

121 Writing Project #1 Fall '20

1) $x^2 + 3x - 28 = 0 \implies$

$a=1, b=3, c=-28 \implies$

$b^2 - 4ac = 3^2 - 4(1)(-28) = 9 + 112 = 121$

Now, $\sqrt{121} = 11 \implies$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm 11}{2(1)} = \begin{cases} \frac{8}{2} = 4 \\ \frac{-14}{2} = -7 \end{cases}$

$\implies x \in \{4, -7\}$

2) $323x^2 + 2132x - 5044 = 0$

$\implies 323x^2 + 2132x - 5044 \implies$

$a=323, b=2132, c=-5044 \implies$

$b^2 - 4ac = 2132^2 - 4(323)(-5044) =$

$= -4545424 + 6516848 = 11062272$

277 is prime, since $\sqrt{277} \approx 16.6$

and we've checked 2, 3, 5, 7, 11, 13

Next prime is 17 > 16.6

So $\sqrt{11062272} = \sqrt{2^{10} \cdot 3 \cdot 13 \cdot 277}$

$= 2^5 \sqrt{10803} = 32 \sqrt{10803}$

So, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2132 \pm 32\sqrt{277}}{2(323)}$

$= -\frac{1066 \pm 16\sqrt{277}}{323} = \left\{ \frac{1066 \pm 16\sqrt{277}}{323} \right\} = x$

#2 $x \in \{-2.4759, -4.1247\}$

2	11062272
2	5531136
2	2765568
2	1382784
2	691392
2	345696
2	172848
2	86424
5	43212
2	21606
3	10803
13	3601
	277

#3

(3) EXACT ANS:

$$x \in \left\{ \frac{-1000 \pm 16\sqrt{277}}{323} \right\}$$

5pb

5pb

(4) $bx^2 + 11wx - 6\pi = 0$

$$B^2 = 4\alpha C, \alpha = b, \beta = 11w, C = -6\pi$$

$$\Rightarrow B^2 = 4\alpha C = (11w)^2 = 4(b)(-6\pi) = 121w^2 + 24b\pi$$

$$\Rightarrow x = \frac{-\beta \pm \sqrt{B^2 = 4\alpha C}}{2\alpha} = \frac{-11w \pm \sqrt{121w^2 + 24b\pi}}{2b} = x$$

A bit awkward, with "b" a coefficient in the original

(5) 5pb $x^2 + 3x - 28 = (x+7)(x-4) = 0$

$$\rightarrow x \in \left\{ -7, 4 \right\}$$

$$\begin{array}{r} -140 \\ 30 \\ \hline -4200 \end{array}$$

-4200 MAGIC

(6) 5pb $30x^2 + 179x - 140 = 0$

$$179 = 180 - 1$$

$$= 190 - 11$$

$$= 200 - 21$$

$$-180$$

$$-2090$$

$$-4200 \text{ sweet!}$$

$$30x^2 + 200x - 21x - 140$$

$$= 10x(3x+20) - 7(3x+20) = (3x+20)(10x-7)$$

$$\rightarrow x \in \left\{ -\frac{20}{3}, \frac{7}{10} \right\}$$

⑥ Sledge hammer version

$$30x^2 + 179x - 140 = 0 \Rightarrow$$

$$a=30, b=179, c=-140 \rightarrow$$

$$b^2 - 4ac = 179^2 - 4(30)(-140) = 32041 + 16800$$

$$= 48841 \quad \sqrt{48841} = 221, 50$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-179 \pm 221}{2(30)} \begin{cases} \nearrow \frac{-400}{60} = -\frac{20}{3} \\ \searrow \frac{42}{60} = \frac{7}{10} \end{cases}$$

So, factored form is

$$30 \left(x + \frac{20}{3}\right) \left(x - \frac{7}{10}\right) = 3 \left(x + \frac{20}{3}\right) (10) \left(x - \frac{7}{10}\right)$$

$= (3x + 20)(10x - 7)$ is factored form for which we're looking, reverse-engineered for exact solutions.

⑦ (5pts) $x^2 - 5x + 5 = 0$

$$x^2 - 5x = -5$$

$$x^2 - 5x + \left(\frac{5}{2}\right)^2 = -5 + \frac{25}{4} = \frac{25 - 20}{4} = \frac{5}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{5}{4}$$

$$x + \frac{5}{2} = \pm \sqrt{\frac{5}{4}} = \pm \frac{\sqrt{5}}{2}$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{5}}{2}$$

$$\Rightarrow x \in \left\{ \frac{-5 \pm \sqrt{5}}{2} \right\}$$

8 (5 pts) $x^2 - 6x - 11 = 0$

$$x^2 - 6x = 11$$

$$x^2 - 6x + 3^2 = 11 + 9 = 20$$

$$(x-3)^2 = 20$$

$$x-3 = \pm\sqrt{20} = \pm 2\sqrt{5}$$

$$x \in \left\{ 3 \pm 2\sqrt{5} \right\}$$

9 (5 pts) $5x^2 + 2x + 11 = 0$

$$5\left(x^2 + \frac{2}{5}x + \frac{11}{5}\right) = 0$$

$$x^2 + \frac{2}{5}x + \left(\frac{1}{5}\right)^2 = -\frac{11}{5} + \frac{1}{25} = \frac{-55+1}{25} = \frac{-54}{25}$$

$$\left(x + \frac{1}{5}\right)^2 = \frac{-54}{25}$$

$$x + \frac{1}{5} = \pm \sqrt{\frac{-54}{25}} = \pm i \frac{3\sqrt{6}}{5}$$

$$\begin{array}{r} 3 \overline{) 54} \\ \underline{30} \\ 24 \\ \underline{21} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

$$x \in \left\{ \frac{-1 \pm 3i\sqrt{6}}{5} \right\}$$

121 WP #1 FALL '20

(10) (5pts) $3x^2 - 4x - 4 = 0 \quad \div 3$

$$x^2 - \frac{4}{3}x = \frac{4}{3}$$

$$x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2 = \frac{4}{3} + \frac{4}{9} = \frac{12+4}{9} = \frac{16}{9}$$

is a perfect square! \Rightarrow Factors by AC method!

$$\left(x - \frac{2}{3}\right)^2 = \frac{16}{9}$$

$$x - \frac{2}{3} = \pm \sqrt{\frac{16}{9}} = \pm \frac{4}{3}$$

$$\Rightarrow x = \frac{2 \pm 4}{3} = \begin{cases} \frac{6}{3} = 2 \\ -\frac{2}{3} \end{cases} \quad x \in \left\{-\frac{2}{3}, 2\right\}$$

Check: $3\left(x + \frac{2}{3}\right)(x - 2) = (3x + 2)(x - 2)$
 $= 3x^2 - 6x + 2x - 4 = 3x^2 - 4x - 4$ ✓