MAT 121-G81/82 Writing Project #2 Spring, 2020

Graphing by Transforming Basic Functions

FORMATTING: This is semi-formal writing, here. You don't have to type it out, but you do need to be very clear. For the formatting guidelines, please see Writing Project #1. They're the same for tests and (face-to-face) homework, except on Tests and Writing Projects, don't waste time writing out the question details, because they come WITH (the cover sheet).

Online Students: Bring your Writing Project with you to the testing center, and turn it in before you take the test. Early Birds may mail the Writing Project to my mailing address, given in the Draft Syllabus. (Recently updated.)

DEADLINE for Early-Bird 10% Bonus is FRIDAY, February 21st. Otherwise, just bring it with you to the test on Wednesday or Thursday, the 26th or 27th for full credit. Solutions will be revealed Monday, February 24th.

Main Resources: Chapter 2 Videos (and notes) and Writing Project 2 Videos (and notes).

Method 1: (See Problem #1 on which the following is based.)

Method 1:

0. $f(x)=\frac{1}{x^2}$ *The basic function.*

1. $3f(x)=3\cdot\frac{1}{x^2}=\frac{3}{x^2}$ $y\mapsto 3y$

2. $3f(x-15)=\frac{3}{(x-15)^2}$ $x\mapsto x+15$

3. $3f(5x-15)=\frac{3}{(5x-15)^2}$ $x\mapsto \frac{1}{5}x$

4. $3f(5x-15)-6=g(x)$ $y\mapsto y-6$

Method 2:

0. $f(x)=\frac{1}{x^2}$ *The basic function.*

1. $3f(x)=3\cdot\frac{1}{x^2}=\frac{3}{x^2}$ $y\mapsto 3y$

2. $3f(5x)=\frac{3}{(5x)^2}$ $x\mapsto \frac{1}{5}x$

3. $3f(5(x-3))=\frac{3}{(5(x-3))^2}$ $x\mapsto x+3$

4. $3f(5(x-3)-6=g(x)$ $y\mapsto y-6$

I prefer Method 2, because of how I want you to think of shape (including stretching and shrinking), first, and position, second, and you'll definitely want to factor out the coefficient of x in trig and calculus, but beginners like Method 1, because they're afraid of fractions, even though it puts a ceiling on their later understanding, which breaks my heart. In #1, by factoring out the 5 inside, you can SEE where the center of the action is, immediately. (Vertical asymptote: $x = 3$! Not $x = 15$!).

Graph the function $g(x)$ by transforming the graph of a basic function, $f(x)$.

1. $g(x)=\frac{3}{(5x-15)^2}-6$ Use $(1,1)$, and $(-1,1)$ as the 2 (x,y)'s in the 1st graph. All functions of the form $f(x)=\frac{1}{x^{even}}$ have the same basic shape, just like $f(x)=\frac{1}{x^2}$

2. $g(x)=-3(7x-35)^{4/7}+4$. All functions of the form $f(x)=x^{\frac{even}{odd}}=\sqrt[odd]{x^{even}}$ have the same basic  shape, when the root is greater than the power. Use $(-1,1),(0,0)$ and $(1,1)$ as the 3 points in the 1st graph.

3. $g(x)=\frac{3}{(5x-15)^3}-6$ Use $(1,1)$, and $(-1,-1)$ for 2 points in the first graph. All functions of the form $f(x)=\frac{1}{x^{odd}}$ have the same shape, just like $f(x)=\frac{1}{x}$

4. $g(x)=-4\sqrt{-3x-27}+12$. All functions of the form $f(x)=\sqrt[even]{x}=x^{\frac{1}{even}}$ and $f(x)=\sqrt{x}=\sqrt[2]{x}=x^{\frac{1}{2}}$ is understood.

5. $g(x)=3\sqrt[3]{2x-30}-11$. All functions of the form $f(x)=\sqrt[odd]{x}$ have the same  shape.

6. $g(x)=-2(3x-12)^5+9$ Power function. Odd power. See Writing Project Videos!

We treat lines and parabolas a little differently. They come up so often – plus the completing-the-square trick – we sidestep the whole $f(bx)$ issue and just work with $g(x)=a(x-h)^2+k$ and $g(x)=m(x-h)+k=m(x-x_1)+y_1$.

7. $g(x)=\frac{7}{3}(x-5)-3$. The goal is to see a line as something with a certain slope passing through a specific point. We're trying to ease you out of pure drill-and-kill on $y=mx+b$ and ease you into $y=m(x-x_1)+y_1$, so your grasp of lines will be more intuitive, and more suited to tangent-line ideas in the Calculus.

8. $g(x)=3(x-5)^2+4$

I expect you to complete the square to re-write these, just like the bonus problems in Test 1 (and tests to come).

9. $g(x)=x^2-6x-11$ (This and the following are *similar* to Test 1 questions, but our purposes are different.)

10. $g(x)=3x^2+5x+1$

One reason I stress point-slope form is that $y=m(x-h)+k$ corresponds to: $y=m(x-x_1)+y_1$.

The "cheat" for completing the square: $g(x)=ax^2+bx+c=a(x-h)^2+k=a\left(x-\frac{-b}{2a}\right)^2+g\left(-\frac{b}{2a}\right)$

I think it's fine to use the "cheat" for a backup/check, but you really want to master the moves. They're much easier to remember, long-term than trying to memorize the $\frac{-b}{2a}$ thing, and how it fits in the formula, and the $g\left(-\frac{b}{2a}\right)$ can be messy and as much or more work than just learning the moves. The moves are demonstrated over and over again in videos. Practice them. Own them. By the time you get to calculus, you want this completing-the-square technique to be quick-twitch, muscle memory. In college algebra, these techniques hone your expression-manipulation chops.