

① $x^2 + 7x - 18 = 0 \rightarrow$ Spring, '20.

$$a=1, b=-7, c=-18 \rightarrow$$

$$b^2 - 4ac = (-7)^2 - 4(1)(-18) = 49 + 72 = 121$$

∵ $121 = 11^2$, it turns out, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm 11}{2(1)} = \frac{7 \pm 11}{2} \begin{matrix} \nearrow \frac{18}{2} \\ \searrow -\frac{4}{2} \end{matrix}$$

$$\rightarrow x \in \{-2, 9\}$$

② $3.62x^2 - 9.71x - 15.68 = 0 \rightarrow$

$$362x^2 - 971x - 1568 = 0 \rightarrow$$

$$a=362, b=-971, c=-1568 \rightarrow$$

$$b^2 - 4ac = (-971)^2 - 4(362)(-1568) = 942,841 + 2,270,464 \\ = 3,213,305$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{971 \pm \sqrt{3,213,305}}{2(362)} \approx \frac{971 \pm 1792.569385}{724}$$

$$\rightarrow x \in \{3.8171, -1.1348\}$$

Bonus

$$\begin{array}{r} 5 \overline{) 3213305} \\ 31 \overline{) 642661} \\ 20731 \end{array}$$

Since no pairs,

$$\sqrt{3113305} \text{ is simplified!}$$

already! But you wouldn't know without breaking it down as far as possible!

$$\begin{array}{l} 2, 3, 5, 7, 11, 13, 17, 19, \\ 23, 29, 31, 37, 43, 47, \\ 53, 59, 61, 67, 71, 73, \\ 79, 83, 89, 97, 101, 103, 107, \\ 109, 113, 127, 131, 137, \\ 139, 149 \end{array}$$

$$\sqrt{20731} \approx 143.9826$$

So we stop @ 139.

So, in lowest terms (simplified radical form), the answer is

$$x \in \left\{ \frac{971 \pm \sqrt{3213305}}{724} \right\}$$

It turns out that the doggone answer doesn't simplify, but you wouldn't know that w/o breaking $\sqrt{3213305}$ down as far as possible!

$$(3) \quad 49x^2 - 28x + 7 = 0 \rightarrow$$

$$a = 49, b = -28, c = 7 \rightarrow$$

$$b^2 - 4ac = (-28)^2 - 4(49)(7) = 784 - 1372 = -588$$

$$\begin{array}{r} 2 \overline{) 588} \\ \underline{2} \\ 294 \\ \underline{3} \\ 147 \\ \underline{7} \\ 49 \\ \underline{7} \\ 0 \end{array}$$

$$\text{So } \sqrt{-588} = 2 \cdot 7 \cdot i\sqrt{3} = 14i\sqrt{3}$$

$$\uparrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{28 \pm 14i\sqrt{3}}{2(49)}$$

$$= \frac{14(2 \pm i\sqrt{3})}{2 \cdot 7 \cdot 7} = \frac{2 \pm i\sqrt{3}}{7}, \text{ so } x \in \left\{ \frac{2 \pm i\sqrt{3}}{7} \right\}$$

$$(4) \quad ax^2 - 5rx - 6z = 0 \rightarrow$$

$$a = a, b = -5r, c = -6z \rightarrow$$

$$b^2 - 4ac = (-5r)^2 - 4(a)(-6z) = 25r^2 + 24az$$

$$\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5r \pm \sqrt{25r^2 + 24az}}{2a} = x$$

$$(5) \quad x^2 - 5x - 24 = x^2 - 8x + 3x - 24$$

$$= x(x-8) + 3(x-8) = (x-8)(x+3) = 0$$

$$\Rightarrow x \in \{-3, 8\}$$

$$(6) \quad 14x^2 + 85x - 150 = 0$$

MAGIC: -2100

$$\rightarrow 14x^2 + 105x - 20x - 150$$

$$85 = -86 + 1 \quad -86$$

$$= -96 + 11 \quad -1056$$

$$= 7x(2x+15) - 10(2x+15)$$

$$= -106 + 21 \quad -2226$$

$$= (2x+15)(7x-10) = 0$$

$$= -100 + 15 \quad -1500$$

$$\rightarrow x \in \left\{ -\frac{15}{2}, \frac{10}{7} \right\}$$

$$= -101 + 16 \quad -1616$$

$$= -102 + 17 \quad -1734$$

$$= -103 + 18 \quad -1854$$

$$= -104 + 19 \quad \text{NO}$$

$$= -105 + 20 \quad -2100$$

Sledgehammer!

$$b^2 - 4ac = 85^2 - 4(14)(-150)$$

$$= 15625 = 125^2, 50$$

$$x = \frac{-85 \pm 125}{2(14)} \rightarrow \frac{40}{28} = \frac{10}{7}$$

$$\frac{-210}{28} = -\frac{15}{2}$$

$$x \in \left\{ -\frac{15}{2}, \frac{10}{7} \right\}$$

$$\text{So } 14\left(x + \frac{15}{2}\right)\left(x - \frac{10}{7}\right) =$$

$$2\left(x + \frac{15}{2}\right)(7)\left(x - \frac{10}{7}\right)$$

$$= (2x+15)(7x-10) \quad \left(\begin{array}{l} \text{instead of} \\ -105+20 \end{array} \right)$$

I should've guessed this quicker, since I knew it ended in "00"

Did my signs wrong for the "85" but this will work if I

fix it: 105-20

12)

WP # 1

(5)

$$\begin{aligned} \textcircled{7} \quad x^2 - 5x - 24 & \\ &= x^2 - 5x + \left(\frac{5}{2}\right)^2 - \frac{25}{4} - 24 \\ &= \left(x - \frac{5}{2}\right)^2 - \frac{121}{4} = 0 \rightarrow \end{aligned}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{121}{4} \rightarrow$$

$$x - \frac{5}{2} = \pm \sqrt{\frac{121}{4}} = \pm \frac{11}{2} \rightarrow$$

$$x = \frac{5 \pm 11}{2} \begin{cases} \nearrow \frac{16}{2} = 8 \\ \searrow \frac{-6}{2} = -3 \end{cases} \rightarrow x \in \{-3, 8\}$$

Check: $x^2 - 5x - 24 = (x-8)(x+3)$ ✓

$$\begin{aligned} \textcircled{8} \quad x^2 - 8x - 17 &= \\ x^2 - 8x + 4^2 - 16 - 17 &= \\ (x-4)^2 - 33 &= 0 \rightarrow \end{aligned}$$

$$(x-4)^2 = 33 \rightarrow$$

$$x-4 = \pm \sqrt{33} \rightarrow$$

$$\boxed{x = 4 \pm \sqrt{33}}$$

(or $x \in \{4 \pm \sqrt{33}\}$)

Scratch =

$$\begin{aligned} &-\frac{25}{4} - \frac{24}{1} \cdot \frac{1}{4} \\ &= \frac{-25 - 96}{4} = \frac{-121}{4} \end{aligned}$$

$$(10) \quad 3x^2 - 4x - 1 =$$

$$3\left(x^2 - \frac{4}{3}x + \right) - 1$$

$$= 3\left(x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2\right) - 1 - 3\left(\frac{4}{9}\right)$$

$$= 3\left(x - \frac{2}{3}\right)^2 - \frac{7}{3} = 0 \rightarrow$$

$$3\left(x - \frac{2}{3}\right)^2 = \frac{7}{3} \rightarrow$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{7}{9} \rightarrow$$

$$x - \frac{2}{3} = \pm \sqrt{\frac{7}{9}} = \pm \frac{\sqrt{7}}{3} \Rightarrow$$

$$x \in \left\{ \frac{2 \pm \sqrt{7}}{3} \right\}$$

I got these last 2 out of sequence. *sigh*

Scratch =

$$-1 - \frac{4}{3} = \frac{-3-4}{3} = -\frac{7}{3}$$

$$(9) \quad 3x^2 + 2x + 5$$

$$= 3\left(x^2 + \frac{2}{3}x + \right) + 5$$

$$= 3\left(x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2\right) + 5 - 3\left(\frac{1}{9}\right)$$

$$= 3\left(x + \frac{1}{3}\right)^2 + \frac{14}{3} = 0$$

Scratch =

$$5 - \frac{1}{3} = \frac{15-1}{3} = \frac{14}{3}$$

$$\rightarrow 3\left(x + \frac{1}{3}\right)^2 = -\frac{14}{3} \rightarrow$$

$$\left(x + \frac{1}{3}\right)^2 = -\frac{14}{9} \rightarrow$$

$$x + \frac{1}{3} = \pm \sqrt{-\frac{14}{9}} = \pm i \frac{\sqrt{14}}{3} \rightarrow$$

$$x = \frac{-1 \pm i\sqrt{14}}{3}$$