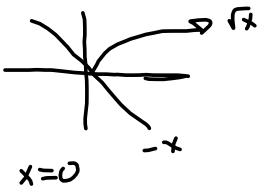
(a) Isolate the radical.

2 (b) Square both sides.

(c) Find the solutions of the resulting quadratic equation. (Enter your answers as a comma-separated list.)

(d) Find the solution(s) that satisfy the original equation. (Enter your answers as a comma-separated list.)

$\Rightarrow \sqrt{x} = -x$



$A = B \Rightarrow$
$A^2 = B^2$, but
$A^2 = B^2$ does
NOT imply
$A = B$.
It implies $A = \pm B$

Their steps:

$\sqrt{3x} = -x$

$(\sqrt{3x})^2 = (-x)^2 = x^2$

$3x = x^2 \Rightarrow x^2 - 3x = 0$

$\Rightarrow x(x-3) = 0$

$\Rightarrow x = 0$ OR $x = 3$

$x = 0$ Yes

$x = 3$ Extraneous

$\sqrt{3(3)} + 3 = 3 + 3 = 6 \neq 0$

$x = 0$ OR

$x \in \{0\}$

Squaring both sides casts a net over the solutions, but that net captures "extraneous" solutions (Fish you throw back), sometimes.

Squaring both sides results in an equation that has MORE solutions than the original, sometimes.

The equation $(x + 1)^2 - 12(x + 1) + 27 = 0$ is of ---Select--- type. To solve the equation, we set $W = x+1$

3

The resulting quadratic equation is $w^2 - 12w + 27 = 0$.

This equation is "quadratic in (x + 1)."

Let W = x + 1.

Solve for W.

Use W to get x.

$w^2 - 12w + 27 = 0$

$w^2 - 9w - 3w + 27$

$= w(w-9) - 3(w-9)$

$= (w-9)(w-3) = 0$

$\Rightarrow w = 3, 9$

$w = x+1$: $\Rightarrow$

$x = w - 1$

$x = 3 - 1 = 2 = x$

$x = 9 - 1 = 8 = x$

$x \in \{2, 8\}$

check:

$(2+1)^2 - 12(2+1) + 27$

$= 9 - 36 + 27 = 0$ ✓

$(8+1)^2 - 12(8+1) + 27$

$= 9^2 - 12(9) + 27 = 81 + 27 - 108 = 0$ ✓

The equation $x^6 + 3x^3 - 5 = 0$ is of ---Select--- type. To solve the equation, we set $W =$ ____ . The result

4

quadratic equation is ____ .

you have to recognize $6 = 3 \cdot 2$, so

$x^6 + 3x^3 - 5 = x^{3 \cdot 2} + 3x^3 - 5 = (x^3)^2 + 3x^3 - 5$

$= w^2 + 3w - 5 = 0$, where $w = x^3$ $\Rightarrow$

$w^2 + 3w = 5$

$w^2 + 3w + \left(\frac{3}{2}\right)^2 = \frac{5}{1} \cdot \frac{4}{4} + \frac{9}{4} = \frac{29}{4}$

$\left(w + \frac{3}{2}\right)^2 = \frac{29}{4}$

$w + \frac{3}{2} = \pm \frac{\sqrt{29}}{2}$

$w = -\frac{3 \pm \sqrt{29}}{2}$

DOH! Didn't ask us to solve for x!

Wanted us to stop here...

Note to finish this solution, you'd have to sub in for x: $x^3 = \frac{-3 \pm \sqrt{29}}{2}$ $\rightsquigarrow$ $x = \sqrt[3]{\frac{-3 \pm \sqrt{29}}{2}}$!

Going beyond our theory, but there are 6 solutions. They're living in this solution, but it involves the 3 cubic roots of a complex number, of which $\sqrt[3]{\frac{-3+\sqrt{29}}{2}}$ is the principal cube root of $\frac{-3+\sqrt{29}}{2}$. There are two more!

Before your brain breaks, let's move on. This is beyond the scope of our course, at present.

Find all real solutions of the equation. (Enter your answers as a comma-separated list.)

5 $3x^3 - 12x^2 = 0$

See #1 Video.

Find all real solutions of the equation. (Enter your answers as a comma-separated list.)

6 $x^3 = 81x$

$x^3 - 81x = x(x^2 - 81) = x(x-9)(x+9) = 0$

$\Rightarrow x \in \{-9, 0, 9\}$

Find all real solutions of the equation. (Enter your answers as a comma-separated list.)

7 $y^5 - 18y^4 + 31y^3 = 0$

$\Rightarrow y^3(y^2 - 18y + 31) = 0$

$\Rightarrow y = 0$, $y^2 - 18y + 31 = 0$

$a = 1, b = -18, c = 31$

$b^2 - 4ac = 18^2 - 4(1)(31)$

$= 324 - 124 = 200$

$2 \mid 200$
$10 \mid 100$
10

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{18 \pm 10\sqrt{2}}{2}$

$= \frac{2(9 \pm 5\sqrt{2})}{2}$

$= 9 \pm 5\sqrt{2}$

$y^2 - 18y = -31$

$y^2 - 18y + 9^2 = -31 + 81 = 50$

$(y - 9)^2 = 50$

$y - 9 = \pm\sqrt{50}$

$y = 9 \pm 5\sqrt{2}$

$2 \mid 50$
$5 \mid 25$
5

Find all real solutions of the equation. (Enter your answers as a comma-separated list.)

8 $(4x + 7)^4 - (4x + 7)^3 = 0$

Let $w = 4x + 7$. Then

$w^4 - w^3 = 0$

$\Rightarrow w^3(w - 1) = 0$

$\Rightarrow w \in \{0, 1\}$ Not done

$w = 4x + 7 = 0 \Rightarrow$

$4x = -7 \Rightarrow$

$x = -\frac{7}{4}$

$w = 4x + 7 = 1$

$\Rightarrow 4x = -6$

$x = -\frac{6}{4} = -\frac{3}{2} = x$

$\Rightarrow x \in \{-\frac{7}{4}, -\frac{3}{2}\}$

Find all real solutions of the equation. (Enter your answers as a comma-separated list.)

9 $x^3 - 2x^2 - 2x + 4 = 0$ FACTOR BY GROUPING

$x^2(x-2) - 2(x-2)$

$= (x-2)(x^2-2)$

$= (x-2)(x-\sqrt{2})(x+\sqrt{2})$

or just

$x-2 = 0$ OR $x^2-2=0$

$x=2$ $x^2 = 2$

$x = \pm\sqrt{2}$

$x \in \{-\sqrt{2}, \sqrt{2}, 2\}$

Find all real solutions of the equation. (Enter your answers as a comma-separated list.)

10 $2x^3 + 3x^2 - 32x - 48 = 0$

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

11 $\frac{4}{x-4} - \frac{8}{x^2} = 0$

DOMAIN: Need $x \neq 0, x=4$

$\mathcal{D} = \mathbb{R} \setminus \{0,4\}$

$= (-\infty,0) \cup (0,4) \cup (4,\infty)$

$= \{x \mid x \neq 0,4\}$

CLEAR FRACTIONS WITH LCD

LCD $= (x-4)x^2$

$(x-4)(x^2)\left(\frac{4}{x-4} - \frac{8}{x^2} = 0\right)$

$\Rightarrow \frac{4}{x-4}(x-4)x^2 - \frac{8}{x^2}(x-4)x^2 = ((x-4)x^2)(0)$

$4x^2 - 8(x-4) = 0$

$4x^2 - 8x + 32 = 0$

$4(x^2 - 2x + 8) = 0$

$a=1, b=-2, c=8$

$b^2 - 4ac = 2^2 - 4(1)(8$

$= 4 - 32 = -28 < 0$

No Real Solutions!

WRITE Everything over the LCD

Old-School, but generally more powerful in the sequel.

$\left(\frac{-1}{x-4}\right)\left(\frac{x^2}{x^2}\right) - \left(\frac{8}{x^2}\right)\left(\frac{x-4}{x-4}\right) =$

$= \frac{4x^2 - 8x + 32}{x^2(x-4)} = 0 \Rightarrow$

$4x^2 - 8x + 32 = 0$

$\frac{A}{B} = 0 \Rightarrow$

$A = 0.$

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

12 $\frac{y+3}{y^2+3} = \frac{2}{y+2}$

$\mathcal{D} = \mathbb{R} \setminus \{-2\}$

$(y^2+3)(y+2) =$ LCD

$(y+2)(y^2+3)\left(\text{EQ'N}\right):$

$(y+2)(y+3) = 2(y^2+3)$

$y^2 + 5y + 6 = 2y^2 + 6$

$-y^2 - 5y - 6 = -y^2 - 5y - 6$

$0 = y^2 - 5y = 0$

$y(y-5) = 0$

$y \in \{0,5\}$

Other Method.

$\left(\frac{y+3}{y^2+3}\right)\left(\frac{y+2}{y+2}\right) = \left(\frac{2}{y+2}\right)\left(\frac{y^2+3}{y^2+3}\right)$

$\Rightarrow \frac{y^2+5y+6}{\text{LCD}} = \frac{2y^2+6}{\text{LCD}} \Rightarrow$

$y^2 + 5y + 6 = 2y^2 + 6$, etc.

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

13 $\dfrac{x + \frac{3}{x}}{6 + \frac{1}{x}} = 5x$

Shortcut Multiply Numerator and denominator by LCD of both. LCD = x

$$\frac{\left(x + \frac{3}{x}\right)x}{\left(6 + \frac{1}{x}\right)x} = \frac{x^2+3}{6x+1} = 5x$$

$\Rightarrow x^2+3 = 5x(6x+1)$

$\Rightarrow x^2+3 = 30x^2+5x$

$\Rightarrow -29x^2-5x+3 = 0$

$\Rightarrow 29x^2+5x-3 = 0$

$b^2-4ac = 5^2-4(29)(-3)$

$= 25+348 = 373$

290
58
348

+ !

\- Negative ! No Real Solution !

wolframalpha.com

```
                353
Ans/17
          20.76470588
5²+4*3*29
                373
Ans^.5
          19.31320792
```

Our upper bound on the list of primes

```
          53.28571429
Ans*7
                373
Ans/11
          33.90909091
Ans*11
                373
```

```
          28.69230769
Ans*13
                373
Ans/17
          21.94117647
Ans*17
                373
```

```
                373
Ans/17
          21.94117647
Ans*17
                373
Ans/19
          19.63157895
```

Slow but Steady

$$\frac{x + \frac{3}{x}}{6 + \frac{1}{x}} = \frac{\frac{x^2+3}{x}}{\frac{6x+1}{x}} = \frac{x^2+3}{6x+1} = 5x$$

$x^2+3 = 5x(6x+1) = 30x^2+5x$

$29x^2+5x-3 = 0$

$\Rightarrow b^2-4ac = 25-4(29)(-3)$ etc.

So $\sqrt{373}$ doesn't simplify

$$x = \frac{-5 \pm \sqrt{373}}{2(29)} = \frac{-5 \pm \sqrt{373}}{58}$$

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

14 $4 = \sqrt{4x - 20}$

$\sqrt{4x-20} = 4$

$(\quad)^2 = (\quad)^2$

$4x - 20 = 4^2 = 16$

$4x - 36 = 0$

$x = 9$

D: Need $4x - 20 \geq 0$

$4x \geq 20$

$\{x \mid x \geq 5\}$ is Domain

$D = [5, \infty)$

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

15 $\sqrt{7x-1} = \sqrt{8x-7}$

Domain:

Need $7x-1 \geq 0$ and $8x-7 \geq 0$

Need $7x \geq 1$ NOT 72 It's 7 =

$x \geq \frac{1}{7}$ and $8x \geq 7$ → Nope

$x \geq \frac{7}{8}$ is correct

$\sqrt{7x-1} = \sqrt{8x-7} \Rightarrow$

$7x-1 = 8x-7$

$-x+6 = 0$

$x = 6 \notin D$

No Real Solution!

$6 \in D$!

$x \geq \frac{1}{7}$ and $x \geq 9$

N Y Y

AND

Intersect / overlap.

$\frac{1}{7}$ $\frac{7}{8}$ $\frac{7}{8}$

$D = [9, \infty)$ $[7/8, \infty)$

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

16 $\sqrt{13+x} = \sqrt{x^2+1}$

D: Need $x+13 \geq 0$ & $x^2+1 \geq 0$

$x \geq -13$ Always > 0

$D = [-13, \infty)$

$13 + x = x^2 + 1$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ OR $x = -3$

Both $\in D$ ✓