MAT 121 Writing Project #2 Fall, 2021

Graphing Functions by Transforming Basic Functions

FORMATTING: This is semi-formal writing, here. That means show some professionalism. You don't have to type it out, but you do need to be very clear. For the formatting guidelines, please see Writing Project #1.

Due: By 5 pm, Friday, October 1st. I discounted 20% for late papers on WP#1. The discount is now the 25% given in the syllabus and schedule. I will still accept late Writing Project #1s until 5 p.m., Friday, October 1st. I will accept late writing Project #2s until 5 p.m. Thursday, October 7th.

Deliver your Writing Project by mail, e-mail or hand-delivery, according to instructions for Writing Project #1. Be sure to check out the mostly correct solutions to Writing Project #1.

Main Resources: Writing Project 2 Videos (and notes) and Chapter 2 Videos (and notes).

Method 1: 0. $f(x) \Rightarrow$ 1. $a\,f(x) \Rightarrow$ 2. $a\,f(x+c) \Rightarrow$ 3. $a\,f(bx+c) \Rightarrow$ 4. $a\,f(bx+c)+d = g(x)$

Method 2: 0. $f(x) \Rightarrow$ 1. $a\,f(x) \Rightarrow$ 2. $a\,f(bx) \Rightarrow$ 3. $a\,f\left(b\left(x+\frac{c}{b}\right)\right) \Rightarrow$ 4. $a\,f\left(b\left(x+\frac{c}{b}\right)\right)+d = g(x)$

Method 2 seems tougher for most beginners, but is more in keeping with what's ahead of you in mathematics, including the fact that Method 2 captures wavelength/frequency information before phase information.

Graph the function $g(x)$ by transforming the graph of a basic function, $f(x)$, as demonstrated in the Writing Project 2 Videos (and notes). Show the basic function and all 4 graphs that get you to $g(x)$ step by step, so we see all the moves.

1. $g(x) = 5\sqrt{3x-21}-2$
2. $g(x) = -5\sqrt{3x-21}+2$
3. $g(x) = 5\sqrt{-3x-21}-11$
4. $g(x) = 3(2x+8)^{2/3}+5$ Ask me about $f(x) = x^{2/3}$ basic function, if you can't find it in my videos.
5. $g(x) = 5\sqrt[5]{3x+21}-6$
6. $g(x) = 5(3x+21)^5-6$

We treat lines and parabolas a little differently. They come up so often - plus the completing-the-square trick – we sidestep the whole $f(bx)$ issue and just work with $g(x) = a(x-h)^2+k$ and $g(x) = m(x-h)+k$.

7. $g(x) = 3(x+5)-7$
8. $g(x) = 3(x+5)^2-7$
9. $g(x) = x^2-4x-7$
10. $g(x) = 4x^2+5x+17$

Point-slope you learned: $y - y_1 = m(x - x_1)$

Point-slope I teach and use: $y = m(x - x_1) + y_1$ which is now $y = m(x-h)+k$

The "cheat" for completing the square: $g(x) = ax^2 + bx + c = a(x-h)^2 + k = a\left(x + \frac{b}{2a}\right)^2 + g\left(-\frac{b}{2a}\right)$

Note that $h = -\frac{b}{2a}$ and $k = g(h) = g\left(-\frac{b}{2a}\right) =$ the output from plugging-in $-\frac{b}{2a}$ into $g(x)$. A student learning to complete the square might better achieve mastery by checking their work by completing the square with and with*out* the cheat. Make sure results match. Find out why they don't, if they don't. Own it.

Here's one way of doing the toughest kind by old-school completing-the-square skill for graphing:

Notice how at the very end, you multiply by the factor of 7 that you divided by in the 1st step. You don't want to forget the leading coefficient of 7 that's out front, because you're trying to graph the original function.

$$P(x) = 7x^2 + 3x + 11 \quad \Rightarrow$$

$$\frac{1}{7}P(x) = x^2 + \frac{3}{7}x + \frac{11}{7}$$

$$= x^2 + \frac{3}{7}x + \left(\frac{3}{14}\right)^2 - \frac{9}{196} + \frac{11}{7}$$

$$= \left(x + \frac{3}{14}\right)^2 + \frac{-9}{2\cdot 2\cdot 7\cdot 7} + \frac{11}{7}\cdot\frac{2\cdot 2\cdot 7}{2\cdot 2\cdot 7}$$

$$= \left(x + \frac{3}{14}\right)^2 + \frac{299}{196} \Rightarrow$$

$$P(x) = 7\left(x + \frac{3}{14}\right)^2 + (7)\left(\frac{299}{2\cdot 2\cdot 7\cdot 7}\right) = 7\left(x + \frac{3}{14}\right)^2 + \frac{299}{28}$$

$$(h,k) = \left(-\frac{3}{14}, \frac{299}{28}\right)$$

$$\approx (-0.2142857143,\ 10.67857143)$$