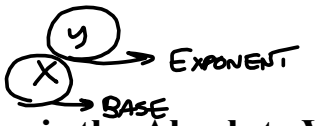
The Quadratic Formula



The Principal Square Root of a Square is the Absolute Value of the Base:

$$\sqrt{A^2} = |A|$$

Recall: $|x| = 7 \Rightarrow x = 7$ OR $x = -7$

i.e. $x = \pm 7$

$\sqrt{3^2} = \sqrt{9} = 3 = |3|$

$\sqrt{(-3)^2} = \sqrt{9} = 3 = |-3|$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Square Root Property: $|-3| = -(-3) = +3$

$$A^2 = B$$

$$\sqrt{A^2} = \sqrt{B}$$

$$|A| = \sqrt{B}$$

$$A = \pm\sqrt{B}$$

$\Rightarrow$ $A^2 = B$, $A = \pm B$

Recall: Completing the Square for Circles. A skill we have!

$x^2 + 10x + y^2 = 0$ $\quad (h,k) = (-5, 0)$, $r = 5$

$x^2 + 10x + 5^2 + y^2 = 25$

$(x+5)^2 + y^2 = 25$

We can use this skill for solving any quadratic equation:

$(x+7)^2 = 11$

$\Rightarrow$ $x + 7 = \pm\sqrt{11}$

$x = -7 \pm \sqrt{11}$

$(x-9)^2 - 5 = 4$

$(x-9)^2 = 9$

$x - 9 = \pm\sqrt{9} = \pm 3$

$x = 9 \pm 3$ $\quad 9+3 = 12$, $9-3 = 6$

SOLUTION SET ANSWER

$x \in \{6, 12\}$

$x^2 + 10x = -3$

$x^2 + 10x + 5^2 = -3 + 25$

$(x+5)^2 = 22$

$x + 5 = \pm\sqrt{22}$

$x = -5 \pm \sqrt{22}$

Handle the case where coefficient of x isn't '1.'

$$2x^2 + 12x - 2 = 0$$
$$x^2 + 6x = 1$$
$$x^2 + 6x + 3^2 = 1 + 9$$
$$(x+3)^2 = 10$$
$$x + 3 = \pm\sqrt{10}$$
$$x = -3 \pm \sqrt{10}$$

Mission One

$$7x^2 - 5x + 2 = 0$$
$$x^2 - \frac{5}{7}x + \frac{2}{7} = 0$$
$$x^2 - \frac{5}{7}x = -\frac{2}{7}$$
$$x^2 - \frac{5}{7}x + \left(\frac{5}{14}\right)^2 = -\frac{2}{7} + \frac{25}{196}$$
$$\left(x - \frac{5}{14}\right)^2 = -\frac{2}{7}\cdot\frac{28}{28} + \frac{25}{196} = \frac{-56+25}{196}$$
$$\frac{5}{14}\quad\frac{28}{7}$$
$$\frac{-31}{196}$$
$$\left(x - \frac{5}{14}\right)^2 = -\frac{31}{196}$$ (No Real Solutions?)
$$x - \frac{5}{14} = \pm\sqrt{\frac{31}{196}} = \pm i\frac{\sqrt{31}}{14}$$
$$x = \frac{5 \pm \sqrt{31}\,i}{14}$$

This is pretty mechanics-dependent. Be nice if we had a formula.

We use what we know to derive the quadratic formula.

$ax^2+bx+c=0$

$\div a$:

$x^2+\frac{b}{a}x = -\frac{c}{a}$

$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2 = -\frac{c}{a}+\frac{b^2}{(2a)^2} = -\frac{c}{a}+\frac{b^2}{4a^2} = \frac{-c}{a}\cdot\frac{4a}{4a}+\frac{b^2}{4a^2}$

$= \frac{-4ac+b^2}{4a^2}$

$= \frac{b^2-4ac}{4a^2}$

$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$x+\frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2|a|}$

BIC of $\pm$, we don't care if $a>0$ OR $a<0$

$\sqrt{4a^2} = \sqrt{4}\sqrt{a^2} = 2|a|$

$x+\frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

b^2-4ac = Discriminant = D

$D>0$ 2 real solns

$D=0$ 1 real soln

$D<0$ { NO REAL SOLN Pre 1.6
2 imaginary (nonreal) solns

From 1.6 on.

$7x^2-5x-13=0$

$a=7, b=-5, c=-13$

The Quadratic Formula gives us the solutions of the equation $ax^2 + bx + c = 0$.

1 (a) State the Quadratic Formula. (Enter your answers as a comma-separated list.)

(b) In the equation $\frac{1}{2}x^2 - x - 2 = 0$, $a =$ 1/2 , $b =$ -1 , and $c =$ -2

What is the solution of the equation? (Enter your answers as a comma-separated list.)

(2) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$2\left(\frac{1}{2}x^2 - x - 2 = 0\right)$

$x^2 - 2x - 4 = 0$

$a = 1, b = -2, c = -4$

$b^2 - 4ac = (-2)^2 - 4(1)(-4)$

$= 4 + 16 = 20$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{20}}{2(1)}$

I should've simplified $\sqrt{20}$ before going to the formula.

$\sqrt{20}$: 2 | 20, 2 | 10, 5

$2\sqrt{5}$, so, ...

$x = \frac{2 \pm 2\sqrt{5}}{2} = \frac{2(1 \pm \sqrt{5})}{2} = 1 \pm \sqrt{5}$

$x^2 - 2x - 4 = 0$

$x^2 - 2x = 4$

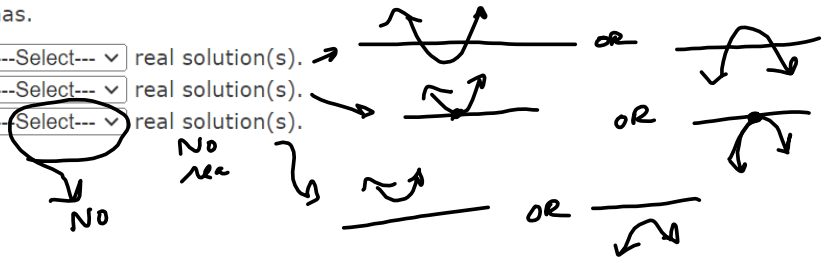
$x^2 - 2x + 1^2 = 4 + 1$

$(x-1)^2 = 5$

$x = 1 \pm \sqrt{5}$

For the quadratic equation $ax^2 + bx + c = 0$, the discriminant is $D =$ b^2-4ac . The discriminant tells us how many real solutions a quadratic equation has.

2 If $D > 0$, the equation has ---Select--- real solution(s).
If $D = 0$, the equation has ---Select--- real solution(s).
If $D < 0$, the equation has ---Select--- real solution(s).



Find all real solutions of the equation by factoring. (Enter your answers as a comma-separated list.)

3 $1x^2 + 11x - 26 = 0$

$(1)(-26) = -26$

$= -(2)(13) = -26$

$-2+13 = 11$!

$x^2 + 13x - 2x - 26$

$= x(x+13) - 2(x+13)$

$= (x+13)(x-2) = 0$

$\Rightarrow x+13 = 0$ OR $x-2 = 0$

by zero-product principle.

$x = -13$ OR $x = -2$

The cheat

$x = 2, -5$ are solutions if and only if $(x-2)$ & $(x+5)$ are factors!

$a=1, b=11, c=-26$

$b^2-4ac = 11^2-4(1)(-26) = 121+104 = 225$!

$225 = 15^2 \rightsquigarrow \sqrt{225} = 15$

$x = \frac{-11 \pm 15}{2}$ ↗ $\frac{4}{2} = 2$ ↘ $\frac{-26}{2} = -13$

$x = -13, 2$

$x \in \{-13, 2\}$

~~$x = \{-13, 2\}$~~ NO.

$(x-(-13))(x-2)$

$= (x+13)(x-2)$

is factored form!

Click Here for Writing Project #1, Problem 5 Video

Click Here for Writing Project #1, Problem 6 Video

Find all real solutions of the equation by factoring. (Enter your answers as a comma-separated list.)

4 $x^2 - 22x + 72 = 0$

Both negative (2) same sign (1)

$2 \mid 72$, $2 \mid 36$, $2 \mid 18$, $3 \mid 9$, 3

$x^2 - 4x - 18x + 72 = 0$

$\Rightarrow x(x-4) - 18(x-4) = 0$

$\Rightarrow (x-4)(x-18) = 0$

$\Rightarrow x-4=0$ or $x-18=0$

$x = 4$ or $x = 18$

Cheat

$a = 1, b = -22, c = 72$

$b^2 - 4ac = 22^2 - 4(1)(72)$

$= 484 - 288 = 196 = 14^2$

$x = \frac{+22 \pm 14}{2(1)}$

-22^2 NO

$(-22)^2 = 22^2$ Yes $= \frac{22 \pm 14}{2}$ → 18, 4

$= 11 \pm 7$

$(x-18)(x-4)$

Find all real solutions of the equation by factoring. (Enter your answers as a comma-separated list

5 $5x^2 - 34x - 7 = 0$

$-35 = -(5)(7) = (-35)(1)$

$5x^2 + x - 35x - 7$

$= x(5x+1) - 7(5x+1)$

$(5x+1)(x-7) = 0$

$x = -\frac{1}{5}, 7$

$5x^2 - 34x - 7 = 0$

$5\left(x^2 - \frac{34}{5}x - \frac{7}{5}\right) = 0$

$x^2 - \frac{34}{5}x - \frac{7}{5} = 0$

$x^2 - \frac{34}{5}x + \left(\frac{17}{5}\right)^2 = \frac{7}{5} + \frac{289}{25}$

$\left(x - \frac{17}{5}\right)^2 = \frac{7}{5}\cdot\frac{5}{5} + \frac{289}{25} = \frac{35+289}{25} = \frac{324}{25}$

$\left(\sqrt{\frac{324}{25}} = \frac{18}{5}\right)$

$\left(x - \frac{17}{5}\right)^2 = \frac{324}{25}$

$x - \frac{17}{5} = \pm\sqrt{\frac{324}{25}} = \pm\frac{\sqrt{324}}{\sqrt{25}} = \pm\frac{18}{5}$

$x = \frac{17 \pm 18}{5}$ → $\frac{35}{5}$, $\frac{-1}{5}$

Find all real solutions of the equation by factoring. (Enter your answers as a comma-separated list.)

$6x^2 - 7x - 20 = 0$

6

Magic # $6(-20) = -120$

$-7 = -8+1 \quad -8$

$= -9+2 \quad -18$

$= -20+13 \quad -260$

$= -15+8 \quad -120$ Sweet!

$6x^2 - 15x + 8x - 20$

$= 3x(2x-5) + 4(2x-5)$

$= (2x-5)(3x+4) = 0$

$x = \frac{5}{2}, -\frac{4}{3}$

so you can spoof how it factors, sufficient to convince me you can factor.

Cheat:

$a=6, b=-7, c=-20$

$b^2-4ac = (-7)^2 - 4(6)(-20)$

$= 49 + 480 = 529$

better be $529 = s^2$ for some s.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...

$5+2+9 = 16$ not divisible by 3

$23^2 = 529$ by dint of lots of work

$13^2 = 169$, $17^2 = 289$, $23^2 = 529$ ✓

$\sqrt{529} = \sqrt{23^2} = 23$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{7 \pm \sqrt{529}}{2(6)}$

$= \frac{7 \pm 23}{12}$ → $\frac{30}{12} = \frac{5}{2}$, $\frac{-16}{12} = -\frac{4}{3}$

2 | 120
2 | 60
2 | 30
5 | 15
5

$-15 + 8 = -7$ ✓

$x = -\frac{4}{3}, \frac{5}{2}$

$\frac{1}{6}(6x^2-7x-20) = (x+\frac{4}{3})(x-\frac{5}{2})$

Leading coefficient $\times 6 = 3 \cdot 2$

$6x^2-7x-20 = 3 \cdot 2 (x+\frac{4}{3})(x-\frac{5}{2})$

$= (3(x+\frac{4}{3}))(2(x-\frac{5}{2}))$

$(3x+4)(2x-5)$

Find all real solutions of the equation by factoring. (Enter your answers as a comma-separated list.)

7 $2s^2 = 5s + 3$ ⇒

$2s^2 - 5s - 3 = 0$

$-(2)(3) = -6 = (-6)(1)$

$-6 + 1$

$2s^2 - 6s + s - 3$

$2s(s-3) + 1(s-3)$

$(s-3)(2s+1) = 0$

⇒ $s = -1, 3$

⇒ $s \in \{-1, 3\}$ is solution-set answer

GOOD STYLE

Find all real solutions of the equation by factoring. (Enter your answers as a comma-separated list.)

8 $16z^2 - 58z = 63$

$16z^2 - 58z - 63 = 0$

$ac = 16(-63) = -1008$ Magic!

$-58 = -58 + 2$ ⇝ -116 small!

$= -100 + 42$ ⇝ -4200 huge

$= -70 + 12$ ⇝ -840 small. go bigger

$= -72 + 14$ ⇝ -1008 sweet!

$16z^2 - 72z + 14z - 63 = 0$

$8z(2z-9) + 7(2z-9) = 0$

$(2z-9)(8z+7) = 0$

$z \in \{\frac{9}{2}, -\frac{7}{8}\}$

OR

$z = -\frac{7}{8}, \frac{9}{2}$

Find all real solutions of the equation by factoring. (Enter your answers as a comma-separated list.)

9 $6x(x - 1) = 1 - 7x$

$6x^2 - 6x = 1 - 7x$

$-1 + 7x = -1 + 7x$

$6x^2 + x - 1 = 0$

$(-1)(6) = -(3)(2)$

$1x \rightarrow +3 - 2 = 1$

$6x^2 + x - 1 = 6x^2 + 3x - 2x - 1 = 3x(2x+1) - 1(2x+1)$

$(2x+1)(3x-1) = 0$

$\Rightarrow x \in \{-\frac{1}{2}, \frac{1}{3}\}$

Find all real solutions of the equation by completing the square. (Enter your answers as a comma-separated list.)

10 $x^2 + 4x - 2 = 0$

$\frac{4}{2} = 2 \rightsquigarrow 2^2 = 4$

$x^2 + 4x + 2^2 = 2 + 4$

$(x+2)^2 = 6$

$x + 2 = \pm\sqrt{6}$

$x = -2 \pm \sqrt{6}$

WebAssign wants

$-2 - \sqrt{6}, -2 + \sqrt{6}$

Find all real solutions of the equation by completing the square. (Enter your answers as a comma-separated list.)

11 $x^2 + 18x + 8 = 0$

$x^2 + 18x = -8$

$\frac{18}{2} = 9 \rightsquigarrow 9^2 = 81 - 8$

$x^2 + 18x + 9^2 = 73$

$(x+9)^2 = 73$

$x + 9 = \pm\sqrt{73}$

$x = -9 \pm \sqrt{73}$

Find all real solutions of the equation by completing the square. (Enter your answers as a comma-separated list.

12 $x^2 - 6x - 3 = 0$

Find all real solutions of the equation by completing the square. (Enter your answers as a comma-separated list

13 $x^2 + 2x - \frac{5}{4} = 0$ This one has fractions.

$x^2+2x = \frac{5}{4}$

$\frac{2}{2} = 1 \rightsquigarrow 1^2 = 1$

$x^2+2x+1^2 = \frac{5}{4} + 1 = \frac{5}{4} + \frac{4}{4} = \frac{9}{4}$

$(x+1)^2 = \frac{9}{4}$

$x+1 = \pm\sqrt{\frac{9}{4}} = \pm\frac{\sqrt{9}}{\sqrt{4}} = \pm\frac{3}{2}$

$x = -1 \pm \frac{3}{2} = -\frac{2}{2} \pm \frac{3}{2}$ $\frac{-2+3}{2} = \frac{1}{2}$, $\frac{-2-3}{2} = -\frac{5}{2}$

$\Rightarrow x \in \{-\frac{5}{2}, \frac{1}{2}\}$

Find all real solutions of the equation by completing the square. (Enter your answers as a comma-separated lis

14 $(8x^2 + 16x = 1) \div 8$

$x^2+2x = \frac{1}{8}$

$x^2+2x+1^2 = \frac{1}{8} + 1 = \frac{1}{8} + \frac{8}{8} = \frac{9}{8}$

$(x+1)^2 = \frac{9}{8}$

$x+1 = \pm\sqrt{\frac{9}{8}} = \pm\frac{3}{\sqrt{8}} = \pm\frac{3}{2\sqrt{2}} = \pm\frac{3\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \pm\frac{3\sqrt{2}}{4}$

$x = -1 \pm \frac{3\sqrt{2}}{4} = \frac{4 \pm 3\sqrt{2}}{4}$

Find all real solutions of the equation by completing the square. (Enter your answers as a comma-separated list.)

15 $4x^2 + 32x + 7 = 0$

Find all real solutions of the equation by completing the square. (Enter your answers as a comma-separated list.

16 $(2x^2 + 7x + 4 = 0) \div 2$

$x^2 + \frac{7}{2}x = -2$

$x^2 + \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = -2 + \frac{49}{16} = -\frac{2}{1}\cdot\frac{16}{16} + \frac{49}{16} = \frac{-32+49}{16} = \frac{17}{16}$

$\left(x + \frac{7}{4}\right)^2 = \frac{17}{16}$

$\Rightarrow x + \frac{7}{4} = \pm\sqrt{\frac{17}{16}} = \pm\frac{\sqrt{17}}{\sqrt{16}} = \pm\frac{\sqrt{17}}{4}$

$\Rightarrow x = \frac{-7 \pm \sqrt{17}}{4}$

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

17 $10x^2 + 13x - 9 = 0$

$b^2 - 4ac = 13^2 - 4(10)(-9) = 169 + 360 = 529 = 23^2$,

so this one factors, since $529 = 23^2$ is a perfect square!

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter N REAL SOLUTION.)

18 $x^2 - 10x + 1 = 0$ $b^2-4ac = (-10)^2 - 4(1)(1) = 100 - 4 = 96$

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

19 $x^2 - \frac{8}{3}x + \frac{16}{9} = 0$ $b^2-4ac = (-\frac{8}{3})^2 - 4(1)(\frac{16}{9}) = \frac{64}{3} - \frac{64}{9} =$

$= \frac{64}{3}\cdot\frac{3}{3} - \frac{64}{9} = \frac{192-64}{9} = \frac{128}{9}$

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

20 $3 + 5z + z^2 = 0$ $b^2-4ac = 5^2 - 4(1)(3) = 25 - 12 = 13$

$z^2 + 5z + 3 = 0$

Find all real solutions of the quadratic equation. (Enter your answers as a comma-separated list. If there is no real solu enter NO REAL SOLUTION.)

21 $z(z - 7) = 5$

$z^2 - 7z - 5 = 0$

Find all real solutions of the equation. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

22 $3x^2 + 2x + 2 = 0$

$a=3, b=2, c=2$ $\quad b^2-4ac = 2^2-4(3)(2) = 4-24 = -20 < 0$

$\Rightarrow$ NO REAL SOLUTION

Use the Quadratic Formula and a calculator to find all real solutions, rounded to three decimals. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

23 $x^2 - 0.011x - 0.063 = 0$ Don't Round Until the End! (Error Propagation)

Approximate Solution

$a=1, b=-0.011, c=-.063$

$b^2-4ac = (.011)^2-4(1)(-.063)$

$= .252121$

Always do b^2-4ac first, if you're only looking for real solutions, especially.

$.011^2-4(1)(-.063)$ OK

$(-.011)^2-4(-.063)$ OK

$-.011^2-4(-.063)$ Terrible

$.011^2+4(.063)$ OK

2 mistakes!

NO

$-.011^2 = .000121$

You dropped the negative!

Graphing/Advanced Calculator

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-.011) \pm \sqrt{.252121}}{2(1)}$$

Exact Solution (See Writing Project #1

$.011 = \frac{11}{1000}$

$.063 = \frac{63}{1000}$

$x^2 - .011x - .063$

$= x^2 - \frac{11}{1000}x - \frac{63}{1000} = 0$

$\Rightarrow 1000x^2 - 11x - 63 = 0$!

$b^2-4ac = 11^2 + 4(1000)(63)$

$= 121 + 252{,}000 = 252121$

$$x = \frac{11 \pm \sqrt{252121}}{2(1000)} = \frac{11 \pm \sqrt{252121}}{2000}$$

Exact!

Simplify $\sqrt{612500}$

612500
2 | 306250
2 | 153125
5 | 30625
5 | 6125
5 | 1225
5 | 245
7 | 49
7

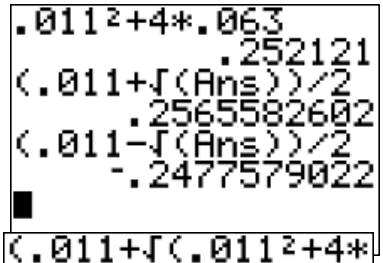
Systematic

1+5+3+1+2+5

= 17 Not divisible by 3

$\sqrt{612500} = 2 \cdot 5 \cdot 5 \cdot 7\sqrt{5}$

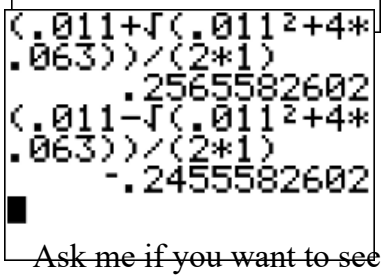
$= 350\sqrt{5}$



1st

$\approx .257 \approx x$

$\approx -.248 \approx x$



~~Ask me if you want to see~~ how to do it with a more basic calculator. It's pretty tedious, because I don't want you rounding anything until the final answer!

Use the Quadratic Formula and a calculator to find all real solutions, rounded to three decimals. (Enter your answers as a comma-separated list. If there is no real solution, enter NO REAL SOLUTION.)

24 $x^2 - 2.459x + 1.250 = 0$

Solve the equation for the indicated variable. (Enter your answers as a comma-separated list.)

25 $h = \frac{1}{2}gt^2 + v_0t$; for t

Quadratic in t

h, g, v_0 variable / indeterminate

$\frac{1}{2}gt^2 + v_0t = h$

$\frac{1}{2}gt^2 + v_0t - h = 0$

$a = \frac{1}{2}g = \frac{g}{2}$

$b = v_0$

$c = -h$

$h = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b^2-4ac = v_0^2 - 4(\frac{g}{2})(-h)$

$= v_0^2 + 2gh$

$h = \frac{-v_0 \pm \sqrt{v_0^2+2gh}}{2(\frac{g}{2})} =$

$\frac{-v_0 \pm \sqrt{v_0^2+2gh}}{g} = -\frac{v_0}{g} \pm \frac{\sqrt{v_0^2+2gh}}{g}$

FALLING BODY

g = acceleration due to gravity ($9.8 \frac{m}{s^2}$ OR $32 \frac{ft}{s^2}$)

v_0 = initial velocity

h = height above ground

(h_0 = initial height)

Model is $h = \frac{1}{2}gt^2 + v_0t + h_0$

Solve the equation for the indicated variable. (Enter your answers as a comma-separated list.)

26 $A = 4x^2 + 8xh$; for x

Use the discriminant to determine the number of real solutions of the equation. Do not solve the equation.

27 $x^2 - 5x + 1 = 0$

- ○ infinitely many real solutions
- ○ two distinct real solutions
- ○ one real solution
- ○ no real solution

Use the discriminant to determine the number of real solutions of the equation. Do not solve the equation.

28 $x^2 = 4x - 4$

- ○ infinitely many real solutions
- ○ two distinct real solutions
- ○ one real solution
- ○ no real solution

Use the discriminant to determine the number of real solutions of the equation. Do not solve the equation.

29 $6x^2 + 3x + \frac{11}{4} = 0$

- ○ infinitely many real solutions
- ○ two distinct real solutions
- ○ one real solution
- ○ no real solution

Use the discriminant to determine the number of real solutions of the equation. Do not solve the equation.

30 $x^2 + rx - s = 0 \quad (s > 0)$

- ○ infinitely many real solutions
- ○ two distinct real solutions
- ○ one real solution
- ○ no real solution

31

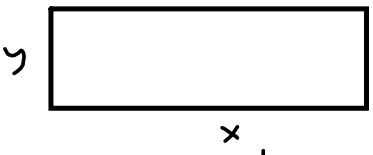
Find two numbers whose sum is 48 and whose product is 407. (Enter your answers as a comma-separated list.)

32

A farmer has a rectangular garden plot surrounded by 200 ft of fence. Find the length and width of the garden (in ft) if its area is 1,275 ft^2.

WANT EXACT ANSWER by the wording

Let: Let x = length of fence in ft
y = width of " " " .



length (larger dimension) ft

width (smaller dimension) ft

$2x + 2y$ = Perimeter = 200

Auxiliary Equation $x + y = 100$ to get rid of y. $y = 100 - x$ is CORRECT!

Area is 1275 ft^2

$xy = 1275$

$x(100 - x) = 100x - x^2 = (-x^2 + 100x = 1275)$ TIMES −1

$x^2 - 100x = -1275$

$x^2 - 100x + (50)^2 = -1275 + 2500$

$(x - 50)^2 = 1225$

$x - 50 = \pm\sqrt{1225} = \pm 35$

$x = 50 \pm 35$ x is the larger

$= 50 + 35 = 85$

$y = 100 - x = 100 - 85 = 15$

$85 = x$

$(50 \pm \sqrt{1225} = 50 \pm 35$ $15 = y$

2500
−1275
1225

1225^(1/2) 35

Sweet! Clean!

33

(a) Find the length y in the figure (in inches). The area of the shaded region is given. (Round your answer to two decimal places.)

Area of triangle $\frac{1}{2}bh$

Area of square: y^2

area = 104 in^2

Lexicon: Let y = the length of the indicated sides in the figure (in).

Area = 104 in^2

= Area of triangle plus Area of square

$= \frac{1}{2}bh + y^2 = \frac{1}{2}(y)(y) + y^2 = \frac{1}{2}y^2 + y^2 = (\frac{1}{2}+1)y^2$

$= \frac{3}{2}y^2 = 104$

$y^2 = \frac{2(104)}{3}$ ⟹

$y = \pm \frac{\sqrt{208}}{\sqrt{3}}$

$= \pm \frac{4\sqrt{13}}{\sqrt{3}}$

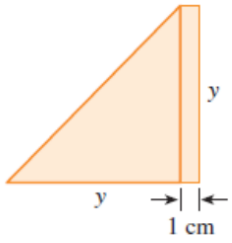
$= \frac{4\sqrt{13}}{\sqrt{3}} \approx$ 8.33 in $= y$

$(y > 0)$

$\sqrt{208} = 4\sqrt{13}$

2 | 208
2 | 104
2 | 52
2 | 26
13

(b) Find the length y in the figure (in cm). The area of the shaded region is given.



area = 1104 cm^2

```
4√(13)/√(3)
      8.326663998
```

34

You and your roommate can clean all the windows in your place in 1 h and 40 min. Working alone, it takes your roommate $2\frac{1}{2}$ h longer than it takes you to do the job. How long (in h) does it take each person working alone to wash all the windows?

you _____ h

your roommate _____ h

Lexicon Let x = # hrs for w.e to do job alone

& y = " " " roomie " " " "

Then $\frac{1}{x}$ is how much of the job I get done in 1 hr

& $\frac{1}{y}$ is " " " " " Roomie gets " " " "

AND since it takes us 1 hr 40 min = $1 + \frac{40\text{ min}}{1} \cdot \frac{1\text{ hr}}{60\text{ min}}$ hours

$= 1 + \frac{2}{3}$ hr $= \frac{3+2}{3}$ hr $= \frac{5}{3}$ hr, and

1 job done = 1 job done, we have

$\left(\frac{1}{x}\right)\left(\frac{5}{3}\right) + \left(\frac{1}{y}\right)\left(\frac{5}{3}\right) = 1$ job done !

$\left(\frac{1}{x}\ \frac{\text{Job done}}{\text{hr}}\right)\left(\#\ \text{of hours}\right) = \frac{1}{x}$ Job

Rate · Time = Amount Done

It takes your roommate 2 1/2 hours longer than you:

$2 + \frac{1}{2}$ hr $= \frac{4}{2} + \frac{1}{2}$ hr $= \frac{5}{2}$ hr, so

$y = x + \frac{5}{2}$

$\left(\frac{1}{x}\right)\left(\frac{5}{3}\right) + \left(\frac{1}{y}\right)\left(\frac{5}{3}\right) = 1$ job done !

$\left(\frac{5}{3}\right)\left(\frac{1}{x}\right) + \frac{5}{3}\left(\frac{1}{x+\frac{5}{2}}\right) = 1$

$\left(\frac{5}{3x} + \frac{5}{3\left(x+\frac{5}{2}\right)} = 1\right)\left(3x\right)\left(3\left(x+\frac{5}{2}\right)\right)$

$\Rightarrow \left(\frac{5}{3x}\right)\left(3x\right)\left(3\left(x+\frac{5}{2}\right)\right) + \left(\frac{5}{3\left(x+\frac{5}{2}\right)}\right)\left(3x\right)\left(3x+\frac{15}{2}\right) = 1\left(3x\right)\left(3x+\frac{15}{2}\right)$

$3(x+\frac{5}{2}) = 3x+\frac{15}{2}$

$\Rightarrow \left(\frac{5}{3x}\right)(3x)(3(x+\frac{5}{2})) + \left(\frac{5}{3(x+\frac{5}{2})}\right)(3x)(3x+\frac{15}{2}) = 1\,(3x)(3x+\frac{15}{2})$

$\Rightarrow 5(3x+\frac{15}{2}) + 5(3x) = 9x^2+\frac{45}{2}x$

$\Rightarrow \left(15x+\frac{75}{2}+15x = 9x^2+\frac{45}{2}x\right)(2)$

$\Rightarrow 30x+75+30x = 18x^2+45x$

$60x+75 = 18x^2+45x$

$-60x-75 = \quad -60x-75$

$0 = 18x^2-15x-75 = 0$

$\Rightarrow 3(6x^2-5x-25) = 0$

$\Rightarrow 6x^2-5x-25 = 0$

$(3)(2)(5)(5)$

$-(15)(10) \rightsquigarrow -15+10 = -5$ ✓

$\Rightarrow 6x^2-15x+10x-25$

$\Rightarrow 3x(2x-5)+5(2x-5)$

$= (2x-5)(3x+5) = 0 \Rightarrow$

$x = \frac{5}{2}$ or $x = -\frac{5}{3}$ → TIME TRAVEL!

$x=\frac{5}{2} \Rightarrow y = x+\frac{5}{2} = \frac{5}{2}+\frac{5}{2} = \frac{10}{2} = 5 = y$

35 Distance-Rate-Time Question

It took a crew 3 h 9 min to row 9 km upstream and back again. If the rate of flow of the stream was 3 km/h, what was the rowing speed (in km/h) of the crew in still water?

Lexicon (3 out of 10 pts)

Let r = rate of speed of boat in still water $\left(\frac{km}{hr}\right)$

Units: hours, Km

	D	r	t
up	9	$r-3$	$\frac{D}{r-3}$
Down	9	$r+3$	$\frac{D}{r+3}$
TOTAL	18	r	$\frac{D}{r-3}+\frac{D}{r+3}$

$\frac{D}{r-3}+\frac{D}{r+3} = 3hr\ 9\ min = \frac{3}{1}+\frac{9}{60} = \frac{3}{1}+\frac{3}{20}$

$= \frac{60+3}{20} = \frac{63}{20}$

$\left(\frac{D}{r-3}+\frac{D}{r+3}=\frac{63}{20}\right)(20)(r-3)(r+3)$

$\Rightarrow$

$\Rightarrow \quad (D)(20)(r+3)+(D)(20)(r-3) = 63(r-3)(r+3)$

$20D(r+3)+20D(r-3) = 63(r^2-9) = 63r^2-9(63)$

Duh.. $D=9$!

$180(r+3)+180(r-3) = 63r^2-567$

$180r+540+180r-540 = 63r^2-567$

$360r = 63r^2-567$

$(63r^2-360r-567=0)\div 3$

$(21r^2-120r-189=0)\div 3$

$7r^2-40r-63=0$

$7(7)(3)(3)$

$7r^2-49r+9r-63=0$

$7r(r-7)+9(r-7)=$

$(r-7)(7r+9)$

$r=7$!