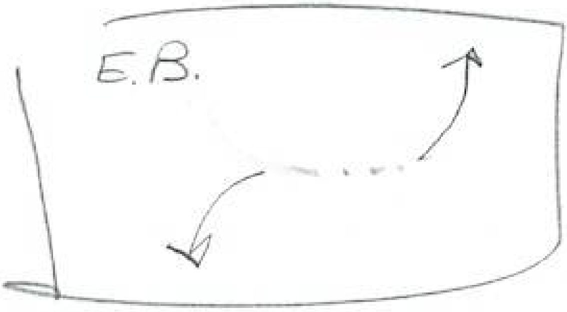
121 Writing Project #3 Spring 2021

(1) $f(x) = 9x^5 - 75x^4 - 224x^3 + 2172x^2 + 1511x + 255$

$9 = 3 \cdot 3$

$$\begin{array}{r|l} 3 & 255 \\ 5 & 85 \\ & 17 \end{array}$$



(2) $\underbrace{9x^5 - 75x^4}_{1} \underbrace{- 224x^3 + 2172x^2}_{2} + 1511x + 255$

$\boxed{\text{2 or 0 positive zeros}}$

$f(-x) = -9x^5 \underbrace{- 75x^4 + 224x^3}_{1} \underbrace{+ 2172x^2 - 1511x}_{2} \underbrace{- 1511x + 255}_{3}$

$\boxed{\text{3 or 1 negative zeros.}}$

(3) p: 255's factors
q: 9's factors.

$\pm 1, \pm \frac{1}{3}, \pm \frac{1}{9}, \pm 3, \cancel{\pm \frac{3}{3}}, \cancel{\pm \frac{3}{9}}, \pm 5, \pm \frac{5}{3}, \pm \frac{5}{9},$

$\pm 15, \cancel{\pm \frac{15}{3} = 5}, \cancel{\pm \frac{15}{9} = \frac{5}{3}},$

$\pm 17, \pm \frac{17}{3}, \pm \frac{17}{9},$

$\pm 51, \cancel{\pm \frac{51}{3} = \pm 17}, \cancel{\pm \frac{51}{9} = \pm \frac{17}{3}}, \pm 85, \pm \frac{85}{3}, \pm \frac{85}{9},$

$\pm 255, \cancel{\pm \frac{255}{3} = \pm 85}, \cancel{\pm \frac{255}{9} = \pm \frac{85}{3}}$

121 wp #3 ②

④ Quick computer sketch:

← Picture shows −5, not +5!

5	9	−75	−224	2172	1511	255
		45	−150	−1870	1510	
	9	−30	−374	302		Nope

−5	9	−75	−224	2172	1511	255	
		−45	600	−1880	−1460	−255	
$-\frac{1}{3}$	9	−120	376	292	51	0	Sweet!
		−3	41	−139	−51		
$-\frac{1}{3}$	9	−123	417	153	0		Sweet!
		−3	42	−153			
	9	−126	459	0			

This work says $f(x) = (x+5)(x+\frac{1}{3})^2(9x^2-126x+459)$

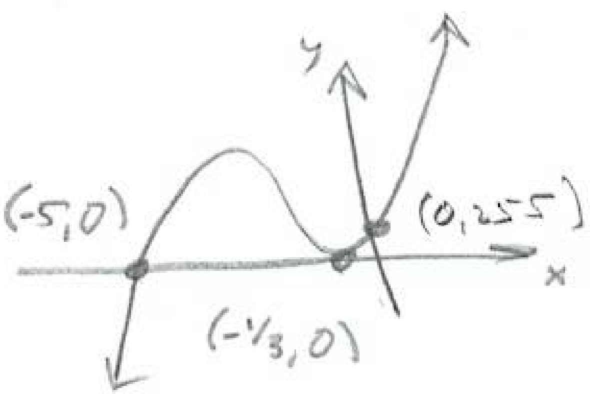
$$\boxed{(x+5)(x+\tfrac{1}{3})^2(9x^2-126x+459)}$$

$b^2-4ac = (-126)^2-4(9)(459) = 15876-16524$

$= -648 < 0 \Rightarrow$ No real zeros

$$\boxed{\begin{array}{l} x=-5 \\ x=-\frac{1}{3} \text{ w/ } m=2 \end{array}}$$

6



7

$2 \lfloor 648$, $2 \lfloor 324$, $2 \lfloor 162$, $3 \lfloor 81$, $3 \lfloor 27$, $3 \lfloor 9$, 3

$\sqrt{648} = 2 \cdot 3^2 \sqrt{2}$

$= 18\sqrt{2}$

$2 \lfloor 126$, $3 \lfloor 63$, $3 \lfloor 21$, 7

$2 \lfloor 18$, $3 \lfloor 9$, 3

$$x = \frac{126 \pm 18i\sqrt{2}}{4(9)} = \frac{18(7 \pm i\sqrt{2})}{36}$$

$$= \frac{7 \pm \sqrt{2}\,i}{2} = x, \text{ so}$$

$a = 9,\ b = -126,\ c = 459$

$$\boxed{f(x) = 9(x+5)(x+\tfrac{1}{3})^2\left(x - \left(\frac{7+i\sqrt{2}}{2}\right)\right)\left(x - \left(\frac{7-i\sqrt{2}}{2}\right)\right)}$$

8

$$R(x) = \frac{6x^2 + 11x - 35}{x^2 + x - 20} = \frac{(2x+7)(3x-5)}{(x+5)(x-4)}$$

$6 = 3 \cdot 2$

$-35 = -5 \cdot 7$

$6x^2 + 21x - 10x - 35 = 3x(2x+7) - 5(2x+7)$

$= (2x+7)(3x-5)$

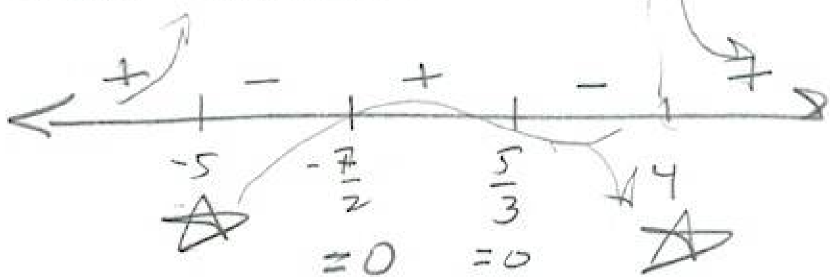
Domain: $\mathbb{R} \setminus \{-5, 4\}$

$\boxed{\text{V.A.: } x = -5,\ x = 4}$

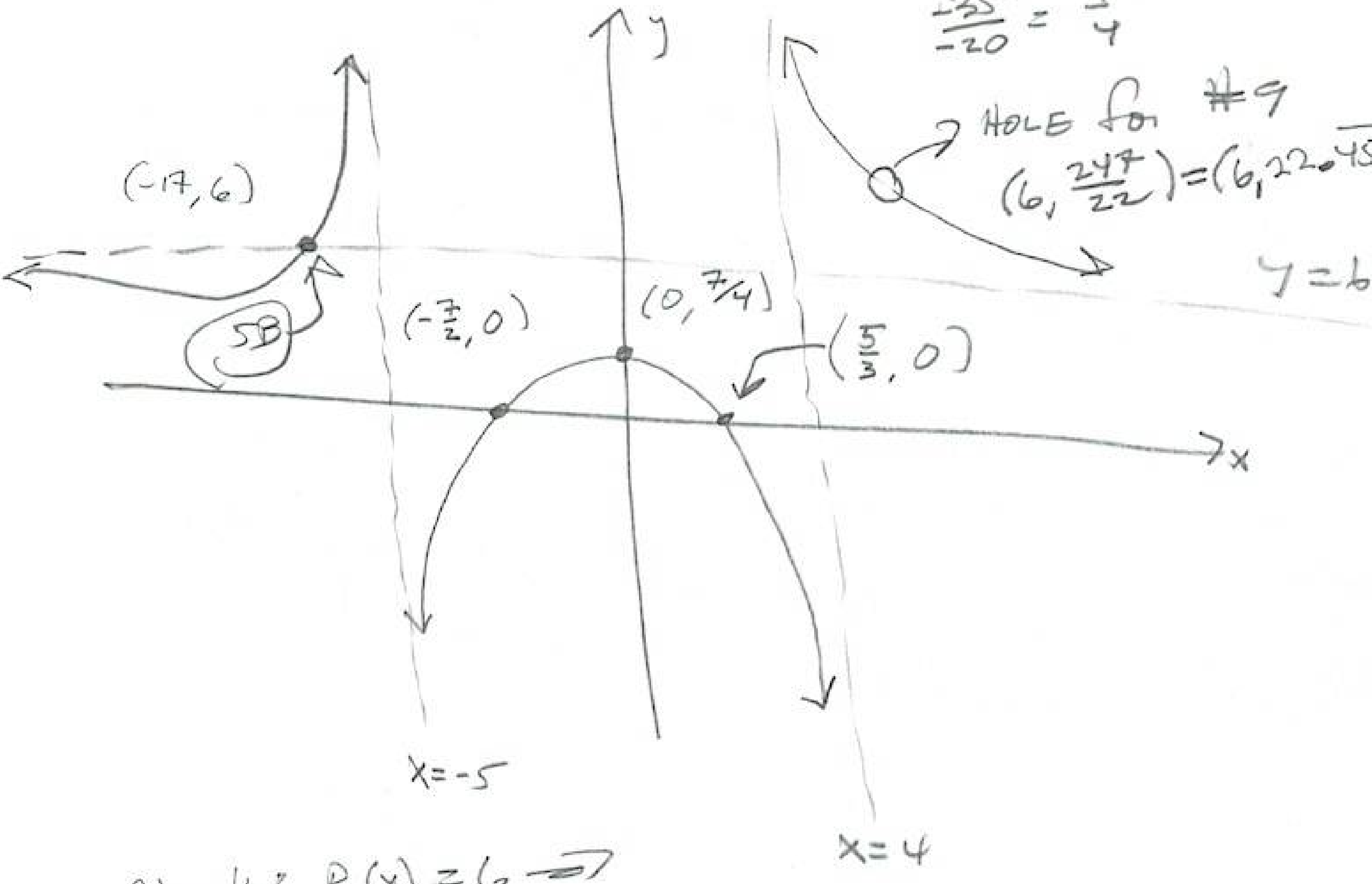
H.A.: $y = \frac{6x^2}{x^2} = \boxed{6 = y}$ H.A.

x-int: $(-\frac{7}{2}, 0), (\frac{5}{3}, 0)$

SIGN PATTERN:



y-int:
$\frac{-35}{-20} = \frac{7}{4}$



Check: $R(x) = 6 \rightarrow$

$$\frac{6x^2+11x-35}{x^2+x-20} = \frac{6}{1} \cdot \frac{x^2+x-20}{x^2+x-20} \rightarrow$$

$$\frac{6x^2+11x-35-6x^2-6x+120}{LCD} = 0 \rightarrow$$

$5x + 85 = 0 \rightarrow$

$5x = -85 \rightarrow x = -17$

(9) $Q(x) = R(x)$ with a hole.

Find it:

$$Q(x) = \frac{6x^3-25x^2-101x+210}{x^3-5x^2-26x+120} = \frac{(6x^2+11x-35)(x-c)}{(x^2+x-20)(x-c)}$$

Denominator's cleaner:

$x=-5$ & $x=4$ are zeros:

$$\begin{array}{r|rrrr} -5 & 1 & -5 & -26 & 120 \\ & & -5 & 50 & -120 \\ \hline 4 & 1 & -10 & 24 & 0 \\ & & 4 & -24 & \\ \hline & 1 & -6 & 0 & \end{array}$$

So $x-c = x-6 \overset{\text{SET}}{=} 0 \longrightarrow$ $x=6$ is hole.

Check: $$\begin{array}{r|rrrr} 6 & 6 & -25 & -101 & 210 \\ & & 36 & 66 & -210 \\ \hline & 6 & 11 & -35 & 0 \end{array}$$ ✓

EVALUATE $R(6)$ (not $Q(6)$)

$$\frac{6(6)^2+11(6)-35}{6^2+6-20} = \frac{216+66-35}{22} = \frac{247}{22}$$

HOLE: $\left(6, \frac{247}{22}\right) \approx (6, 22.\overline{45}$

See #8 (HOLE PART)

10 $T(x) = \dfrac{6x^3 - 25x^2 - 101x + 210}{3x^2 + x - 10} = \dfrac{(2x+7)\cancel{(3x-5)}(x-6)}{\cancel{(3x-5)}(x+2)}$

$-30 = (-5)(6)$

$3x^2 - 5x + 6x - 10 = x(3x-5) + 2(3x-5)$

$= (3x-5)(x+2)$

Relied on #s 8, 9 for factorization of the numerator.

$\mathcal{D} = \mathbb{R} \setminus \{-2, \frac{5}{3}\}$ V.A.: $\boxed{x = -2, \ x = \frac{5}{3}}$

x-int: $(-\frac{7}{2}, 0), (\frac{5}{3}, 0), (6, 0)$

Hole!

Teacher tricked me! This one has a hole, too! $x = \frac{5}{3}$?!

OK.

$T^*(x) = \dfrac{(2x+7)(x-6)}{x+2}$

$= \dfrac{2x^2 - 5x - 42}{x+2}$

$$\begin{array}{r|rrrr} 6 & 6 & -25 & -101 & 210 \\ & & 36 & 66 & -210 \\ \hline -\frac{7}{2} & 6 & 11 & -35 & 0 \\ & & -21 & 35 & \\ \hline & 6 & -10 & 0 & \end{array}$$

✓

O.A.:

$$\begin{array}{r|rrr} -2 & 2 & -5 & -42 \\ & & -4 & 18 \\ \hline & 2 & -9 & -24 \end{array}$$

$y = \boxed{2x - 9 \text{ is O.A.}}$

#10 cont'd

Check for intersection w/ O.A.:

$$T^{*}(x) = \frac{2x^2-5x-42}{x+2} = 2x-9 \Rightarrow$$

$$\frac{2x^2-5x-42}{x+2} = \frac{2x-9}{1}\cdot\frac{x+2}{x+2} = \frac{2x^2-5x-18}{x+2} \Rightarrow$$

$$\frac{2x^2-5x-42-2x^2+5x+18}{LCD} = 0 \Rightarrow$$

$$\frac{-24}{LCD} = 0 \Rightarrow \text{Never, so no surprises, there}$$

Now, for the hole @ $x = \frac{5}{3}$:

$$T^{*}\left(\frac{5}{3}\right) = \frac{2\left(\frac{5}{3}\right)^2-5\left(\frac{5}{3}\right)-42}{\frac{5}{3}+2}$$

$$= \frac{\frac{50}{9}-\frac{25}{3}\cdot\frac{3}{3}-\frac{42}{1}\cdot\frac{9}{9}}{\frac{5}{3}+\frac{2}{1}\cdot\frac{3}{3}} = \frac{\frac{50-75-378}{9}}{\frac{5+6}{3}}$$

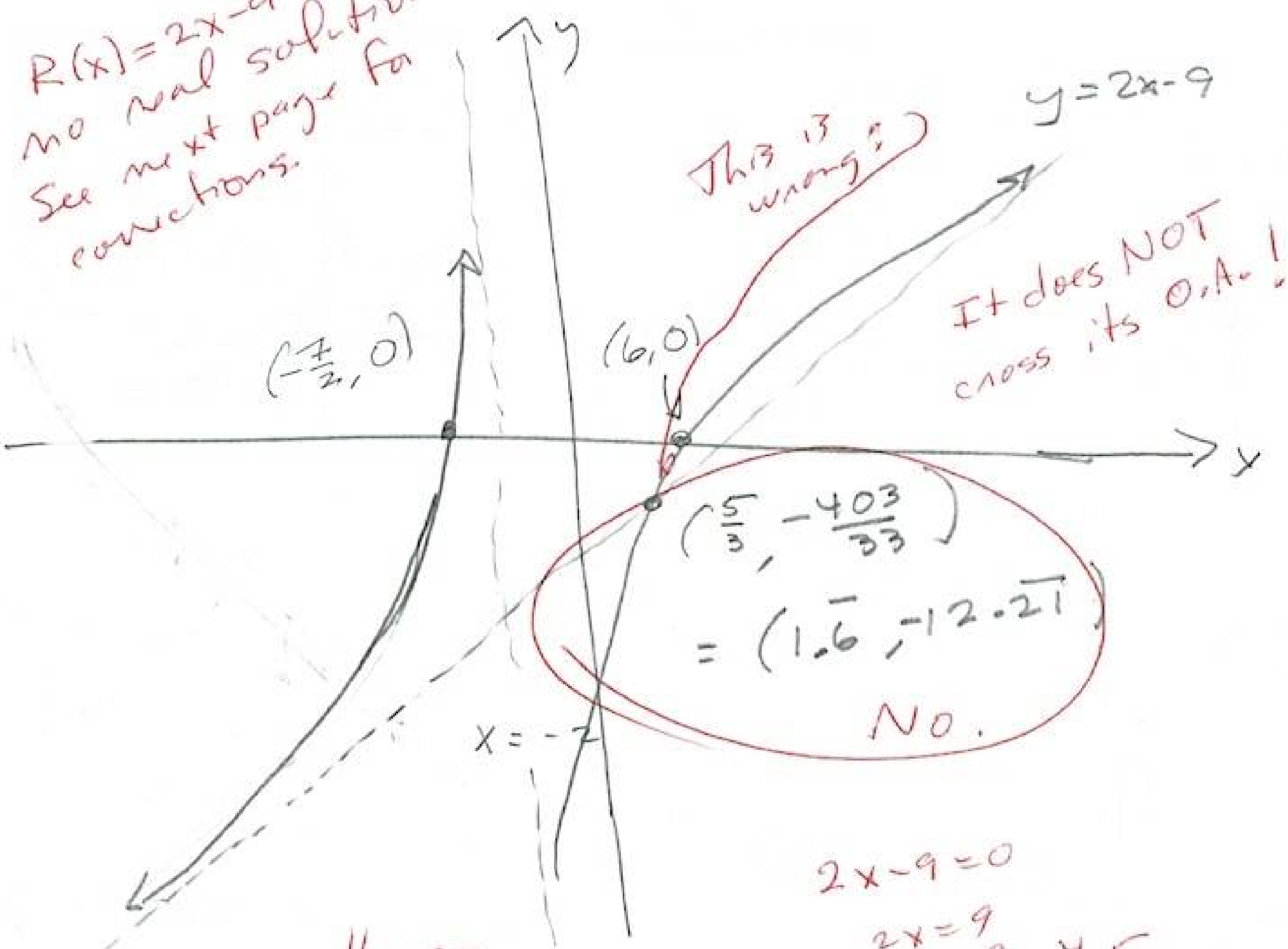
$$= \frac{\frac{-403}{9}}{\frac{11}{3}} = \left(\frac{-403}{9}\right)\left(\frac{3}{11}\right) = \frac{-403}{33}$$

$$\boxed{\left(\frac{5}{3}, -\frac{403}{33}\right) = \left(1.\overline{6}, -12.\overline{21}\right) \text{ HOLE}}$$

121 WP#3

#10 cnt'd

$R(x) = 2x-9$ has no real solution. See next page for corrections.



This is wrong!

It does NOT cross its O.A.!

No.

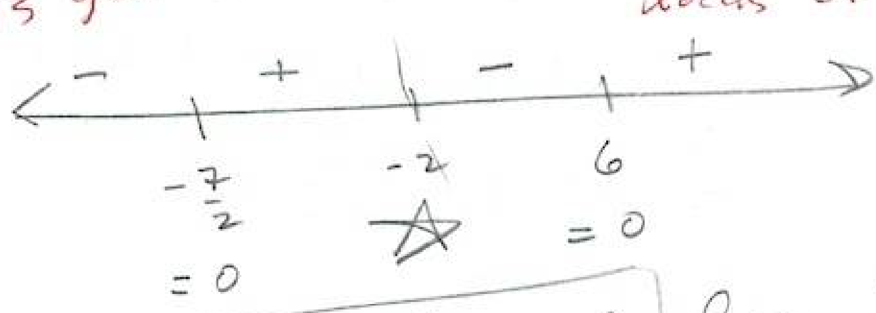
$2x-9=0$

$2x=9$

$x = \frac{9}{2} = 4.5$

This sign pattern is good!

$x = \frac{5}{3}$ is the locus of the hole.

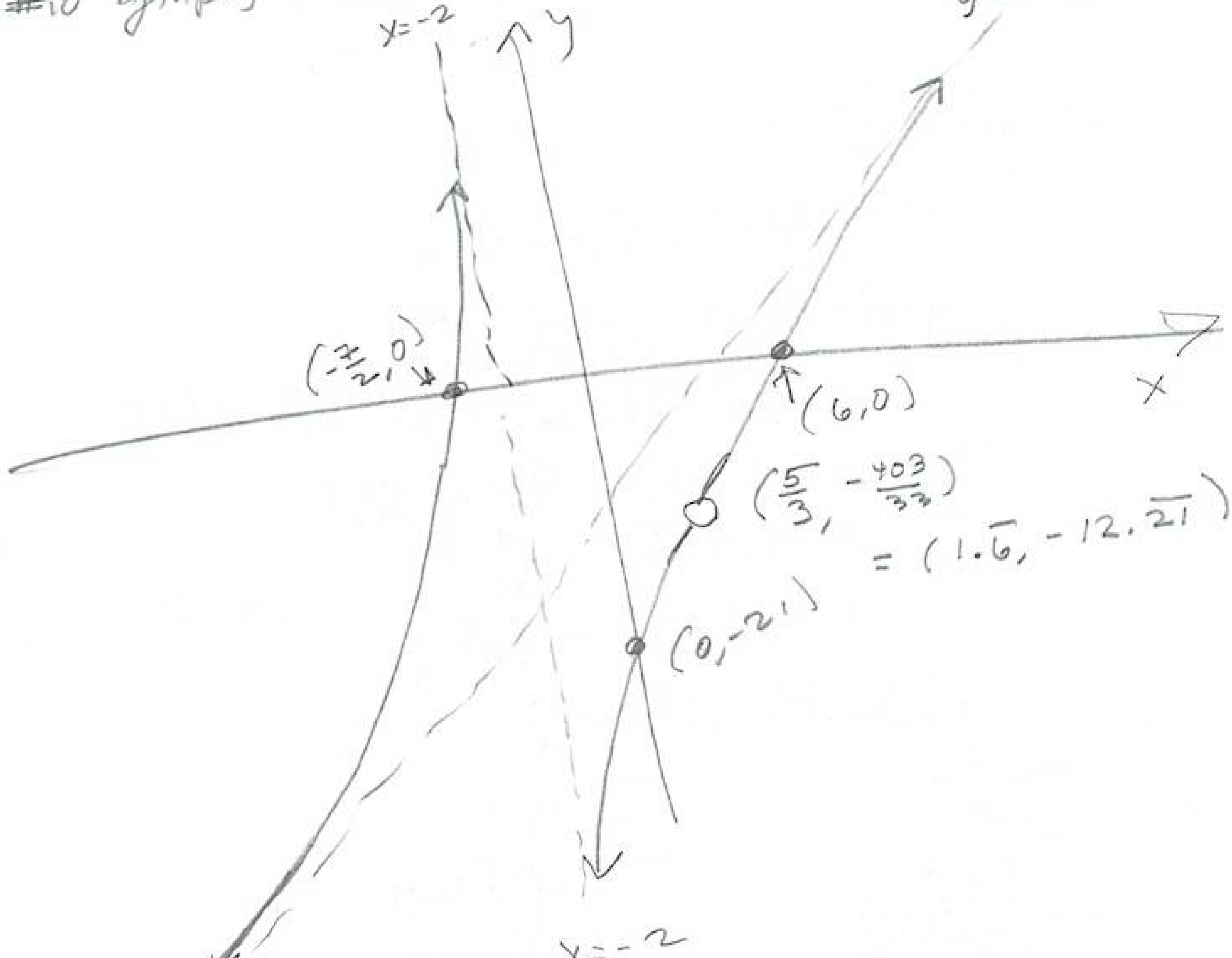


$\boxed{\text{y-int: } (0, -21)}$ from $\frac{-42}{2} = -21$

121 WP #3

(8.5)

#10 graph, CORRECTED!



Q(x)=2x-9 has no sol'n

$$\frac{2x^2-5x-42}{x+2} = \frac{2x-9}{1}\cdot\frac{x+2}{x+2} \longrightarrow$$

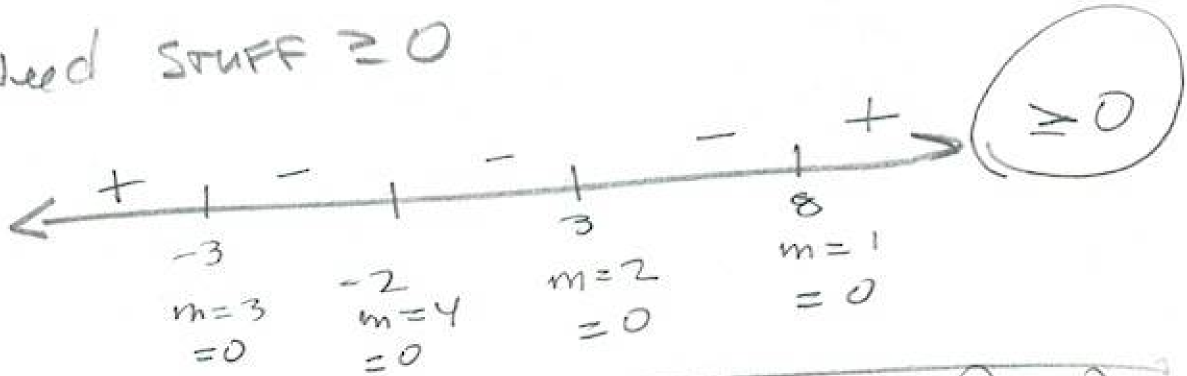
$$\frac{2x^2-5x-42-(2x^2-5x-18)}{LCD} = 0 \longrightarrow -5x+5x-42+18=0$$

$$\longrightarrow -24 = 0 \;\cancel{\longrightarrow}$$

Left half of graph stays above 2x-9; Right half stays below

(11) $w(x) = \sqrt{(x-3)^2(x+3)^3(x+2)^4(x-8)} = \sqrt{\text{STUFF}}$

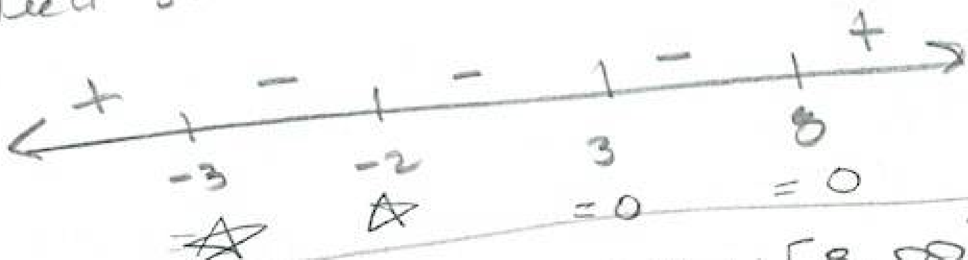
Need STUFF ≥ 0



$\boxed{(-\infty, -3] \cup \{-2, 3\} \cup [8, \infty) = \mathcal{D}(w)}$

(12) $\sqrt{\dfrac{(x-3)^2(x-8)}{(x+3)^3(x+2)^4}} = \sqrt{\text{STUFF}} = K(x)$

Need stuff ≥ 0 <u>AND</u> $(x+3)^3(x+2)^4 \neq 0$



$\boxed{\mathcal{D}(K) = (-\infty, -3) \cup \{3\} \cup [8, \infty)}$