MAT 121 Take-Home Test 3 – Spring, 2022 Name_____
30 Points WRITING PROJECT #3 GRAPHING UTILITY RECOMMENDED

FORMATTING: This is semi-formal writing, here. That means show some professionalism. You don't have to type it out, but you do need to be very clear. See Course Schedule for due dates. **Staple this page, with your name on it, as a cover sheet for your project. Do not staple your project to your test. This project is due Friday, March 25th.**

1. Write on only one side of each page. I will not award (or deduct) points for anything on the backs of pages.
2. Plain white paper without lines (8 ½ x 11-inch A4 copier paper works just fine). Paper with lines:
3. Staple top left corner. Do NOT staple over problem numbers or any of your work. If I can't see it, you didn't do it.
4. Leave margins. "MAT 121" in big letters in top left corner of every page solves all problems with margins.
5. Write DARK. I don't mind if you use pen. Just put a line through mistakes. Pencil's good, but make sure you're getting it DARK, i.e., BLACK, with a white background.
6. Leave ROOM between problems and between steps on your work. I have bad eyes, so being stingy with space and paper is a mistake on Writing Projects. **Don't do work in 2 columns!**

For early feedback, make a black-and-white, multi-page PDF and upload it to the D2L drop-box for Writing Project #3. Otherwise, mail your neat, clear, black-and-white, one-side-of-each-page work to me at:

Harry Mills
EDBH 134K
Aims Community College
5401 West 20th Street
Greeley, CO 80634

Alternatively, you may just slide it under my office door in Ed Beaty by or before the deadline: EDBH 134K

Mail or E-Mail your Writing Project 2 by or before Friday, March 25th. Late work accepted as late as Thursday, March 31st, at a 20% discount.

Main Resources: Chapter 3 Videos (and notes), Writing Project 2 Videos (and notes), and a selection of Old Writing Projects.

BEGIN TEST:

We will be working with $f(x) = 6x^5 - 5x^4 - 44x^3 + 49x^2 + 84x - 108$ for most of this test. We'll say everything about this polynomial that's worth saying.

1. (2 pts) Describe the end behavior of f with a simple graphic.

2. (2 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeros.

3. (2 pts) Use the Rational Zeros Theorem to determine the *possible* rational zeros (roots) of f.

4. (2 pts) Using a graphing utility (only smart to do so on a take-home) and the information, above, to find all the *real* roots of f. This will involve using synthetic division to split off one factor at a time, and, at each step, working with the remaining, very depressed polynomial.

5. (2 pts) From your work, above, factor f over the real numbers. This will involve an irreducible quadratic factor that your grapher has no way of helping you to see, without the synthetic divisions in #4, bringing

you closer and closer, step by step, to the irreducible quadratic.

6. (2 pts) Give a rough sketch of f from all of the above information. This is an *art* whose essence is really only found in my videos. If you're too tied to your grapher's output, you'll not capture the real essence of what's going on, or the key features I'm always looking for.

7. (2 pts) Now we've covered everything *real* about f. Let's use that work to find *all* the roots of f and *split* f *into linear factors*. 5 roots are *guaranteed by the Fundamental Theorem of Algebra,* and we have found the 3 real ones. The other 2 are nonreal, hiding inside the irreducible quadratic polynomial that remains as the last, very very depressed piece that's not broken all the way down in #5. Now do your quadratic equation thing to *find* the 2 nonreal roots. *Finally*, apply the Factor Theorem to *all* the above work, and represent f as a product of linear factors, $f(x)=a(x-r_1)^{m_1}(x-r_2)^{m_2}\cdots(x-r_w)^{m_w}$. Don't forget the leading coefficient, a.

   This wrings (almost) every useful drop of the Theorems on Polynomials out of f, so now on to Rational Functions, which are *quotients* of polynomials!

8. (5 pts) Sketch the graph of $R(x)=\dfrac{3x^2-13x-10}{x^2+4x-21}$, showing all intercepts, asymptotes, and capturing the *essential features* including the shape of the graph. If you're a slave to your grapher, and oblivious to the features I'm looking for, it'll jump off the page at me (and be bad).

Note: There *is* a subtle feature to this graph that I downplay on tests, but you should pick up on with a take-home, namely, the horizontal asymptote *does* intersect the graph of the function

I'm willing to part with **5 bonus points** if you can find the point of intersection of $R(x)$ and label it with an ordered-pair label.

9. (2 pts) Sketch the graph of $Q(x)=\dfrac{3x^3+11x^2-114x-80}{x^3+12x^2+11x-168}$. All the work you did for #8 applies to this one, *except* for the *hole* in the graph of Q, which I expect you to find and clearly label in your graph.

10. (5 pts) Sketch the graph of $T(x)=\dfrac{x^3+12x^2+11x-168}{3x^2-13x-10}$, showing all intercepts and asymptotes.

Now for a pair of questions many struggle with on the sit-down test, but which are actually *very simple* if you can synthesize your skills and *apply* them to these sorts of questions. Often the downfall of people on the sit-down, but designed to be easy points for people who are putting things together.

For HELP on these problems, you want to look at Test Prep Videos, in particular the Test-Prep Videos for the SIT-DOWN Test 3, because the old Take-Home 3/Writing Project #3 didn't have these type-questions.

11. (2 pts) What is the domain of $W(x)=\sqrt{(x-2)(x+1)(x-7)(x+3)^2}$?

12. (2 pts) What is the domain of $K(x)=\sqrt{\dfrac{(x-2)(x-7)}{(x+1)(x+3)^2}}$?