1 The imaginary number i has the property that $i^2 = -1$

$\sqrt{-1} = i$ is DEFINITION $\quad (\sqrt{A})^2 = A$

$(\sqrt{-1})^2 = i^2 = -1 \quad \sqrt{A^2} = |A|$

2 For the complex number 7 + 9i the real part is $\boxed{7}$ and the imaginary part is 9

3 (a) The complex conjugate of 5 + 6i is $\overline{5+6i}$ = $\boxed{5-6i = \bar{z}}$

$z = 5+6i$

(b) $(5+6i)(\overline{5+6i}) = \boxed{(5+6i)(5-6i)} = 5^2+6^2 = 25+36 = 61$

$z = a+bi$

Then $z\bar{z} = a^2+b^2$

$(a+bi)(a-bi) = a^2 - abi + bia + (bi)(-bi)$

$= a^2 - abi + abi + (b)(-b)(i)(i)$

$= a^2 - b^2i^2 = a^2 - b^2(-1) = a^2+b^2$

Conjugate Pairs Theorem

4 If 7 + 9*i* is a solution of a quadratic equation with real coefficients, then $7-9i$ is also a solution of the equation.

Recall: The (embedded) Factor Theorem in my factoring "cheat."

Click Here for video #6 in 1.5, where I worked one.

FACTOR THEOREM

If $x=A$ & $x=B$ are solutions of

$ax^2+bx+c=0$, then

$ax^2+bx+c = a(x-A)(x-B)$

$x=e+fi$ & $x=g+hi$ are both solutions of

$ax^2+bx+c=0$, then

$$ax^2+bx+c = a(x-(e+fi))(x-(g+hi)) = a(x-A)(x-B)$$

$$= a(x^2-(g+hi)x-(e+fi)x+(e+fi)(g+hi))$$

$$= ax^2-a(g+hi)x-a(e+fi)x+a(e+fi)(g+hi)$$

$$= ax^2-agx-ahix-aex-afix+a(eg+ehi+fgi+fhi^2)$$

$$= ax^2-agx-ahix-aex-afix+aeg+aehi+afgi+afhi^2$$

$$= ax^2-agx-ahix-aex-afix+aeg+aehi+afgi-afh$$

For Everything to turn out ax^2+bx+c, & everything real, then a is real ✓

$A = e+fi$

$B = g+hi$

$a(x-A)(x-B)$ &

we want to expand this and recapture ax^2+bx+c, where a,b,c are real.

x's

$-ag - ahi - ae - afi =$

$= -ag - ae - ahi - afi$

$= -a(g-a) - \underline{a(h+f)}i$

needs to go away

That says $h+f=0$.

i.e. $h=-f$

so $A = e+fi$

$B = g-fi$

Also

$+aeg + aehi + afgi - afh$

has to be real. That means

$aehi + afgi = 0$

$ai(eh+fg) = 0$

since $h=-f$ by previous work,

$e(-f) + fg = 0$

$f(-e+g) = 0$

$-e+g = 0$

$g = e$, so $B = g+hi = e-fi = B = \overline{e+fi}$

$A = e+fi$, so

$B = \overline{A}$.

5 *Yes or No?*

Is every real number also a complex number?

SOME NOTATION

The set of real numbers is $\mathbb{R} = \{ x \mid x \text{ is a real number} \}$

" " " Complex Numbers is $\mathbb{C} = \{ a+bi \mid a, b \in \mathbb{R}$, where i is the imaginary unit, with $i^2 = -1 \}$

$\mathbb{R} \subset \mathbb{C}$

$7 = 7 + 0i,\ 0 \in \mathbb{R}$

$\Rightarrow 7 \in \mathbb{C} \supset \mathbb{R}$

Find the real and imaginary parts of the complex number.

9 $\dfrac{-5 - 11i}{7} = z = -\frac{5}{7} - \frac{11}{7}i \Rightarrow$

$\mathrm{Re}(z) = -\frac{5}{7}$ & $\mathrm{Im}(z) = -\frac{11}{7}$.

Find the real and imaginary parts of the complex number.

11

$z = 6 - \sqrt{-7} = 6 - \sqrt{7}\,i$

$Re(z) = 6$ & $Im(z) = -\sqrt{7}$

Evaluate the sum and write the result in the form *a* + *bi*. (Simplify your answer completely.)

14

$(6 - 5i) + (-5 - 9i) = 6 + (-5) - 5i - 9i = 1 - 14i$

Evaluate the difference and write the result in the form *a* + *bi*. (Simplify your answer completely.)

15 $(-5 + 2i) - (4 - 3i)$

$= -5 + 2i - 4 + 3i = -9 + 5i$

Evaluate the product and write the result in the form $a + bi$. (Simplify your answer completely.)

17 $-7(6 - 5i) = \boxed{-42 + 35i}$

Evaluate the product and write the result in the form $a + bi$. (Simplify your answer completely.)

18 $(7 - 6i)(4 + 9i)$

FOIL / Distributive Law

$-54i^2 = +54$

$28 + 63i - 24i - 54i^2$

$= 28 + 54 + 39i = \boxed{82 + 39i}$

Evaluate the product and write the result in the form $a + bi$. (Simplify your answer completely.)

23 $(11 + i)^2$

$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

$(a-b)^2 = (a-b)(a-b) = a^2 - 2a + b^2$

$(11 + i)^2 =$

$(a+bi)^2 = a^2 + 2abi + (bi)^2$

$(11+i)^2 = 11^2 + 2(11)(i) + i^2$

$= a^2 + 2abi - b^2$

$b^2 i^2 = -b^2$!

$= \boxed{121 + 22i - 1}$

Evaluate the quotient and write the result in the form $a + bi$. (Simplify your answer completely.)

24 $\frac{3}{i} = \frac{3}{i} \cdot \frac{i}{i} = \frac{3i}{i^2} = \frac{3i}{-1} = -3i$

Evaluate the quotient and write the result in the form $a + bi$. (Simplify your answer completely.)

26 $\frac{1 - 21i}{1 + 5i} = \left(\frac{1 - 21i}{1 + 5i}\right)\left(\frac{1 - 5i}{1 - 5i}\right) = \frac{1 - 5i - 21i + 105i^2}{1^2 + 5^2} =$

$= \frac{1 - 105 - 26i}{1 + 25} = \frac{-104}{26} - \frac{26i}{26}$

$\frac{-104}{26} = -\frac{52}{13} = -4$

$= -4 - i$

Evaluate the quotient and write the result in the form $a + bi$. (Simplify your answer completely.)

28 $\left(\frac{34i}{1-4i}\right)\left(\frac{1+4i}{1+4i}\right) = \frac{34i + (34i)(4i)}{1^2+4^2} = \frac{34i - 136}{17}$

$= -\frac{136}{17} + \frac{34i}{17} = \boxed{-8+2i}$

Evaluate the quotient and write the result in the form $a + bi$. (Simplify your answer completely.)

29 $(6-7i)^{-1} = \left(\frac{1}{6-7i}\right)\left(\frac{6+7i}{6+7i}\right) = \frac{6+7i}{6^2+7^2} = \frac{6+7i}{36+49}$

$= \boxed{\frac{6}{85} + \frac{7}{85}i}$

Evaluate the quotient and write the result in the form $a + bi$. (Simplify your answer completely.)

31 $\frac{6}{1+i} - \frac{6}{1-i} = \left(\frac{6}{1+i}\right)\left(\frac{1-i}{1-i}\right) - \left(\frac{6}{1-i}\right)\left(\frac{1+i}{1+i}\right)$

LCD $= (1+i)(1-i) = 1^2+1^2 = 2$

$= \frac{6(1-i) - 6(1+i)}{\text{LCD}} = \frac{6-6i-[6+6i]}{2} = \frac{6-6i-6-6i}{2}$

$= \frac{-12i}{2} = -6i$

Evaluate the power, and write the result in the form $a + bi$. (Simplify your answer completely.)

34 i^{18} $i^2 = -1$

$i^{18} = i^{2\left(\frac{18}{2}\right)} = (i^2)^9 = (-1)^9 = -1$

$i^{19} = i^{18+1} = i^{18}i^1 = i^{18}i = (i^2)^9 i = (-1)^9 i = -i$

$i^{373} = i^{372+1} = i^{372}i = (i^2)^{\frac{372}{2}} i = (-1)^{186} i = i$

Evaluate the radical expression and express the result in the form $a + bi$. (Simplify your answer completely.)

38 $\dfrac{\sqrt{-18}}{\sqrt{2}} = \dfrac{\sqrt{18}\, i}{\sqrt{2}} = \sqrt{\dfrac{18}{2}}\, i = \sqrt{9}\, i = 3i$

Evaluate the radical expression and express the result in the form $a + bi$. (Simplify your answer completely.)

39 $\sqrt{-64}\sqrt{-16} = \sqrt{64 \cdot 16} = \sqrt{8^2 \cdot 4^2} = 8 \cdot 4 = 32$ BAD!

Take care of $\sqrt{-\text{STUFF}}$ 1^{st}:

$= (i\sqrt{64})(i\sqrt{16}) = \sqrt{64 \cdot 16}\, i^2 = (\sqrt{8^2 \cdot 4^2})(-1) = \boxed{-32}$ GOOD!

Evaluate the radical expression and express the result in the form $a + bi$. (Simplify your answer completely.)

40 $\sqrt{\dfrac{1}{2}}\sqrt{-162} = \sqrt{\dfrac{1}{2}}\sqrt{162}\, i = \sqrt{\dfrac{162}{2}}\, i = \sqrt{81}\, i = \boxed{9i}$

Evaluate the radical expression and express the result in the form $a + bi$. (Simplify your answer completely.)

41 $\left(3 + \sqrt{-1}\right)\left(9 - \sqrt{-5}\right) = (3 + i)(9 - i\sqrt{5}) = 27 - 3i\sqrt{5} + 9i - \sqrt{5}\,i^2$

$= (27 + \sqrt{5}) + -3\sqrt{5}\,i + 9i$

$= (27 + \sqrt{5}) + (9 - 3\sqrt{5})i$

webAssign's ok, here

Evaluate the radical expression and express the result in the form $a + bi$. (Simplify your answer completely.)

42 $\left(\sqrt{5} - \sqrt{-4}\right)\left(\sqrt{35} - \sqrt{-28}\right) = \sqrt{5}\sqrt{35} - \sqrt{5}\sqrt{-28} - \sqrt{-4}\sqrt{35} + \sqrt{-4}\sqrt{-28}$

$= \sqrt{5 \cdot 5 \cdot 7} - \sqrt{5}\, i\sqrt{28} - i\sqrt{4}\sqrt{35} + \sqrt{4}\, i\sqrt{28}\, i$

$= 5\sqrt{7} - \sqrt{5}\, i\,(2\sqrt{7}) - i(2)\sqrt{35} + 2i\,(2\sqrt{7})i$

$= 5\sqrt{7} - 2\sqrt{5}\sqrt{7}\, i - 2\sqrt{35}\, i + 4\sqrt{7}\, i^2$

$= 5\sqrt{7} - 4\sqrt{7} - 2\sqrt{35}\, i - 2\sqrt{35}\, i$

$= \sqrt{7} - 4\sqrt{35}\, i$

Find all solutions of the equation and express them in the form $a + bi$. (Enter your answers as a comma-separated list. Simplify your answer completely.)

43 $x^2 + 16 = 0$

Find all solutions of the equation and express them in the form $a + bi$. (Enter your answers as a comma-separated list. Simplify your answer completely.)

44 $2x^2 + 13 = 0$

They want you to rationalize the denominator.

($\sqrt{2}$ is not rational)

$2x^2 = -13$

$x^2 = \frac{-13}{2}$

$x = \pm\sqrt{-\frac{13}{2}} = \pm\sqrt{\frac{13}{2}}\, i = \pm\sqrt{\frac{13}{2}\cdot\frac{2}{2}}\, i = \pm\frac{\sqrt{26}}{\sqrt{4}}\, i = \boxed{\pm\frac{\sqrt{26}}{2} i}$

Find all solutions of the equation and express them in the form $a + bi$. (Enter your answers as a comma-separated list. Simplify your answer completely.)

45 $x^2 - 14x + 74 = 0$

$x^2 - 14x + 7^2 = -74 + 49$

$(x-7)^2 = -25$

$x = 7 \pm \sqrt{-25} = 7 \pm 5i$

Find all solutions of the equation and express them in the form $a + bi$. (Enter your answers as a comma-separated list. Simplify your answer completely.)

50 $\left(t + 2 + \frac{7}{t} = 0\right) t$

$2|24$, $2|12$, $2|6$, 3 $\quad \sqrt{24} = 2\sqrt{6}$

$t^2 + 2t + 7 = 0$

$a = 1, b = 2, c = 7$

$b^2 - 4ac = 2^2 - 4(7) = 4 - 28 = -24$

$t = \frac{-2 \pm 2\sqrt{6}\, i}{2} = -1 \pm \sqrt{6}\, i$

Evaluate the given expression for $z = 4 - 5i$ and $w = 5 + 3i$. (Simplify your answer completely.)

52 $\bar{z} + \bar{w}$

$z = 4 - 5i \Rightarrow \bar{z} = 4 + 5i$

$w = 5 + 3i \Rightarrow \bar{w} = 5 - 3i$

$\Rightarrow \bar{z} + \bar{w} = 4 + 5i + (5 - 3i) = \boxed{9 + 2i}$

$\bar{z} + \bar{w} = \overline{z + w}$

$\bar{z} + \bar{w} = \overline{(z + w)}$

$= \overline{(4 - 5i) + (5 + 3i)}$

$= \overline{9 - 2i} = 9 + 2i = \overline{z + w}$. $= \bar{z} + \bar{w}$.

Evaluate the given expression for $z = 5 - 6i$. (Simplify your answer completely.)

54 $z \cdot \bar{z}$

55 Evaluate the given expression for $z = 2 - 3i$ and $w = 5 + 3i$. (Simplify your answer completely.)

If $z = a + bi$ and $w = c + di$, show that the statement is true.

56 $\bar{z} + \bar{w} = \overline{z + w}$

The sum of the conjugates is the conjugate of the sum.

$$\overline{z+w} = \overline{(a+bi)+(c+di)}$$
$$= \overline{(a+c)+(b+d)i}$$
$$= \boxed{(a+c)-(b+d)i = \overline{z+w}}$$

$$\bar{z}+\bar{w} = (a-bi)+(c-di)$$
$$= (a+c)+(-b-d)i$$
$$= \boxed{(a+c)-(b+d)i = \bar{z}+\bar{w}}$$

They're the same! Woo-hoo!

If $z = a + bi$ and $w = c + di$, show that the statement is true.

57 $\overline{zw} = \bar{z} \cdot \bar{w}$

The conjugate of the product is the product of the conjugates.

$$\overline{zw} = \overline{(a+bi)(c+di)}$$
$$= \overline{ac+adi+bci+bdi^2}$$
$$= \overline{(ac-bd)+(ad+bc)i}$$
$$= (ac-bd)-(ad+bc)i$$
$$= \overline{zw}$$

Same!

$$\bar{z}\bar{w} = (\overline{a+bi})(\overline{c+di})$$
$$= (a-bi)(c-di)$$
$$= ac-adi-bci+bdi^2$$
$$= (ac-bd)-(ad+bc)i$$
$$= \bar{z}\bar{w}$$