MAT 121 Take-Home Test 3 – Fall, 2021 Name_____
30 Points WRITING PROJECT #3 GRAPHING UTILITY RECOMMENDED

Remember to put your name on your work. This class seems to think I'm psychic. Leave a margin in the top left corner. Spread out your work. Work in one column. Just because there's room on the right doesn't mean you have to fill all of it. Submit problems in order.

For max learning and best efficiency, see Writing Project Videos and Test-Prep Videos than just about anything else you can do. They contain pretty much everything on Writing Project #3 and Test #3 that you need. Find an old Writing Project and an old Test 3 in the Old Writing Projects directory and the Old Tests directory, respectively. Make sure their solutions are there, so you can compare what you would do with what you should be doing.

There are a lot of details involved, but a limited number of ideas. Learn the ideas and the quickest, most efficient ways to use them, to avoid getting bogged down in the details.

End behavior is a small, simple idea, but you can use it to kick-start a sign pattern and know more about how polynomials and rational functions look than if you plotted 50 random points. Graphing functions and solving inequalities are very closely linked in a beautiful way. If you can see it, you'll like it. If you can't, then it's overwhelming. See the beauty!

A grapher (calculator or online or computer algebra system) will give you some quick, instant information about where zeros and asymptotes are located, but will not give you the essence of the shape, without proper interpretation. For that, you need to have good understanding about smoothness of these functions and the ideas I present in videos.

If I've been complaining about faint work, I urge you to either switch to black pen OR drop off your finished Writing Project #3 at my office: EDBH 134K OR mail it to me, all by Friday, October 22nd. The postmark will serve as the submission date, although if it comes in later than Friday the 22nd, I can't guarantee that it'll be graded before you take Test 3 on MyMathLab.

I'll check my mail at home and under my door at AIMs Greeley campus (EDBH 134K) on Friday the 22nd.

My address: Harry Mills, 2358 50th Ave, Greeley, CO, 80634

We will be working with $f(x) = 9x^5 - 84x^4 + 298x^3 - 528x^2 + 453x - 148$ for most of this test. We'll say everything about this polynomial that's worth saying.

1. (2 pts) Describe the end behavior of f with a simple graphic.

2. (2 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeros.

3. (2 pts) Use the Rational Zeros Theorem to determine the *possible* rational zeros (roots) of f.

4. (2 pts) Using the information, above, find all real zeros of f. Finding all zeros includes finding the multiplicity of each. This means performing multiple synthetic divisions. Always check for multiplicity greater than 1 with another synthetic division, just in case.

5. (2 pts) From your work, above, factor f over the real numbers. This will involve an irreducible quadratic factor that your grapher has no way of helping you to see. without the synthetic divisions in #4, bringing you closer and closer, step by step, to the irreducible quadratic.

6. (2 pts) Give a rough sketch of f from all of the above information. This is an *art* whose essence is really only found in my videos. If you're too tied to your grapher's output, you'll not capture the real essence of what's going on, or the key features I'm always looking for. Your picture will be more "vertical" than it should be.

7. (2 pts) Now we've covered everything *real* about f. Let's use that work to find *all* the roots of f and *split* f *into linear factors*. 5 roots are *guaranteed by the Fundamental Theorem of Algebra,* and we have found the 3 real ones. The other 2 are nonreal, hiding inside the irreducible quadratic polynomial that remains as the last, very very depressed piece that's not broken all the way down in #5. Now do your quadratic equation thing to *find* the 2 nonreal roots. *Finally*, apply the Factor Theorem to *all* the above work, and represent f as a product of linear factors, $f(x)=a(x-r_1)^{m_1}(x-r_2)^{m_2}\cdots(x-r_w)^{m_w}$. Don't forget the leading coefficient, *a*.

   This wrings (almost) every useful drop of the Theorems on Polynomials out of f, so now on to Rational Functions, which are *quotients* of polynomials!

8. (5 pts) Sketch the graph of $R(x)=\dfrac{3x^2+13x-56}{x^2-5x-14}$, showing all intercepts, asymptotes, and capturing the *essential features* of the shape of the graph. If you're a slave to your grapher, and oblivious to the features I'm looking for, it'll jump off the page at me (and be bad).

Note: There *is* a subtle feature to this graph that I downplay on tests, but you should pick up on with a take-home, namely, the horizontal asymptote *does* intersect the graph of the function.

I'm willing to part with **5 bonus points** if you can find the point of intersection of $R(x)$ with its horizontal asymptote and label it with an ordered-pair label. I'm also looking for its effect on the graph. There's a little wiggle to this graph in the 1st quadrant.

9. (2 pts) Sketch the graph of $Q(x)=\dfrac{3x^3+46x^2+87x-616}{x^3+6x^2-69x-154}$. All the work you did for #8 applies to this one, *except* for the *hole* in the graph of *Q*, which I expect you to find and clearly label in your graph.

10. (5 pts) Sketch the graph of $T(x)=\dfrac{3x^3+46x^2+87x-616}{x^2-5x-14}$, showing all intercepts and asymptotes.

Now for a pair of questions many struggle with on the sit-down test, but which are actually *very simple* if you can synthesize your skills and *apply* them to these sorts of questions. Often the downfall of people on the sit-down, but designed to be easy points for people who are putting things together.

For HELP on these last three problems, you want to look at the Test-Prep Videos for the SIT-DOWN Test 3, because the old Take-Home 3/Writing Project #3's didn't have these type-questions.

11. (2 pts) What is the domain of $W(x)=\sqrt{(x+11)^2(x-4)^5(x+3)(x-8)}$?

12. (2 pts) What is the domain of $\mathrm{K}(x)=\sqrt{\dfrac{(x+11)^2(x-8)}{(x-4)^5(x+3)}}$?

13. (2 pts bonus) What is the domain of $\mathrm{T}(x)=\log_5\left(\dfrac{(x+11)^3(x+5)}{(x-4)^2(x-2)}\right)$