121 Writing Project #3 Spring, 2020

$f(x) = 9x^5 + 3x^4 - 54x^3 - 474x^2 - 1872x - 2040$

(1) $9x^5$

(2) $9x^5 + \underbrace{3x^4 - 54x^3}_{1} - 474x^2 - 1872x - 2040$

1 positive root

$f(-x) = \underbrace{-9x^5 + 3x^4}_{1} \underbrace{+ 54x^3 - 474x^2}_{2} \underbrace{- 474x^2 + 1872x}_{3} \underbrace{+ 1872x - 2040}_{4}$

4, 2, or 0 negative roots.

(3) $2040 = p$'s

$9 = q$'s

$$\begin{array}{r|l} 2 & 2040 \\ 2 & 1020 \\ 2 & 510 \\ 3 & 255 \\ 5 & 85 \\ & 17 \end{array}$$

$\pm 2, \pm 4, \pm 8, \pm 6, \pm 12, \pm 24, \pm 3, \pm 5, \pm 10, \pm 20, \pm 40,$
$\pm 17, \pm 34, \pm 68, \pm 136,$ wow! This is huge!

(4)

$$\begin{array}{r|rrrrrr}
\boxed{5} & 9 & 3 & -54 & -474 & -1872 & -2040 \\
 & & 45 & 240 & 930 & 2280 & 2040 \\ \hline
-2 & 9 & 48 & 186 & 456 & 408 & 0 \\
 & & -18 & -60 & -252 & -408 & \\ \hline
-2 & 9 & 30 & 126 & 204 & 0 & \\
 & & -18 & -24 & -204 & & \\ \hline
 & 9 & 12 & 102 & 0 & &
\end{array}$$

(5) $f(x) = (x-5)(x+2)^2(9x^2 + 12x + 102)$

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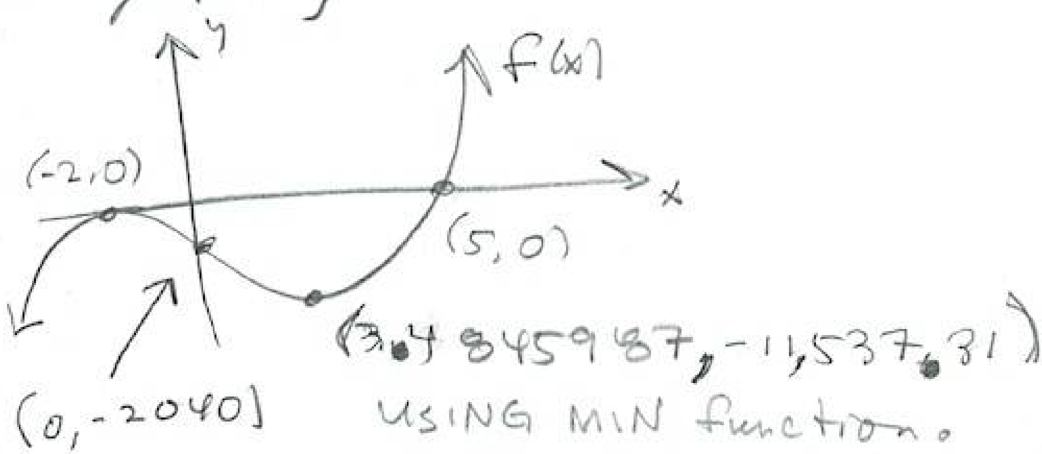
(6) From graphing calculator:



USING MIN function.
NOT REQUIRED, BUT
NICE.

(7) From #5, we have

$f(x) = (x-5)(x+2)^2(9x^2+12x+102)$

we find the roots of the irreducible

$9x^2+12x+102=0$ over the reals:

Common factor of 3:

$3(3x^2+4x+34)$

$a=3, b=4, c=34$

$b^2-4ac = 4^2-4(3)(34)$

$= 16-408 = -392$

$$\begin{array}{r|l} 2 & 392 \\ 2 & 196 \\ 2 & 98 \\ 7 & 49 \\ & 7 \end{array}$$

$x = \dfrac{-4 \pm 14\sqrt{2}\,i}{2(3)} = \boxed{\dfrac{-2 \pm 7\sqrt{2}}{3}}$

$= x$

$\cancel{f(x) = 9(x+2)^2(x-5)\left(x-\left(\frac{-2+\sqrt{2}i}{3}\right)\right)}$

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#7 Final :

$$\boxed{f(x) = 9(x+2)^2(x-5)\left(x-\left(\frac{-2+\sqrt{7}i}{3}\right)\right)\left(x-\left(\frac{-2-\sqrt{7}i}{3}\right)\right)}$$

#8 $R(x) = \dfrac{21x^2-26x-15}{x^2+x-12}$ FACTOR

$21x^2-26x-15$

$-26 = -35+9$, so

$(21)(-15) = -315$

$3 \lfloor 315$

$3 \lfloor 105$

$5 \lfloor 35$

7

$21x^2-35x+9x-15$

$= 7x(3x-5)+3(3x-5)$

$= (3x-5)(7x+3) \overset{SET}{=} 0$

$\Rightarrow x \in \{\frac{5}{3}, -\frac{3}{7}\}$

$x^2+x-12 = x^2+4x-3x-12$

$= x(x+4)-3(x+4)$

$= (x+4)(x-3) \overset{SET}{=} 0$

$\Rightarrow x \in \{-4, 3\}$

DOMAIN: $\mathbb{R} \setminus \{-4, 3\}$

Vertical Asymptotes : $\boxed{x=-4,\ x=3}$

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#8 Critical x-intercepts:

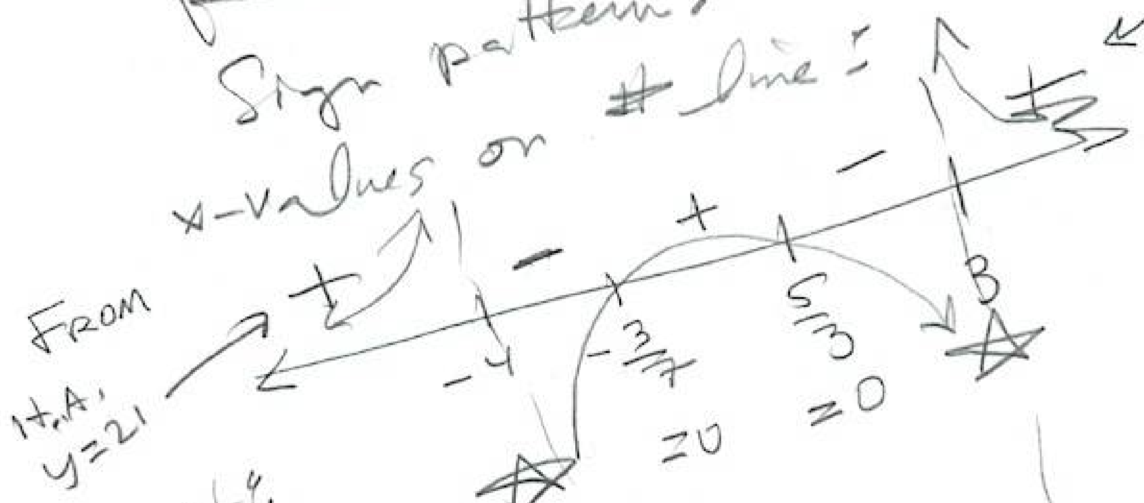
$\boxed{\left(\frac{5}{3}, 0\right), \left(-\frac{3}{7}, 0\right)}$

End Behavior

$\frac{21x^2}{x^2} = 21 \xrightarrow{|x| \to \infty} 21$

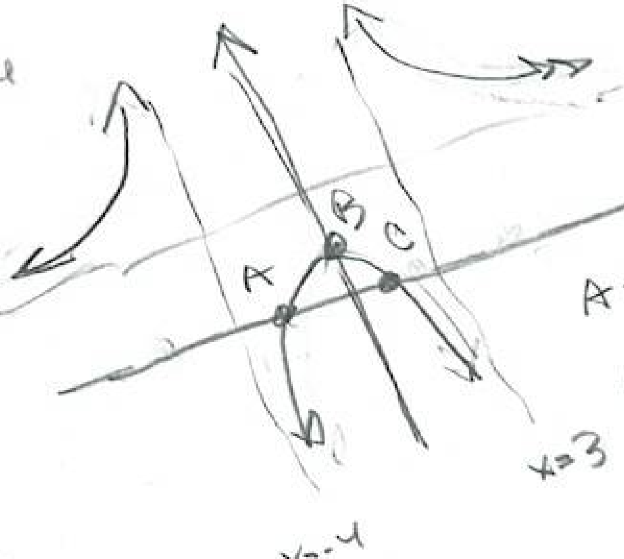
$\boxed{y = 21}$ is horizontal Asymptote.

Sign pattern: arrange critical x-values on # line:



y-int: $\frac{15}{12} = \frac{5}{4}$

y=21



$A = \left(-\frac{3}{7}, 0\right)$

$B = \left(0, \frac{5}{4}\right)$

$C = \left(\frac{5}{3}, 0\right)$

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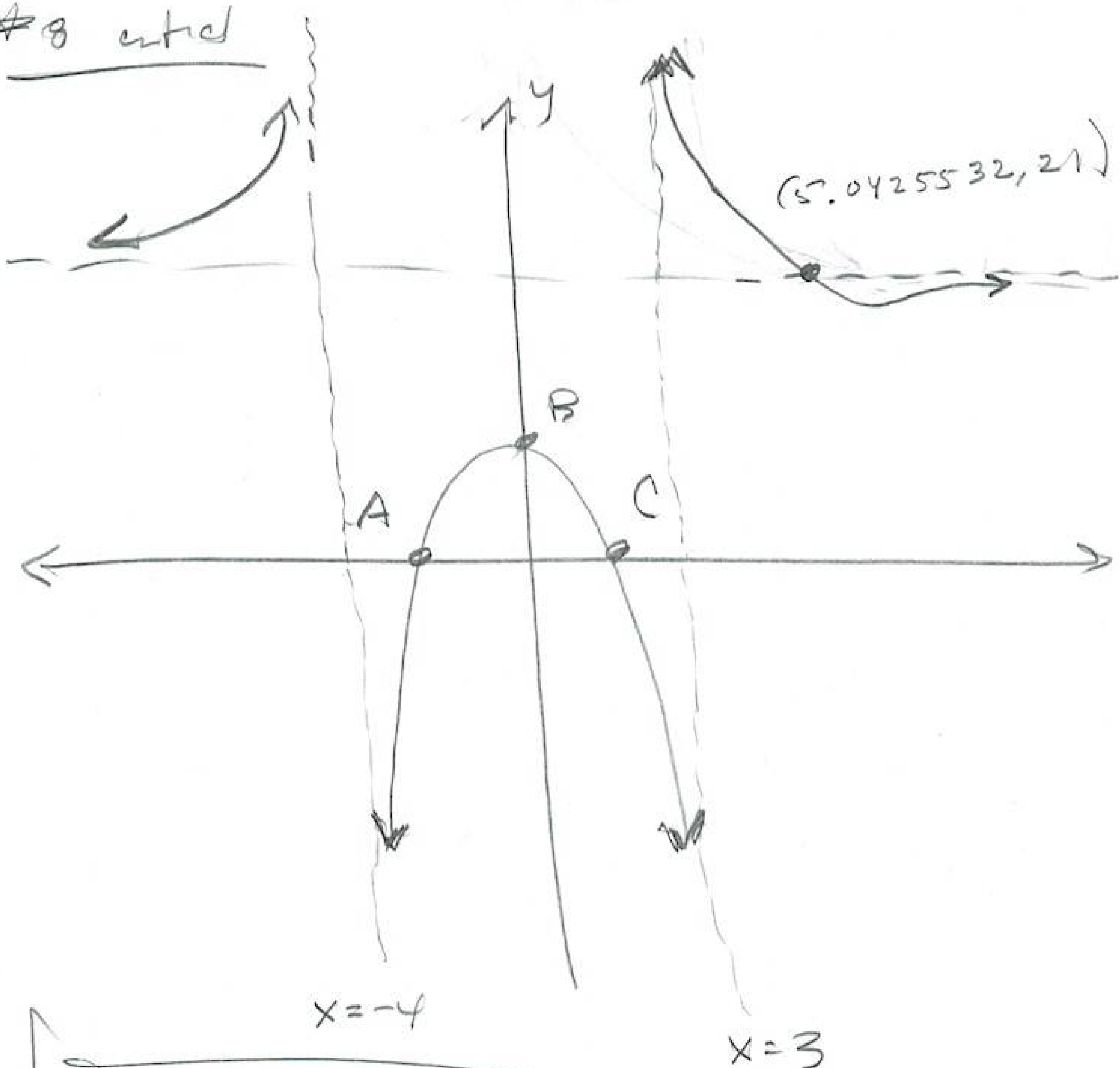
#8 cntd



$A = \left(-\frac{3}{7}, 0\right)$

$B = \left(0, \frac{5}{4}\right)$

$C = \left(\frac{5}{3}, 0\right)$

121 WP#3

(9) $Q(x) = \dfrac{21x^3+100x^2-171x-90}{x^3+5x^2-18x-72} = \dfrac{r(x)}{s(x)}$

$s(x) = (x+4)(x-3)(x-?)$

$$\begin{array}{r|rrrr} 3 & 1 & 5 & -18 & -72 \\ & & 3 & 24 & 18 \\ \hline & 1 & 8 & 6 & -54 \end{array}$$

$x^2+x-12 = (x+4)(x-3)$

Dang! See if the numerator's just as big of a blunder:

$$\begin{array}{r|rrrr} \frac{5}{3} & 21 & 100 & -171 & -90 \\ & & 35 & 225 & 90 \\ \hline & 21 & 135 & 54 & 0 \\ -\frac{3}{7} & & -9 & -54 & \\ \hline & 21 & 126 & 0 & \end{array}$$

$21(x+6)(x+\frac{3}{7})(x-\frac{5}{3})$

$= (7x+3)(3x-5)(x+6)$

$21x+126=0$

$x = -\frac{126}{21} = -6 = x$

12.1 WP #3

9 cntd Teacher fears he got #9 wrong!

$x=3$ & $x=-4$ should yield a zero, but his synthetic division the denominator says otherwise.

Let's see if $x+6$ divides evenly into the denominator of $Q(x) = x^3+5x^2-18x-72$

$$\begin{array}{r|rrrr} -6 & 1 & 5 & -18 & -72 \\ & & -6 & 6 & 72 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

Hmmmm This says x^2-x-12, not x^2+x-12, like I wanted.

My glaucoma got me. Not you.

I'll graph this one

$$Q(x) = \frac{(7x+3)(3x-5)(x+6)}{(x-4)(x+3)(x+6)}$$

Changed the V.A.'s unintention-ally. Very similar sketch.

12)   WP #3

*9 contd

$x = -\frac{3}{7}, \frac{5}{3}, -6, -3, 4, -6$

$Y_2 = \frac{\sim}{x^2 - x - 12}$

$Y_3 = \frac{\sim}{x^2 + x - 12}$

$Y_2(-6) = \frac{299}{10}$

Hole @ $x = -6$.

$Y_3(-6) = 49.8\overline{3}$

$= \frac{299}{6}$

$$R(-6)^* = \frac{21(-6)^2 - 26(-6) - 15}{(-6)^2 - (-6) - 12}$$

* Switching the sign in the x-term, in keeping with the (slightly) different Q(x) in #9

$$= \frac{756 + 156 - 15}{36 + 6 - 12} = \frac{897}{30} = \frac{299}{10} = 29.9$$

$\boxed{\text{HOLE} = (-6, 29.9)}$ ← Version 1 = as given.

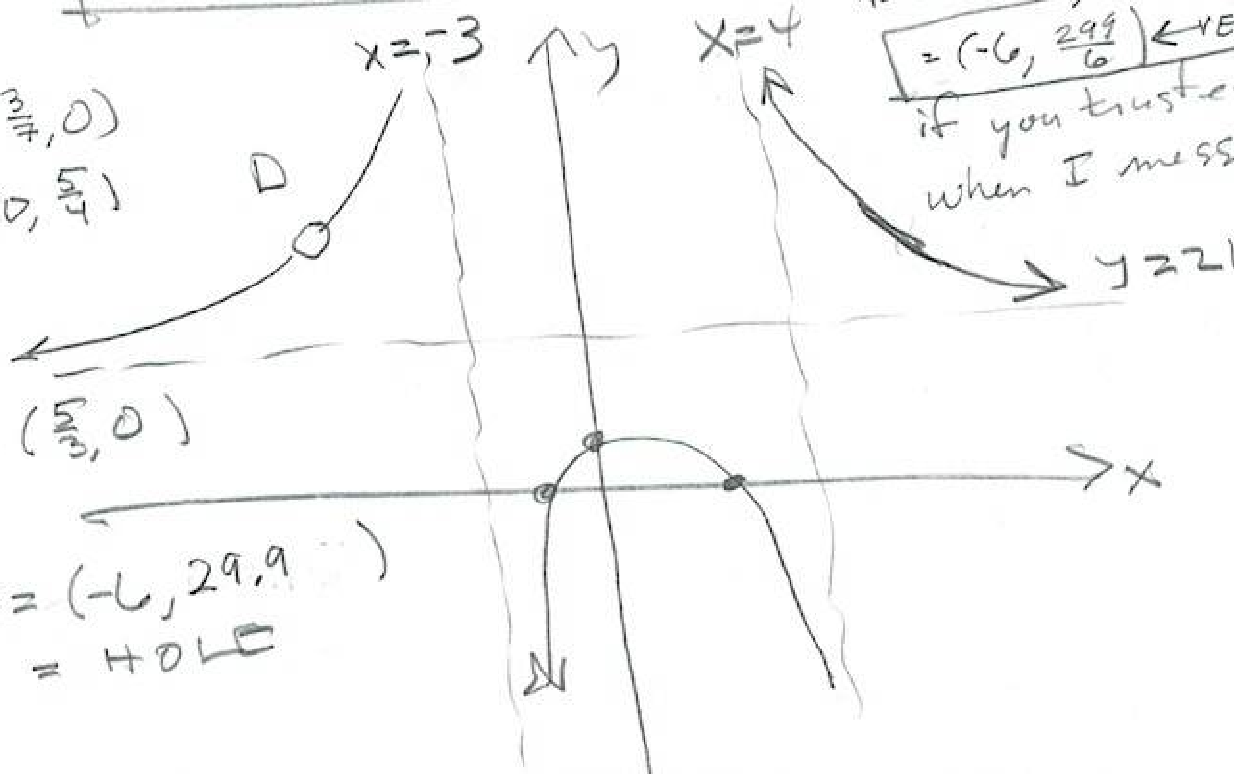
HOLE: $(-6, 49.8\overline{3})$

$\boxed{= \left(-6, \frac{299}{6}\right)}$ ← VERSION 2

if you trusted me, when I messed up

$A = \left(-\frac{3}{7}, 0\right)$

$B = \left(0, \frac{5}{4}\right)$



$C = \left(\frac{5}{3}, 0\right)$

$D = (-6, 29.9)$

= HOLE

(121) WP ~~#~~3

10) $T(x) = \dfrac{21x^3+100x^2-171x-90}{x^2-x-12}$

$$\begin{array}{r} 21x \quad +121 \\ x^2-x-12 \overline{\smash{)}\, 21x^3+100x^2-171x-90} \\ -(21x^3-21x^2-252x\quad) \\ \hline 121x^2+ \ldots \end{array}$$

$21x=-121$

$x=\frac{-121}{21}$

≈ -10.08333

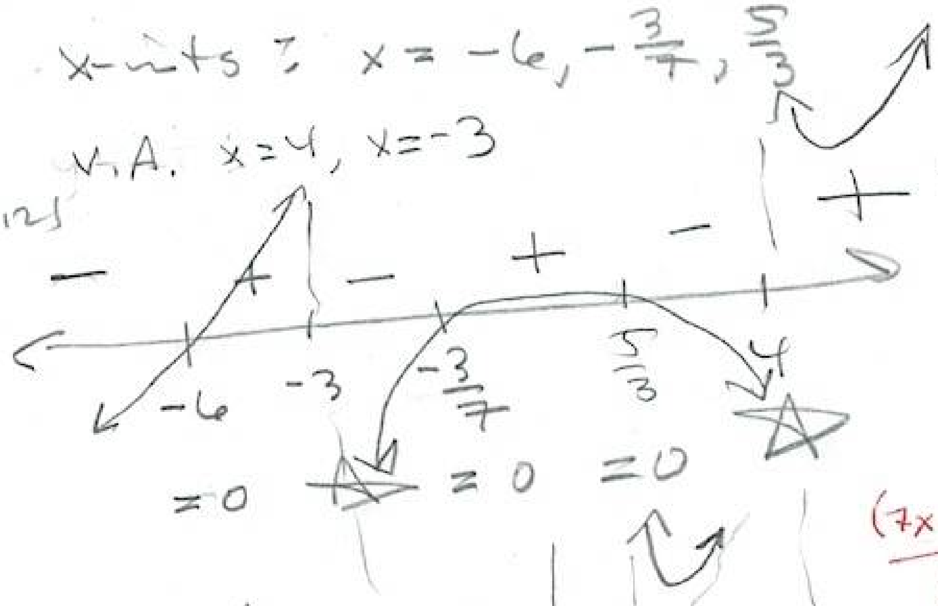
$y=21x+121$ is oblique asymptote.

x-ints: $x=-6, -\frac{3}{7}, \frac{5}{3}$

V.A. $x=4$, $x=-3$

$y=21x+121$ says $-$

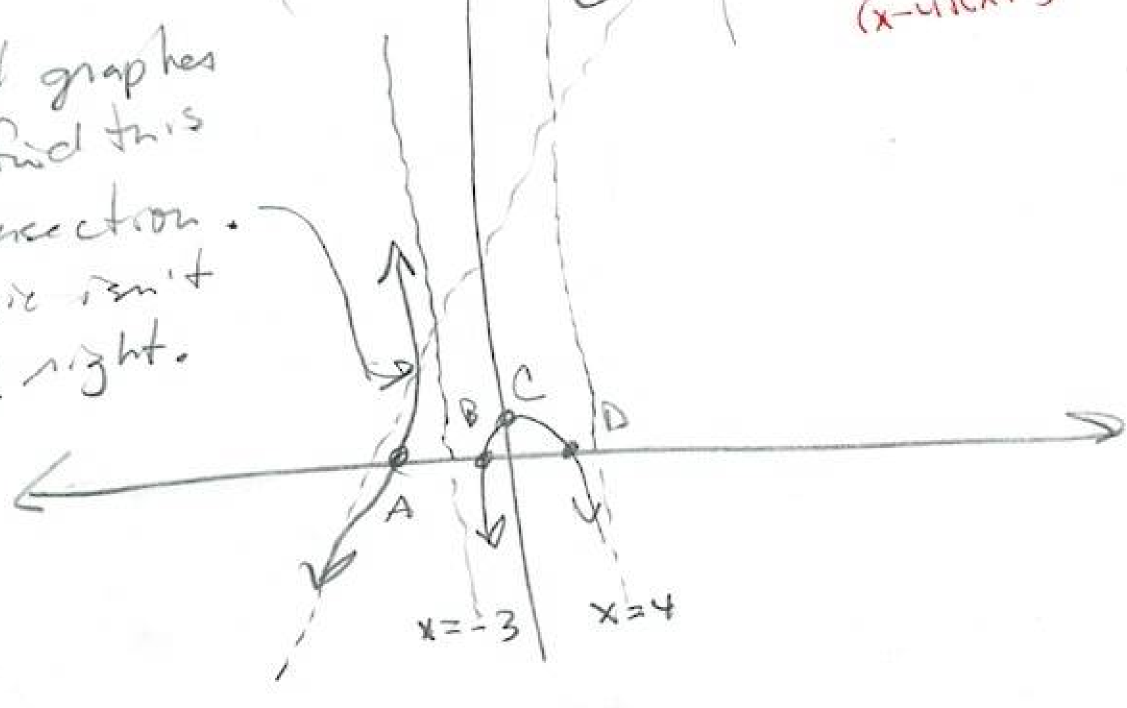
$y=21x+121$ says $+$



$\dfrac{(7x+3)(3x-5)(x+6)}{(x-4)(x+3)}$

Used graphes to find this intersection.

This pic isn't quite right.



121 WP #3

#10 Final Graph

$A \approx (-6.74, -20.59)$

$B \approx (-5.76, 0)$ Bonus

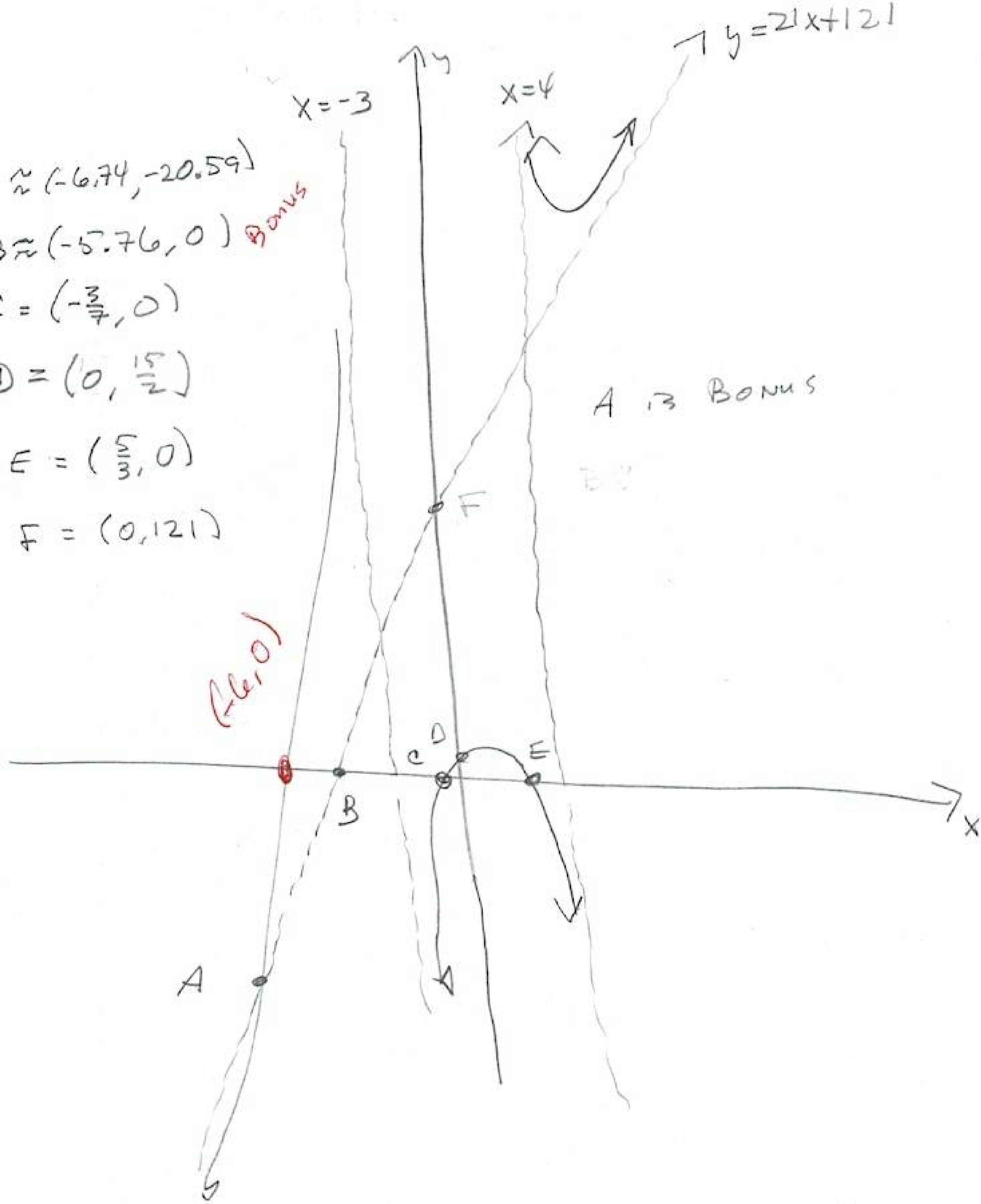
$C = (-\frac{3}{7}, 0)$

$D = (0, \frac{15}{2})$

$E = (\frac{5}{3}, 0)$

$F = (0, 121)$

A is Bonus



12] WP#3

(11) (2 pts) $w(x) = \sqrt{(x+11)^3(x-4)^2(x+5)(x-2)} = \sqrt{P(x)}$

$\mathcal{D}(w) = \{x \mid w(x) \geq 0\}$

zeros: $-11, 4, -5, 2$

x: $-11, -5, 2, 4$

m: $3, 1, 1, 2$

w(x) ≥ 0

− N | Y | Y + | Y | N − | Y | Y + | Y | Y +

-11 $=0$, -5 $=0$, 2 $=0$, 4 $=0$

$(x+11)^3(x-4)^2(x+5)(x-2) = x^3 \cdot x^2 \cdot x \cdot x +$ ...

$= x^7 +$ smaller degree

+

−

$\boxed{\mathcal{D}(\ldots) = [-11, -5] \cup [2, \infty)}$

Need $g(x) \geq 0$ and $x \neq 4, 2$

(12) $\sqrt{\dfrac{(x+11)^3(x+5)}{(x-4)^2(x-2)}} = K(x) = \sqrt{g(x)}$

$g(x) \geq 0$

N − | Y | Y + | Y | N − | N | Y + | N | Y +

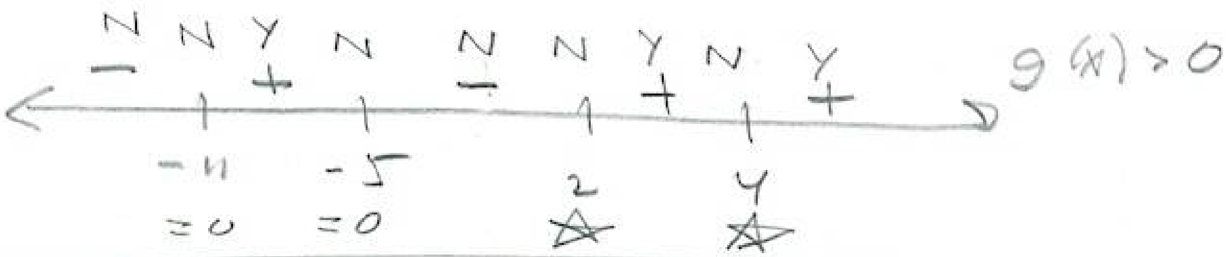
-11 $=0$, -5 $=0$, 2 ☆, 4 ☆

$\boxed{\mathcal{D}(K(x)) = [-11, -5] \cup (2, 4) \cup (4, \infty)}$

12-1 WP#3

(13) (2pts) $T(x) = \log_5\left(\dfrac{(x+11)^3(x+5)}{(x-4)^2(x-2)}\right) = \log_5(g(x))$

Need $g(x) > 0$ and $x \neq 2, 4$



$\boxed{D(T) = (-11,-5) \cup (2,4) \cup (2,\infty)}$