121 WRITING PROJECT #1 Spring, 2021

(1) $x^2 - x - 20 = 0 \rightarrow a=1, b=-1, c=-20$

$\Rightarrow b^2 - 4ac = (-1)^2 - 4(1)(-20) = 1 + 80 = 81$

$\sqrt{81} = 9 \Rightarrow$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm 9}{2(1)} = \begin{cases} \frac{10}{2} = 5 \\ -\frac{8}{2} = -4 \end{cases}$$

$\Rightarrow \boxed{x \in \{-4, 5\}}$

(2) $2.12x^2 - 5.23x - 4.27 = 0 \rightarrow$

$\Rightarrow 212x^2 - 523x - 427 = 0 \rightarrow$

$a = 212, b = -523, c = -427 \Rightarrow$

$b^2 - 4ac = (-523)^2 - 4(212)(-427)$

$= 273529 + 362096$

$= 635625$ & $\sqrt{635625} \approx 797.2609$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{523 \pm \sqrt{635625}}{2(212)}$$

$\approx$ 3.113822962 or $-.6468418301$

$\Rightarrow \boxed{x \in \{3.1138, -.6468\}}$ to 4 places

Bonus

$$x = \frac{523 \pm \sqrt{635625}}{2(212)}$$

3	635625
3	211875
5	70625
5	14125
5	2825
5	565
	113

checked $113 \div 7, 11$

$11 > \sqrt{113}$, so we're done looking for prime factors;

hence $\sqrt{b^2-4ac} = 3 \cdot 5 \cdot 5 \sqrt{113}$

$= 75\sqrt{113}$ ⟶

$$x = \frac{523 \pm 75\sqrt{113}}{424}$$

(3) $7x^2 - 4x + 1 = 0 \Rightarrow a = 7,\ b = -4,\ c = 1$ ⟶

$b^2 - 4ac = (-4)^2 - 4(7)(1) = 16 - 28 = -12$

2	12
2	6
	3

$$x = \frac{4 \pm 2i\sqrt{3}}{2(-4)} = \frac{2(2 \pm i\sqrt{3})}{4(-2)} = \frac{2 \pm i\sqrt{3}}{-4} = x$$

OR $\frac{2 \pm i\sqrt{3}}{4}$ is OK, too.

$\sqrt{12} = 2\sqrt{3}$

2	12
2	6
	3

(4) $2x^2 - 11rx - 6\pi = 0 \Rightarrow$

$a = 2,\ b = -11r,\ c = -6\pi \Rightarrow$

$b^2 - 4ac = (-11r)^2 - 4(2)(-6\pi)$

$= 121r^2 + 24\pi \Rightarrow$

$$x = \frac{11r \pm \sqrt{121r^2 + 24\pi}}{2}$$

(5) $x^2 - x - 20 = x^2 - 5x + 4x - 20$

$= x(x-5) + 4(x-5) = (x-5)(x+4) = 0 \Rightarrow$

$x \in \{-4, 5\}$

(6) $60x^2 - 103x + 44 = 0$ ← signs same.

$ac = +2640$

Want $a + c = -103$

a & c both negative.

$-103 = -102 - 1$ — 102, 204

$= -101 - 2$

$= -53 - 50$ — 2650

$= -54 - 49$ — 2646

$= -52 - 51$ — 2652

$= -55 - 48$ — 2640 ! Sweet!

$60x^2 - 55x - 48x + 44 =$

$5x(12x - 11) - 4(12x - 11) = (12x - 11)(5x - 4) = 0 \Rightarrow$

$x \in \left\{\frac{11}{12}, \frac{4}{5}\right\}$

#6 METHOD 2 2C

2	2640
2	1320
2	660
2	330
5	165
3	33
	11

I skipped doing "3," which should come before "5" in the breakdown.

Guess and check 'til two factors sum to 103. This method's a pain for big #s. By previous work, though,

$(5)(11) + (2^4 \cdot 3) = 55 + 48 = 103$

Finding factors of 2640 that sum to 103 is easier to be systematic about.

Now, for the sledgehammer:

$a = 60, b = -103, c = 44 \Rightarrow$

$b^2 - 4ac = (-103)^2 - 4(60)(44) = 10609 - 10560$

$= 49 \Rightarrow$

$x = \frac{103 \pm \sqrt{49}}{2(60)} = \frac{103 \pm 7}{120}$

$\frac{110}{120} = \frac{11}{12}$

$\frac{96}{120} = \frac{48}{60} = \frac{24}{30} = \frac{12}{15} = \frac{4}{5}$

$\Rightarrow \boxed{x \in \left\{\frac{11}{12}, \frac{4}{5}\right\}}$

Now, reverse-engineer factored form:

$60\left(x - \frac{11}{12}\right)\left(x - \frac{4}{5}\right) = 12\left(x - \frac{11}{12}\right)(5)\left(x - \frac{4}{5}\right)$

$= \boxed{(12x - 11)(5x - 4)}$

(7) $x^2-5x+5 = x^2-5x+\left(\frac{5}{2}\right)^2-\frac{25}{4}+5$

$= \left(x-\frac{5}{2}\right)^2-\frac{25}{4}+\frac{20}{4}$

$= \left(x-\frac{5}{2}\right)^2-\frac{5}{4}=0 \Rightarrow$

$\left(x-\frac{5}{2}\right)^2=\frac{5}{4} \Rightarrow$

$x-\frac{5}{2}=\pm\frac{\sqrt{5}}{2} \Rightarrow$

This answer's form is fine.

$$x=\frac{5}{2}\pm\frac{\sqrt{5}}{2}$$

(8) $x^2-10x-11 = x^2-10x+5^2-25-11$

$= (x-5)^2-36=0 \Rightarrow$

$(x-5)^2=36 \Rightarrow$

$x-5=\pm\sqrt{36}=\pm 6 \Rightarrow$

$x=5\pm 6 \begin{cases} \nearrow 11 \\ \searrow -1 \end{cases} \Rightarrow x\in\{-1,11\}$

(9) $7x^2+4x+13=0 \Rightarrow$

$x^2+\frac{4}{7}x+\frac{13}{7}=0$

$\frac{4}{7}\div 2=\frac{2}{7} \rightsquigarrow \left(\frac{2}{7}\right)^2=\frac{4}{49} \Rightarrow$

$x^2+\frac{4}{7}x+\left(\frac{2}{7}\right)^2-\frac{4}{49}+\frac{13}{7}\cdot\frac{7}{7}=$

$\left(x+\frac{2}{7}\right)^2+\frac{-4+91}{49}=\left(x+\frac{2}{7}\right)^2+\frac{87}{49}=0$

$\Rightarrow \left(x+\frac{2}{7}\right)^2=-\frac{87}{49} \Rightarrow$

$x+\frac{2}{7}=\pm i\frac{\sqrt{87}}{7} \Rightarrow$

$$x=\frac{-2\pm i\sqrt{87}}{7}$$

3 | 87
2 9

121 WP #1

(6)

(10)

$$7x^2 - 33x - 10 = 0$$

$$x^2 - \frac{33}{7}x - \frac{10}{7} = 0$$

$7 \overline{)196}$ 28

$$\frac{\frac{33}{7}}{2} = \frac{33}{14} \rightsquigarrow \left(\frac{33}{14}\right)^2 = \frac{1089}{196} \Longrightarrow$$

$$x^2 - \frac{33}{7}x + \left(\frac{33}{14}\right)^2 - \frac{1089}{196} - \frac{10}{7} \cdot \frac{28}{28}$$

$$= \left(x - \frac{33}{14}\right)^2 + \frac{-1089 - 280}{196} = 0 \Longrightarrow$$

$$\left(x - \frac{33}{14}\right)^2 - \frac{1369}{196} = 0 \Longrightarrow$$

$37 \overline{)1369}$ 37

$$\left(x - \frac{33}{14}\right)^2 = \frac{1369}{196} \Longrightarrow$$

$$x - \frac{33}{14} = \pm \frac{\sqrt{1369}}{14} = \pm \frac{37}{14} \Longrightarrow$$

$$x = \frac{33 \pm 37}{14}$$

$\frac{70}{14} = \frac{10}{2} = 5$

$\frac{33-37}{14} = \frac{-4}{14} = -\frac{2}{7}$

$$x \in \left\{-\frac{2}{7}, 5\right\}$$