

# WP #3 / TAKE-HOME TEST 3

Fall '21

$$f(x) = 9x^5 - 84x^4 + 298x^3 - 528x^2 + 453x - 148$$

End Behavior  $\sim 9x^5$

② 5, 3, or 1 positive zeros

$$f(-x) = -9x^5 - 84x^4 - 298x^3 - 528x^2 - 453x - 148$$

0 negative roots

③  $\frac{p}{q} = \frac{\text{factors of } -148}{\text{factors of } 9}$

$$\begin{array}{r} 2 \overline{) 148} \\ \underline{2} \phantom{0} \\ 74 \\ \underline{2} \phantom{0} \\ 37 \end{array}$$

- $\pm 1, \pm 2, \pm 4, \pm 37, \pm 74, \pm 148,$
- $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{37}{3}, \pm \frac{74}{3}, \pm \frac{148}{3},$
- $\pm \frac{1}{9}, \pm \frac{2}{9}, \pm \frac{4}{9}, \pm \frac{37}{9}, \pm \frac{74}{9}, \pm \frac{148}{9}$

18 of them

④

|    |   |     |     |      |     |      |     |
|----|---|-----|-----|------|-----|------|-----|
| 11 | 9 | -84 | 298 | -528 | 453 | -148 |     |
|    |   |     |     |      |     |      | 148 |
|    |   |     |     |      |     |      | 0   |
|    | 9 | -75 | 223 | -305 | 148 |      |     |
|    |   |     |     |      |     |      | 148 |
|    |   |     |     |      |     |      | 0   |
|    | 9 | -66 | 157 | -148 |     |      |     |
|    |   |     |     |      |     |      | 148 |
|    |   |     |     |      |     |      | 0   |
|    | 9 | -66 | 36  | -128 | 148 |      |     |
|    |   |     |     |      |     |      | 148 |
|    |   |     |     |      |     |      | 0   |
|    | 9 | -30 | 37  | 0    |     |      |     |

WP #3

121

(4) cont'd  $9x^2 - 30x + 37 = 0 \rightarrow$

$a=9, b=-30, c=37 \rightarrow$

$b^2 - 4ac = 30^2 - 4(9)(37) = 900 - 1332$

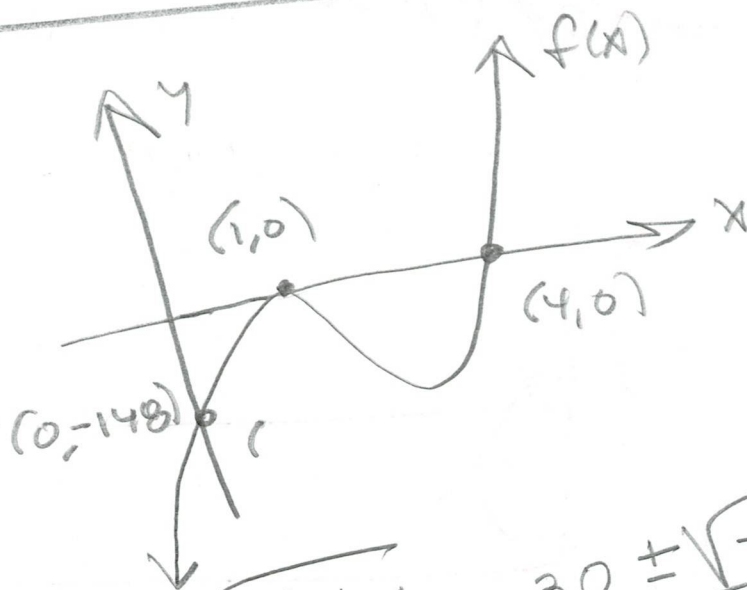
$= -432 < 0 \rightarrow$  No real roots  $\rightarrow$

Real zeros (roots) are  $x=1, 4$ .

$x=1$  has multiplicity  $m=2$ .

(5)  $f(x) = (x-1)^2(x-4)(9x^2 - 30x + 37)$

(6)



(7)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{30 \pm \sqrt{-432}}{2(9)}$$

$$= \frac{30 \pm 12i\sqrt{3}}{2(9)} = \frac{6(5 \pm 2i\sqrt{3})}{2(3)(3)}$$

$\frac{5 \pm 2i\sqrt{3}}{3} = x$

Handwritten calculations for  $\sqrt{432}$ :

$$\sqrt{432} = 12\sqrt{3}$$

\*7 critical

$$So, f(x) = 9(x-1)^2(x-4)\left(x - \left(\frac{5+2\sqrt{3}}{3}\right)\right)\left(x - \left(\frac{5-2\sqrt{3}}{3}\right)\right)$$

(8)  $R(x) = \frac{3x^2 + 13x - 56}{x^2 - 5x - 14} = \frac{(3x-8)(x+7)}{(x-7)(x+2)}$

2 | 56  
 2 | 28  
 2 | 14  
 7

SPB

$3(2)(2)(7)(\cancel{2})$   
 $3x^2 + 21x - 8x - 56$   
 $= 3x(x+7) - 8(x+7) = (x+7)(3x-8)$

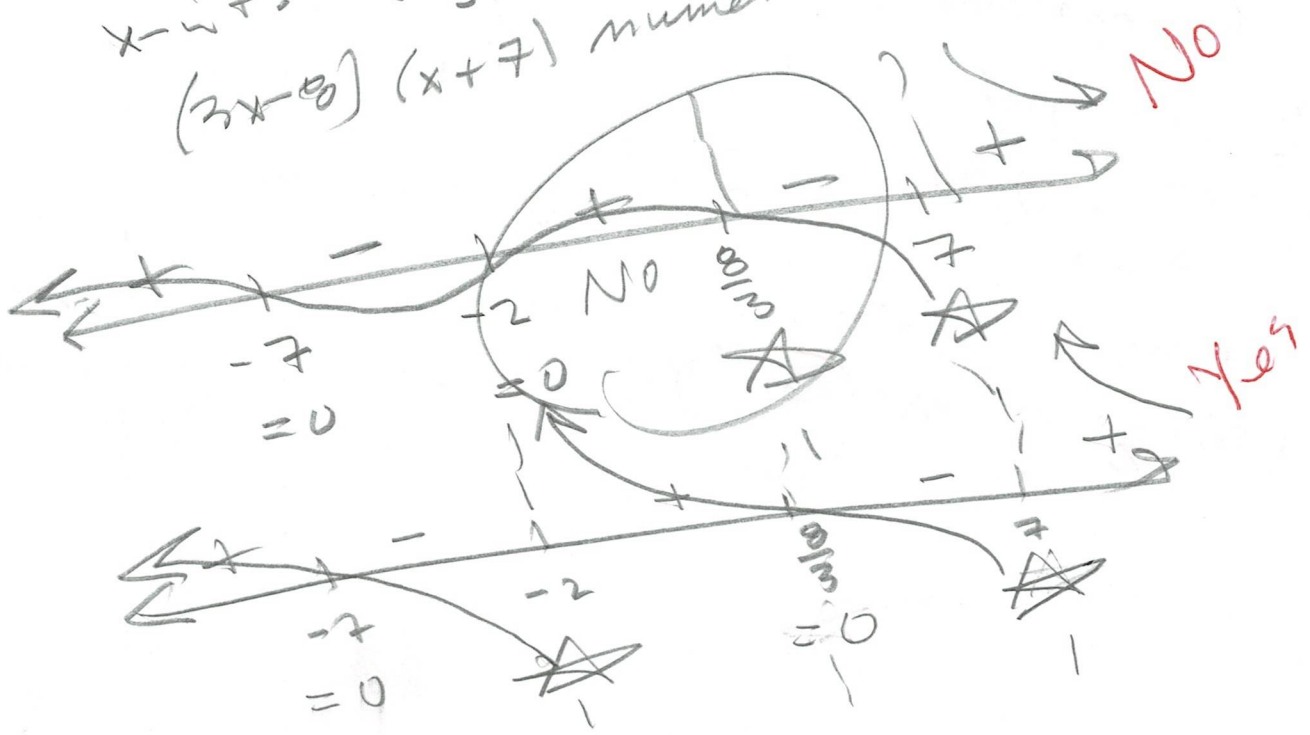
$\downarrow (R) = \mathbb{R} \setminus \{-2, 7\}$

VA:  $x = -2, x = 7$

H.A:  $\frac{3x^2}{x^2} = 3 = y$

y-int:  $\frac{-56}{-14} = -\frac{8}{-2} = 4 \rightarrow (0, 4)$

x-int:  $(\frac{8}{3}, 0), (-7, 0)$ , from  $(3x-8)(x+7)$  numerator.



#8 cont'd

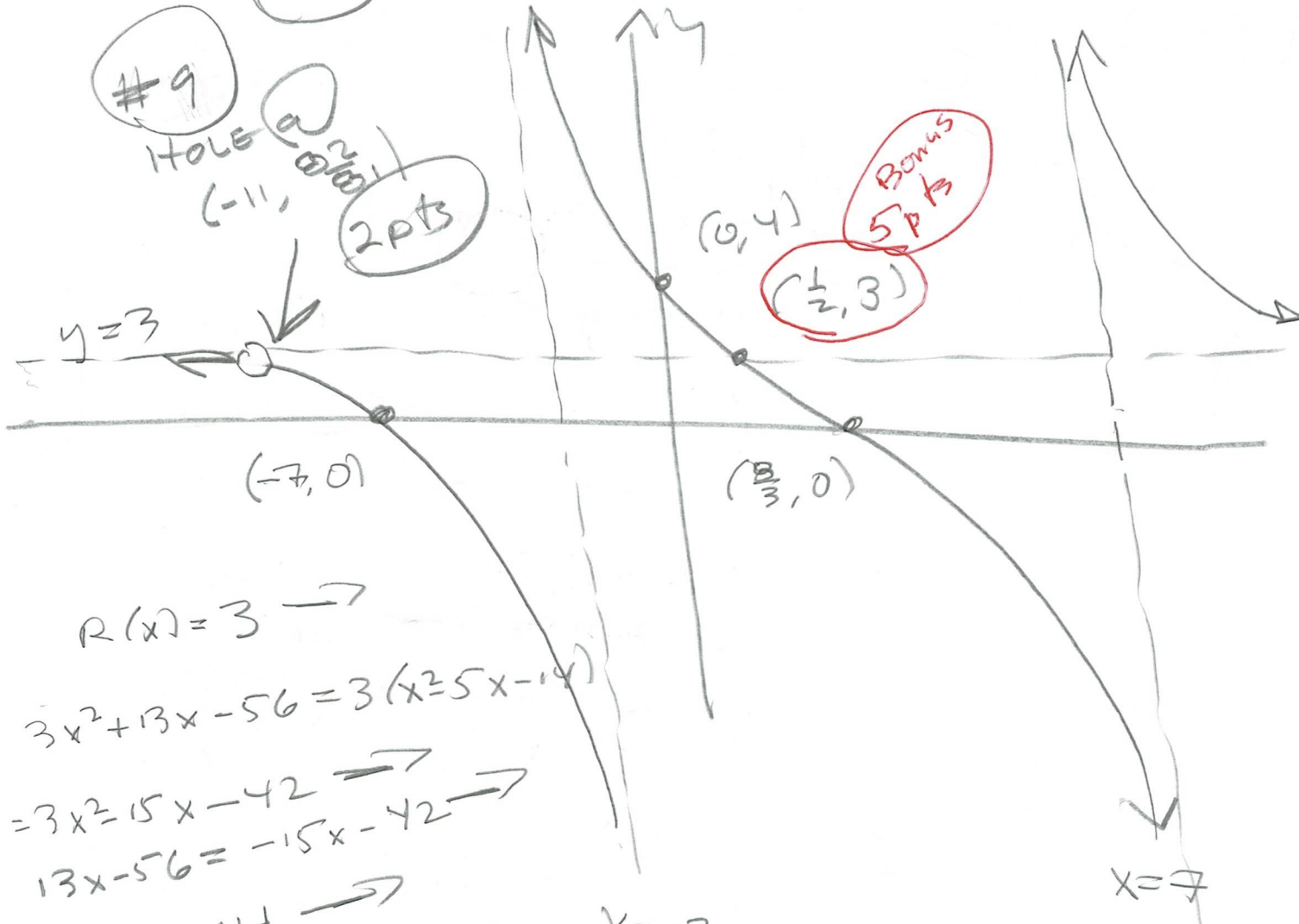
#8 5pts

#9

hole

(-11, 0)

2pts



$R(x) = 3 \rightarrow$

$3x^2 + 13x - 56 = 3(x^2 - 5x - 14)$

$= 3x^2 - 15x - 42 \rightarrow$

$13x - 56 = -15x - 42 \rightarrow$

$28x = 14 \rightarrow$

$x = \frac{1}{2} \rightsquigarrow (\frac{1}{2}, 4)$

$x = -2$

$x = 7$

12)

WP #3

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$$Q(x) = \frac{3x^3 + 46x^2 + 87x - 616}{x^3 + 6x^2 - 69x - 154} \rightarrow$$

$$\begin{array}{r} x-2 \overline{) 1 \quad 6 \quad -69 \quad -154} \\ \underline{\phantom{x-2} \phantom{)} \phantom{1} \phantom{6} \phantom{-69} \phantom{-154}} \\ \phantom{x-2} \phantom{)} \phantom{1} \phantom{6} \phantom{-69} \phantom{-154} \end{array}$$

$$\begin{array}{r} 7 \overline{) 1 \quad 4 \quad -77} \\ \underline{\phantom{7} \phantom{)} \phantom{1} \phantom{4} \phantom{-77}} \\ \phantom{7} \phantom{)} \phantom{1} \phantom{4} \phantom{-77} \end{array}$$

$$\begin{array}{r} \phantom{7} \phantom{)} \phantom{1} \phantom{4} \phantom{-77} \\ \phantom{7} \phantom{)} \phantom{1} \phantom{4} \phantom{-77} \end{array}$$

2PB  
see #8's graph

so  $x+11$  is the factor,  
 $x=-11$  is where the hole is, &

R(-11) from #8 is

$$\frac{(3(-11)-8)(-11+7)}{(-11-7)(-11+2)} = \frac{(-33-8)(-4)}{(-19)(-9)}$$

$$= \frac{(-41)(-2)}{(9)(9)} = \frac{82}{81} \rightarrow (-11, \frac{82}{81})$$

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WP #3

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$$(10) \quad \nabla(x) = \frac{3x^3 + 46x^2 + 87x - 616}{x^2 - 5x - 14}$$

$$= \frac{(3x-8)(x+7)(x+11)}{(x-7)(x+2)}$$

$$x\text{-int: } (-11, 0), (-7, 0), \left(\frac{8}{3}, 0\right)$$

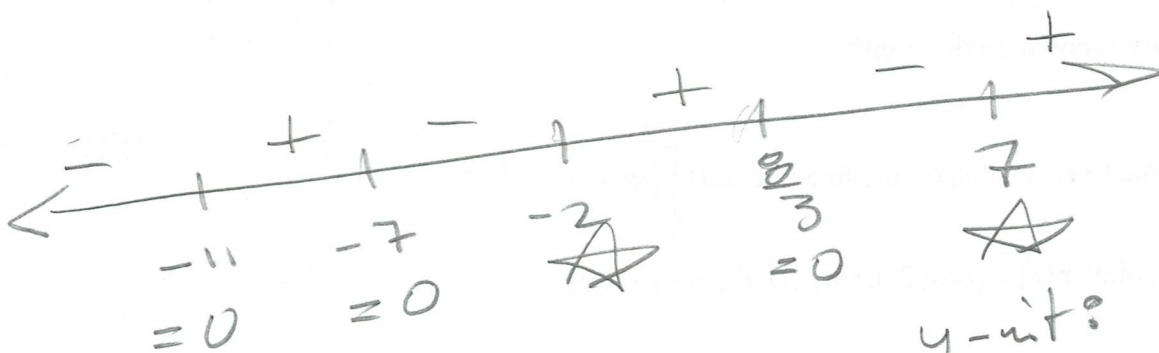
$$VA: (x = -2, x = 7)$$

SPB

S.A.

$$x^2 - 5x - 14 \quad \begin{array}{r} 3x + 61 \\ \hline 3x^3 + 46x^2 + 87x - 616 \\ - (3x^3 - 15x^2 - 42x) \\ \hline 61x^2 + 129x \end{array}$$

$$y = 3x + 61 \quad \text{is S.A.}$$



$$y\text{-int: } \frac{-616}{-14} = \frac{308}{7}$$

$$= 44$$

$(0, 44)$   
 $y\text{-int}$



~~10~~

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WP #3

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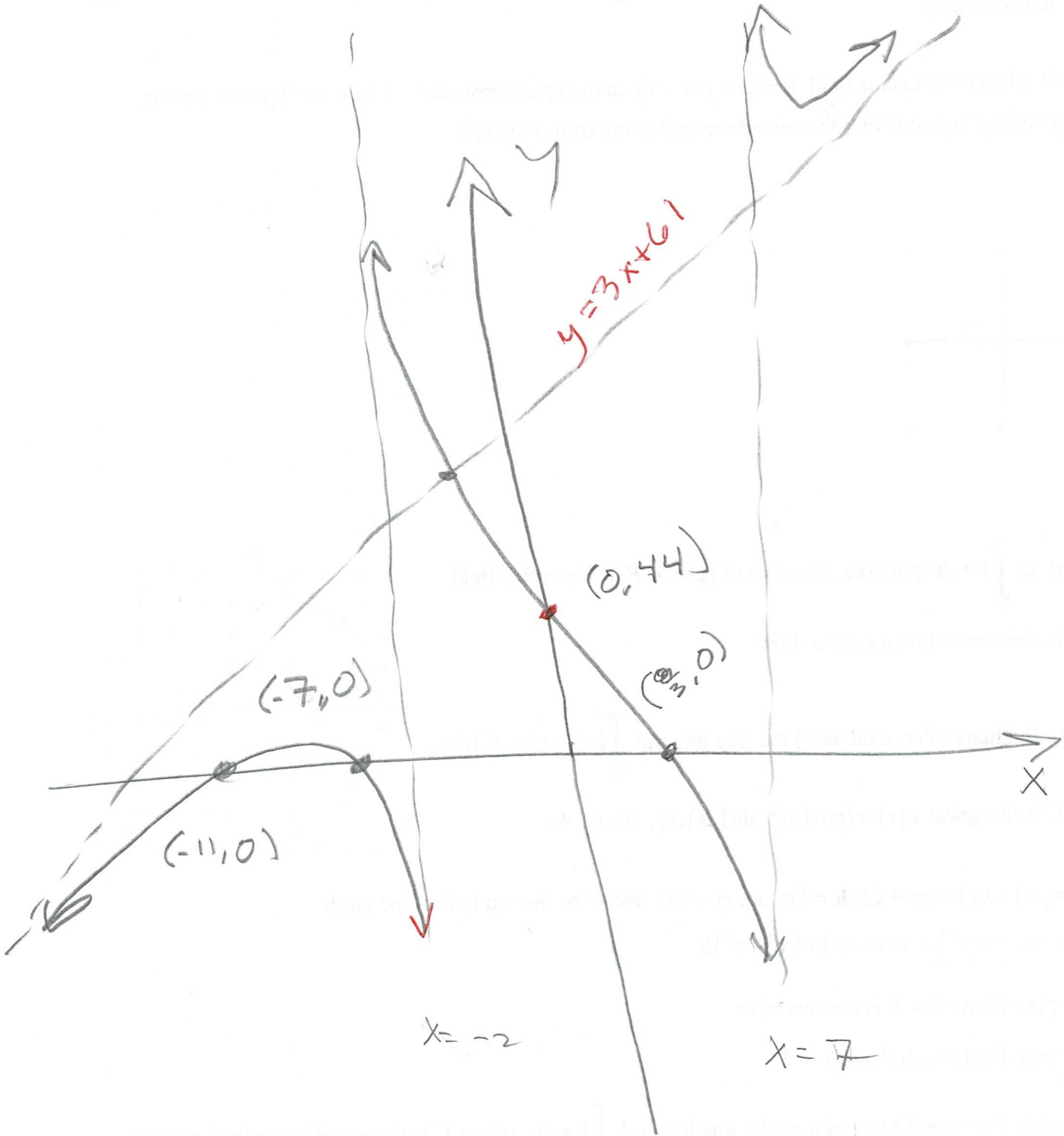
#10 critical

$$3x+61 = R(x)$$

$$3x+61 = 0$$

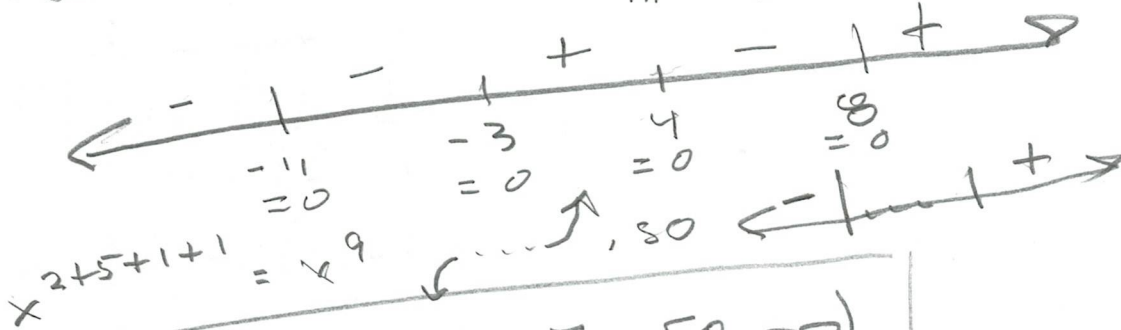
$$3x = -61$$

$$x = -\frac{61}{3} < -11$$



$$(11) \quad W(x) = \sqrt{(x+11)^2(x-4)^5(x+3)(x-8)} = \sqrt{\text{stuff}}$$

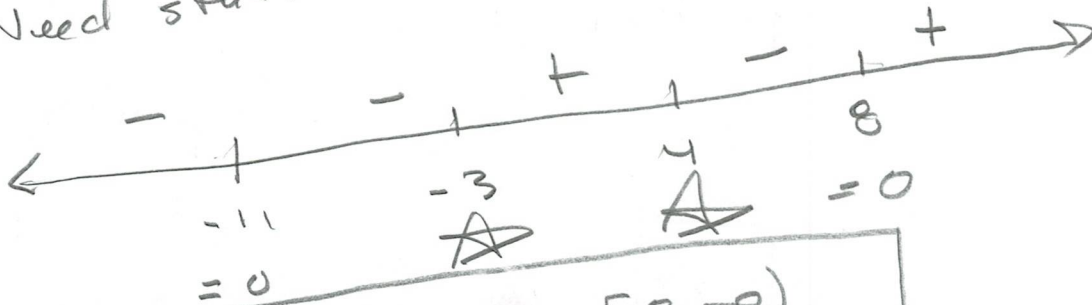
Need stuff  $\geq 0$ .  $x = -11, -3, 4, 8$   
 $m = 2, 1, 5, 1$



$$D = \{-11\} \cup [-3, 4] \cup [8, \infty)$$

$$(12) \quad K(x) = \sqrt{\frac{(x+11)^2(x-8)}{(x-4)^5(x+3)}} = \sqrt{\text{stuff}}$$

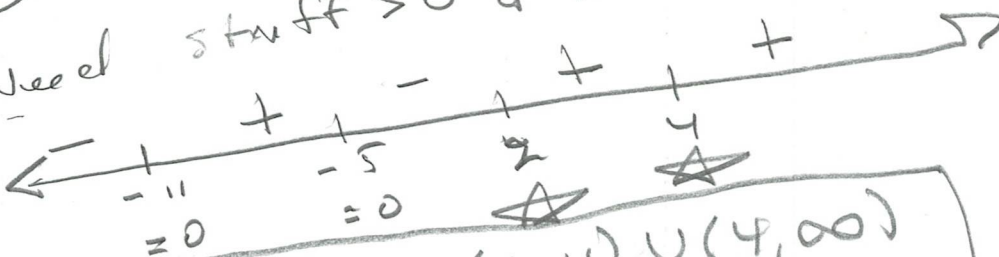
Need stuff  $\geq 0$  &  $(x-4)^5(x+3) \neq 0$



$$D = \{-11\} \cup (-3, 4) \cup [8, \infty)$$

$$(13) \quad T(x) = \log_5 \left( \frac{(x+11)^3(x+5)}{(x-4)^2(x-2)} \right) = \log_5(\text{stuff})$$

Need stuff  $> 0$  &  $(x-4)^2(x-2) \neq 0$



$$D = (-11, -5) \cup (2, 4) \cup (4, \infty)$$