12 Writing Project #3 Fall '18

30 Points towards Test 3

(1) $f(x) = 6x^5 - 5x^4 - 44x^3 + 49x^2 + 84x - 108$

2pts

End Behavior: $6x^5$

(2) Descartes':

$f(x) = 6x^5 - 5x^4 - 44x^3 + 49x^2 + 84x - 108$

2pts

1 2 3

3 or 1 pos. zeros

$f(-x) = -6x^5 - 5x^4 + 44x^3 + 49x^2 - 84x - 108$

1 2

2 or 0 neg. zeros

(3) Rational zeros: $\frac{p's}{q's} : \frac{108}{6}$

2pts

2 | 108	2 | 6
2 | 54	3
3 | 27	
3 | 9	
3	

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm 18, \pm 24, \pm 36, \pm 54, \pm 108, \pm 9$

$\pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{3}{2}, \pm \frac{4}{2}, \pm \frac{6}{2}, \pm \frac{12}{2}, \pm \frac{18}{2}, \pm \frac{24}{2}, \pm \frac{36}{2}, \pm \frac{54}{2}, \pm \frac{108}{2}, \pm \frac{9}{2}$

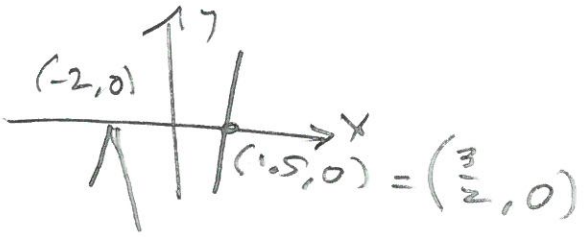
$\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{3}{3}, \pm \frac{4}{3}, \pm \frac{6}{3}, \pm \frac{12}{3}, \pm \frac{18}{3}, \pm \frac{24}{3} = \pm 8, \pm \frac{36}{3}, \pm \frac{54}{3}, \pm \frac{108}{3}, \pm \frac{9}{3}$

$\pm \frac{1}{6}, \pm \frac{2}{6}, \pm \frac{3}{6}, \pm \frac{4}{6}, \pm \frac{6}{6}, \pm \frac{12}{6}, \pm \frac{18}{6}, \pm \frac{24}{6}, \pm \frac{36}{6}, \pm \frac{54}{6}, \pm \frac{108}{6}, \pm \frac{9}{6}$

I count 21 possibles.

121 wp #3

(4) 20 pts

Grapher shows 

Suggests $x=-2$, $m=$ even

$\frac{3}{2}=$ $x=1.5$, $m=$ odd

$$\begin{array}{r|rrrrr} -2 & 6 & -5 & -44 & 49 & 84 & -108 \\ & & -12 & 34 & 20 & -138 & 108 \\ \hline -2 & 6 & -17 & -10 & 69 & -54 & 0 \end{array}$$ sweet!

$$\begin{array}{r|rrrr} & & -12 & 58 & -96 & 54 \\ \hline -2 & 6 & -29 & 48 & -27 & 0 \end{array}$$ sweet!

$$\begin{array}{r|rrrr} & & -12 & 82 & -260 \\ \hline & 6 & -41 & 130 & \text{No} \end{array}$$

$$\begin{array}{r|rrrr} \frac{3}{2} & 6 & -29 & 48 & -27 \\ & & 9 & -30 & 27 \\ \hline & 6 & -20 & 18 & 0 \end{array}$$ sweet!

x^2 x c r

Now the depressed poly. is quadratic, & we put the hammer down.

$6x^2-20x+18 = 2(3x^2-10x+9)$

$a=3,\ b=-10,\ c=9$

$b^2-4ac = (-10)^2-4(3)(9) = 100-108 = -8 < 0$

$\Rightarrow$ No real roots.

20 pts

#5 So $(x+2)^2\left(x-\frac{3}{2}\right)(6x^2-20x+18)$ is factored over $\mathbb{R}$. $6x^2-20x+18$ is irreducible over $\mathbb{R}$.

#4 $x=-2, m=2, x=1, m=$

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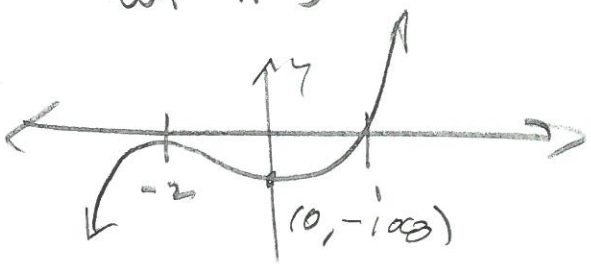
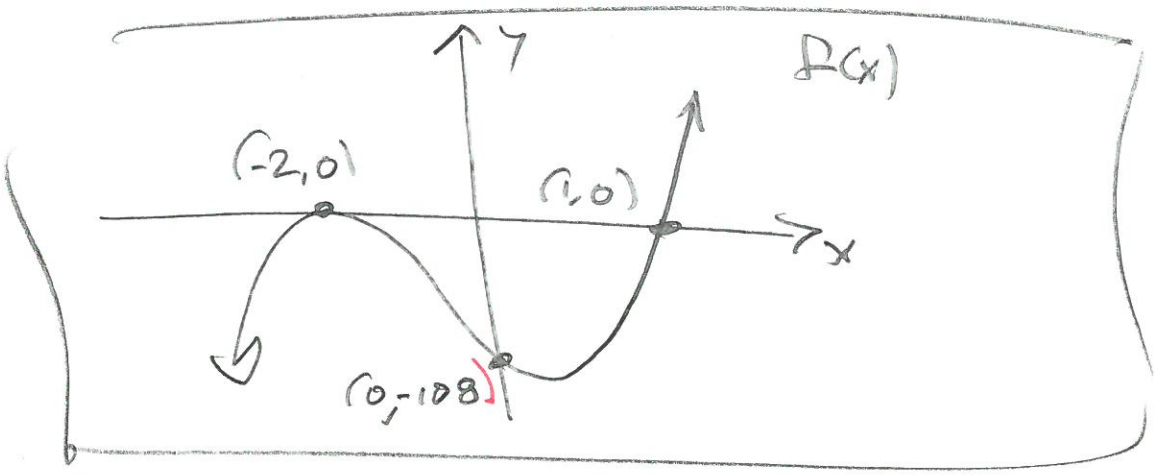
WP #3

(6) 20 pts





(7) 20 pts

$6x^2-20x+18 = 2(3x^2-10x+9) \overset{\text{SET}}{=} 0$

$\rightarrow 3x^2-10x+9=0 \rightarrow$

$b^2-4ac = -8 \quad @ \quad \sqrt{-8} = 2i\sqrt{2}$

$\Rightarrow x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{10 \pm 2i\sqrt{2}}{2(3)} = \boxed{\dfrac{5 \pm i\sqrt{2}}{3} = x}$

$\Rightarrow f(x) = 6(x+2)^2(x-1)\left(x-\left(\dfrac{5+i\sqrt{2}}{3}\right)\right)\left(x-\left(\dfrac{5-i\sqrt{2}}{3}\right)\right)$

121 WP #3

(8) $R(x) = \dfrac{3x^2+13x-10}{x^2+4x-21} = \dfrac{(x-5)(3x+2)}{(x+7)(x-3)}$

-30 = MAGIC

$-13 = -14+1$

$= -15+2$ ✓

$3x^2-13x-10$

$= 3x^2-15x+2x-10$

$= 3x(x-5)+2(x-5)$

$= (x-5)(3x+2)$

2 | 30
3 | 15
5

$\mathcal{D} = \mathbb{R} \setminus \{-7, 3\}$

$x=-7$, $x=3$ V.A.

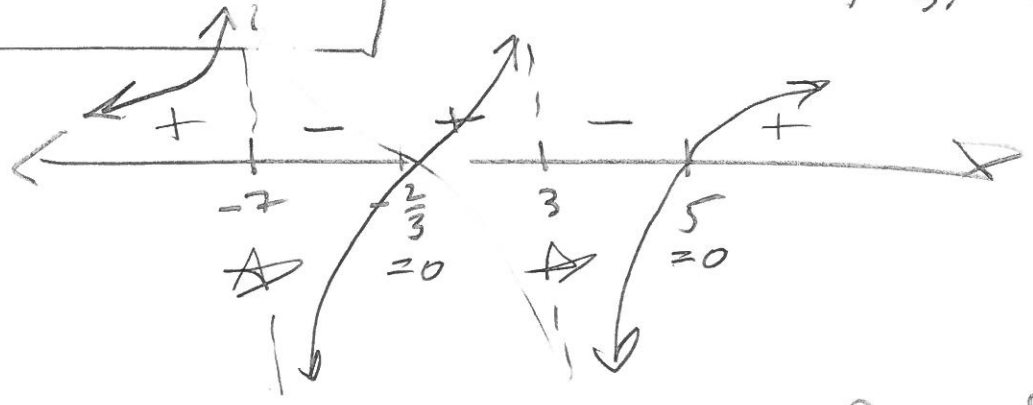
H.A.: $\dfrac{3x^2+\dots}{x^2+\dots} \xrightarrow{x\to\infty} 3 = y$ H.A

x-int: $(-\frac{2}{3}, 0)$, $(5, 0)$

y-int: $(0, \frac{10}{21})$

$-\frac{2}{3}, 5, -7, 3$

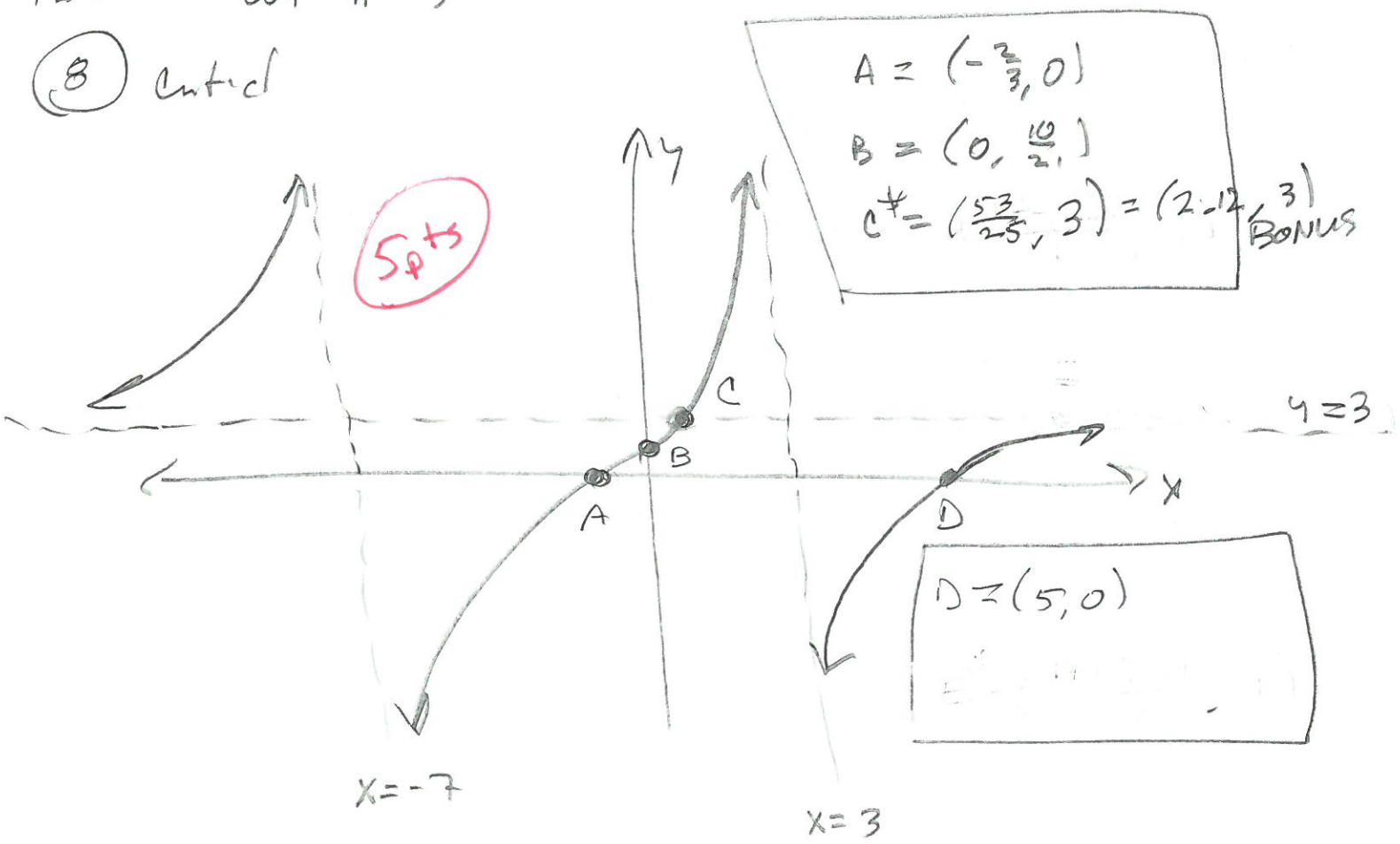
$-7, -\frac{2}{3}, 3, 5$



SEE NEXT PAGE FOR FINISHED GRAPH

121 WP # 3

(8) Critical



$A = (-\frac{2}{3}, 0)$

$B = (0, \frac{10}{21})$

$C^* = (\frac{53}{25}, 3) = (2.12, 3)$ BONUS

$D = (5, 0)$

BONUS: $R(x) = 3 = H.A.$

$$\frac{3x^2-13x-10}{x^2+4x-21} = 3 = \frac{3x^2+12x-63}{x^2+4x-21} \longrightarrow$$

$$3x^2 - 13x - 10 = 3x^2 + 12x - 63$$

$$-13x - 10 = 12x - 63$$

$$-25x = -53$$

$$x = \frac{53}{25} = 2.12 \longrightarrow C = \left(\frac{53}{25}, 3\right)$$

121

WP #3

(7) (2pts) $\dfrac{3x^3+11x^2-114x-80}{x^3+12x^2+11x-168}$

$-\frac{2}{3}$ | 3 11 −114 −80

−2 −6 80

5 | 3 9 −120 0

15 120

3 24 0

$3x+24 = 3(x+8) \Rightarrow x+8$ is the hidden factor

y=

−7 | 1 12 11 −168

−7 −35 168

3 | 1 5 −24 0

3 24

1 8 0

$x+8$. Good!

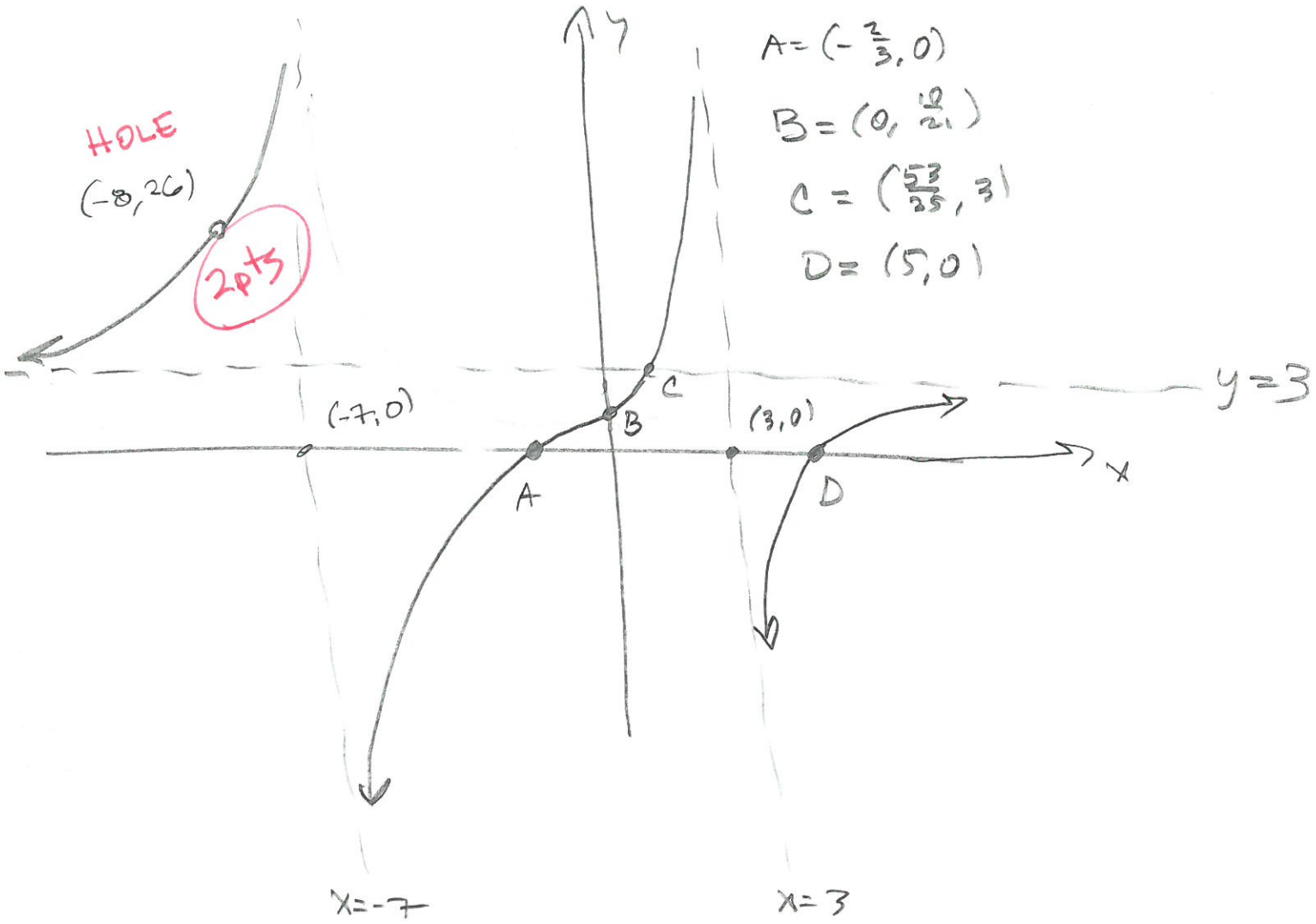
So, $Q(x) = \dfrac{(3x+2)(x-5)(x+8)}{(x+7)(x-3)(x+8)} = R(x)$, with hole @ $x=-8$

$R(-8) = \dfrac{(3(-8)+2)(-8-5)}{(-8+7)(-8-3)} = \dfrac{(-22)(-13)}{(-1)(-11)} = 26$

HOLE @ $(-8, 26)$

121 WP #3

(9) cntʼcl



121 WP #3

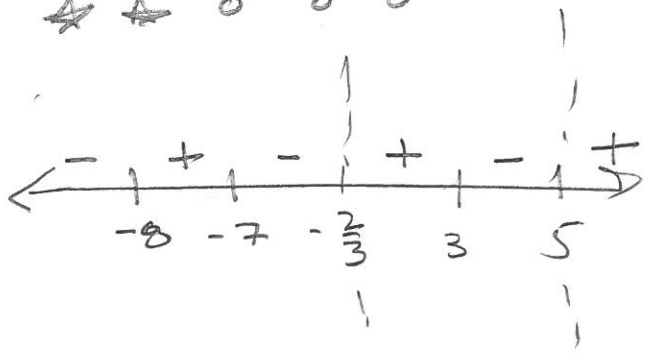
(10) $T(x) = \dfrac{x^3+12x^2+11x-168}{3x^2-13x-10} = \dfrac{(x+7)(x-3)(x+8)}{(3x+2)(x-5)}$

by previous work.

$-\frac{2}{3}, 5, -8, -7, 3$

$\not\exists \quad \not\exists \quad 0 \quad 0 \quad 0$

$D = \mathbb{R} \setminus \{-\frac{2}{3}, 5\}$



V.A.: $x = -\frac{2}{3}, x = 5$

H.A.: NONE

x-int: $(-8,0), (-7,0), (3,0)$

y-int: $(0, \frac{168}{10}) = (0, \frac{84}{5})$

$\frac{x^3}{3x^2} = \frac{1}{3}x$

Slant asymptote:

$$\begin{array}{r|l} & \frac{1}{3}x + \frac{49}{9} \\ 3x^2-13x-10 & x^3+12x^2+11x-168 \\ & -\left(x^3 - \frac{13}{3}x^2 - \frac{10}{3}x\right) \\ \hline & \frac{49}{3}x^2 \end{array}$$

$\frac{12.3}{3} + \frac{13}{3}$

$= \frac{36+13}{3} = \frac{49}{3}$

$\frac{\frac{49}{3}x^2}{3x^2} = \frac{49}{9}$

S.A. $y = \frac{1}{3}x + \frac{49}{9} = .3x + 5.\overline{4}$

$(0, \frac{49}{9}) \notin \frac{1}{3}x + \frac{49}{9} = 0$

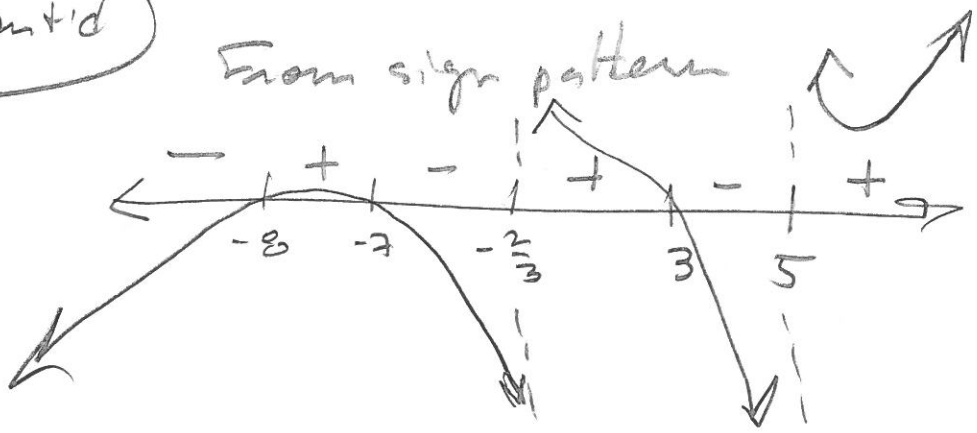
$\Rightarrow \frac{1}{3}x = -\frac{49}{9}$

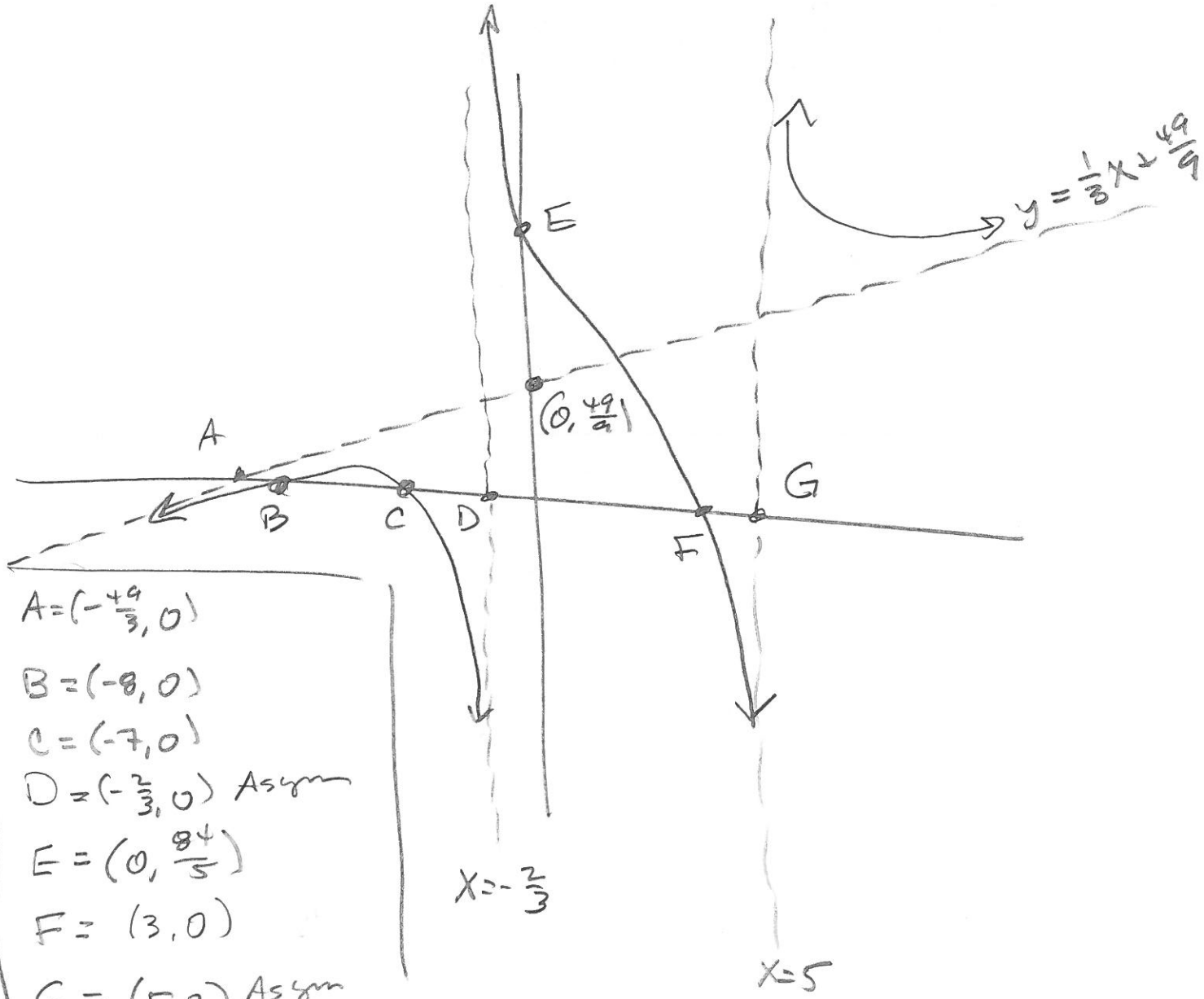
$\Rightarrow x = -\frac{49}{3}$

Intercepts for Asymptote $(-\frac{49}{3}, 0) = (16.\overline{3}, 0)$

121 WP #3

10 cont'd





$A = (-\frac{49}{3}, 0)$

$B = (-8, 0)$

$C = (-7, 0)$

$D = (-\frac{2}{3}, 0)$ Asym

$E = (0, \frac{84}{5})$

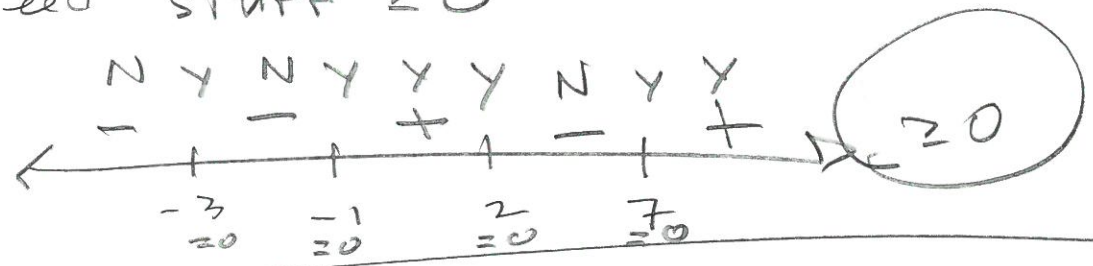
$F = (3, 0)$

$G = (5, 0)$ Asym

121 WP #3

(11) (2pts) $W(x) = \sqrt{(x-2)(x+1)(x-7)(x+3)^2} = \sqrt{\text{STUFF}}$

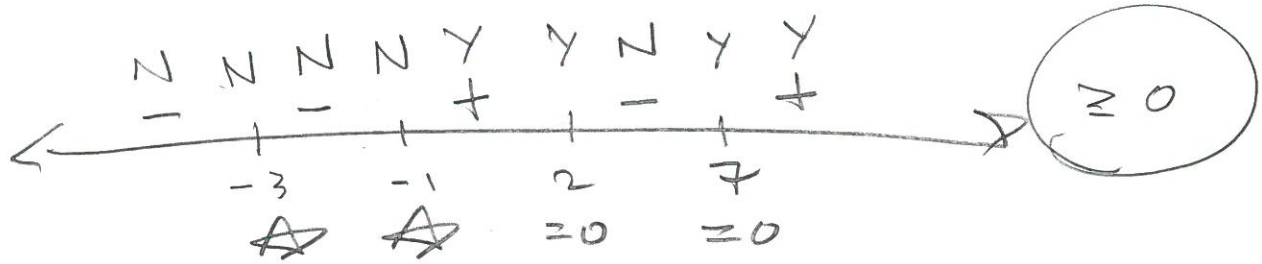
Need STUFF ≥ 0



$\Rightarrow \boxed{\mathcal{D}(W) = \{-3\} \cup [-1, 2] \cup [7, \infty)}$

(12) (2pts) $K(x) = \sqrt{\dfrac{(x-2)(x-7)}{(x+1)(x+3)^2}} = \sqrt{\text{STUFF}}$

NEED STUFF ≥ 0



$\Rightarrow \mathcal{D}(K) = (-1, 2] \cup [7, \infty)$