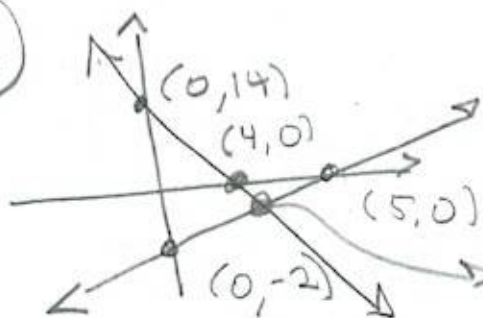


①
 a) $2x + 5y = 10$
 $7x + 2y = 28$

x	y	
0	-2	$-5y = 10$
5	0	$2x = 10$

x	y	
0	14	$2y = 28$
4	0	$7x = 28$

10 pts



Solution is

$$\left(\frac{160}{39}, -\frac{14}{39}\right)$$

$$\approx (4.102564103, -0.358974359)$$

b) 10 pts

$$2x = 5y + 10$$

$$x = \frac{5y + 10}{2} \rightarrow$$

$$7x + 2y = 7\left(\frac{5y + 10}{2}\right) + 2y = 28 \rightarrow$$

$$\frac{35y + 70}{2} + \frac{2y}{1} \cdot \frac{2}{2} = \frac{28}{1} \cdot \frac{2}{2} \rightarrow$$

$$35y + 70 + 4y = 56 \rightarrow$$

$$39y = 56 - 70 = -14 \rightarrow$$

$$y = \frac{-14}{39} \Rightarrow$$

$$x = \frac{5y + 10}{2} = \frac{5\left(\frac{-14}{39}\right) + 10}{2} = \frac{-\frac{70}{39} + \frac{10 \cdot 39}{1 \cdot 39}}{2}$$

$$= \frac{-70 + 390}{39} = \frac{320}{39} = \frac{160}{39} = x$$

$$(x, y) = \left(\frac{160}{39}, -\frac{14}{39}\right) \approx (4.1, -0.36)$$

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121 WP #4 Fall '20

① ② ③ 10pts

$$\begin{aligned}2x - 5y &= 10 \\ 7x + 2y &= 28\end{aligned}$$

$$\begin{array}{r} -7E1 \quad -14x + 35y = -70 \\ 2E2 \quad \underline{14x + 4y = 56} \end{array}$$

$$39y = -14$$

$$y = -\frac{14}{39}$$

$$-2x - 5y = 2x - 5\left(-\frac{14}{39}\right) = 2x + \frac{70}{39} = 10$$

$$\Rightarrow \frac{2x}{1} \cdot \frac{39}{39} + \frac{70}{39} = \frac{10}{1} \cdot \frac{39}{39} \Rightarrow$$

$$78x + 70 = 390 \Rightarrow$$

$$78x = 320 \Rightarrow$$

$$x = \frac{320}{78} = \frac{160}{39} = x$$

121 WP #4 Fall '20

(2)

$$x - 2y + z = 18$$

$$2x - 3y + 3z = 38$$

$$2x - 6y + z = 37$$

$$-2E1 - 2x + 4y - 2z = -36$$

$$E2 \quad 2x - 3y + 3z = 38$$

$$y + z = 2$$

$$x - 2y + z = 18$$

$$y + z = 2$$

$$2y + z = -1$$

$$-2E1 - 2x + 4y - 2z = -36$$

$$E3 \quad 2x - 6y + z = 37$$

$$-2y - z = 1$$

$$-2E2 \quad -2y - 2z = -4$$

$$E3 \quad 2y + z = -1$$

$$-z = -5$$

$$z = +5$$

$$y + z = y + 5 = 2 \rightarrow y = -3$$

$$2y + z = 2y + 5 = -1 \rightarrow 2y = -6$$

$$x - 2y + z = x - 2(-3) + 5 = x + 6 + 5 = x + 11 = 18$$

$$x = 7$$

Matrix Multiplication (optional)

Check:

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & 3 \\ 2 & -6 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 + 6 + 5 \\ 14 + 9 + 15 \\ 14 + 18 + 5 \end{bmatrix} = \begin{bmatrix} 18 \\ 38 \\ 37 \end{bmatrix} \checkmark$$

(3) (2) (10pts)

$$3x + 12y - 8z = 3$$

$$2x + 7y - 5z = 4$$

$$-x - 5y + 3z = 1$$

$$-E3 \quad x + 5y - 3z = -1$$

$$E1 \quad 3x + 12y - 8z = 3$$

$$E2 \quad 2x + 7y - 5z = 4$$

$$-3E1 \quad -3x - 15y + 9z = -3$$

$$E2 \quad 3x + 12y - 8z = 3$$

$$\hline -3y + z = 6$$

$$-2E1 \quad -2x - 10y + 6z = 2$$

$$E3 \quad 2x + 7y - 5z = 4$$

$$\hline -3y + z = 6$$

$$\text{So, } x + 5y - 3z = -1$$

$$-3y + z = 6 \rightarrow$$

$$-3y + z = 6$$

$$x + 5y - 3z = -1$$

$$-3y + z = 6$$

$$0 = 0!$$

$$\text{So } -3y = -z + 6 \rightarrow y = \frac{-z + 6}{-3} = \frac{z - 6}{3} = y$$

$$x + 5y - 3z = x + 5\left(\frac{z - 6}{3}\right) - 3z = -1$$

$$x + \frac{5z - 30}{3} - 3z = -1$$

$$\frac{3x + 5z - 30 - 9z}{3} = \frac{-3}{3}$$

$$3x - 4z - 30 = -3$$

$$3x = 4z + 30 - 3 = 4z + 27$$

$$\boxed{x = \frac{4z + 27}{3}}$$

$$(x, y, z) \in \left\{ \left(\frac{4z + 27}{3}, \frac{z - 6}{3}, z \right) \mid z \in \mathbb{R} \right\}$$

3b 10pts

$$z=0 \rightarrow (x, y, z) = (9, -2, 0)$$

$$z=1 \rightarrow (x, y, z) = \left(\frac{4(1)+27}{3}, \frac{1-6}{3}, 1 \right) = \left(\frac{31}{3}, -\frac{5}{3}, 1 \right)$$

$$z=-1 \rightarrow (x, y, z) = \left(\frac{4(-1)+27}{3}, \frac{-1-6}{3}, -1 \right) = \left(\frac{23}{3}, -\frac{7}{3}, -1 \right)$$

4 10pts

This is the same system as #3, except the "1" on the right-hand side of Equation 3 is now a "3". This changes the last system in #3

$$\begin{array}{l} \text{to} \\ x+5y-3z = -1 \\ -3y+z = 6 \\ -3y+z = 8 \end{array} \Rightarrow \begin{array}{l} x+5y-3z = -1 \\ -3y+z = 6 \\ 0 = 2? \end{array}$$

$0 = 2$ is absurd \Rightarrow #4 has no solution
 (or I made a mistake!)