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Find common ratio

1 $4, 2, 1, \frac{1}{2}, \ldots$

$\frac{2}{4} = \frac{1}{2}, \frac{1}{2}, \frac{\frac{1}{2}}{1} = \boxed{\frac{1}{2} = r}$

2 $10^2, 10^3, 10^4, \ldots$

$\frac{10^3}{10^2} = 10, \frac{10^4}{10^3} = \boxed{10 = r}$

3 $-1, 2, -4, 8, \ldots$

$\frac{2}{-1} = -2, \frac{-4}{2} = \boxed{-2 = r}$

4 $1, -1, 1, -1, \ldots$ $\boxed{r = -1}$

Write nth term

5 $\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}$

$\frac{\frac{1}{3}}{\frac{1}{6}} = \frac{1}{3} \cdot \frac{6}{1} = 2$

$\frac{\frac{2}{3}}{\frac{1}{3}} = \boxed{2 = r}$

$a = \frac{1}{6}$

So, we have $a_n = \boxed{\frac{1}{6}(2)^{n-1}}$

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6 $.9, .09, .009, .0009$

$\frac{.09}{.9} = \frac{9}{90} = \frac{1}{10} = r$ $\boxed{a_n = .9\left(\frac{1}{10}\right)^{n-1}}$

$a = .9$

7 $4, -12, 36, -108$

$-\frac{12}{4} = -3, \frac{36}{-12} = -3 = r$ $\boxed{a_n = 4(-3)^{n-1}}$

$a = 4$

Find the sum

8 $3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \ldots$

$\frac{-1}{3} = -\frac{1}{3}, \frac{\frac{1}{3}}{-1} = -\frac{1}{3}, \frac{-\frac{1}{9}}{\frac{1}{3}} = -\frac{1}{9} \cdot \frac{3}{1} = \boxed{-\frac{1}{3} = r}$

$a = 3$

$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$

with the "n"

$= 3\left(\frac{\left(-\frac{1}{3}\right)^n - 1}{-\frac{1}{3} - 1}\right) \xrightarrow{n \to \infty} 3\left(\frac{-1}{-\frac{4}{3}}\right)$

$= 3\left(\frac{1}{\frac{4}{3}}\right) = 3\left(\frac{3}{4}\right) = \boxed{\frac{9}{4} = S}$

w/o the "n"
after $n \to \infty$

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$.9 + .09 + .009 + \cdots$

$\frac{.09}{.9} = .1 = \frac{1}{10} = r \qquad S_n$

$a = .9 = \frac{9}{10}$

$$S_n = \frac{9}{10}\left(\frac{\left(\frac{1}{10}\right)^n - 1}{\frac{1}{10} - 1}\right) = \frac{9}{10}\left(\frac{\left(\frac{1}{10}\right)^n - 1}{-\frac{9}{10}}\right) \xrightarrow{n \to \infty} \frac{9}{10}\left(\frac{-1}{-\frac{9}{10}}\right)$$

$$= \left(\frac{9}{10}\right)\left(\frac{10}{9}\right) = \boxed{1 = S}$$

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$-9.9 + 3.3 - 1.1 + \cdots$

$\frac{3.3}{-9.9} = -\frac{1}{3}, \quad \frac{-1.1}{3.3} = -\frac{1}{3} = r$

$a = -9.9$

$$S_n = -9.9\left(\frac{-1}{-\frac{1}{3} - 1}\right) = -9.9\left(\frac{-1}{-\frac{4}{3}}\right) = -9.9\left(\frac{3}{4}\right) = \boxed{7.425}$$

Same

$$= -9.9\left(\frac{3}{4}\right)$$

$$= -\left(9 + \frac{9}{10}\right)\left(\frac{3}{4}\right) = -\frac{99}{10}\left(\frac{3}{4}\right) = \boxed{-\frac{297}{40}}$$

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11 $\sum_{k=1}^{\infty} 34(0.01)^k$ is trick question

is the trick.

Need $k-1$ up there for our $a\left(\frac{-1}{r-1}\right)$ thing

$$\sum_{k=1}^{\infty}(34)(0.01)^k = \sum_{k=1}^{\infty}(34)(0.01)(0.01)^{k-1}$$

$$= \sum_{k=1}^{\infty} .34(.01)^{k-1} = .34\left(\frac{-1}{.01-1}\right) = .34\left(\frac{-1}{-.99}\right)$$

$$= \frac{.34}{.99} = \boxed{\frac{34}{99} = .3\overline{4}\overline{3}\overline{4}}$$

$r = .01$

$a = .34$ is the trick

Use S_n formula. Check by long way

12 $1.5 - 3 + 6 - 12 + 24 - 48 + 96 - 192 = -127.5$

$a = 1.5,\ r = -2,\ n = 8$ terms

$$a\left(\frac{r^n - 1}{r-1}\right) = 1.5\left(\frac{(-2)^8 - 1}{-2-1}\right) = \frac{3}{2}\left(\frac{256-1}{-3}\right)$$

$$= \frac{3}{2}\left(\frac{255}{-3}\right) = \boxed{-\frac{255}{2}} = -127.5$$

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$$\sum_{k=1}^{12} 2(1.05)^{k-1} \qquad a=2,\ r=1.05,\ n=12$$

$$S_n = 2\left(\frac{(1.05)^{12}-1}{1.05-1}\right) \approx 2\left(\frac{1.795856326-1}{.05}\right)$$

$$=2\left(\frac{.795856326}{.05}\right) \approx 31.83425304$$

$$\approx 31.8343$$

$$2(1.05)^{1-1} + 2(1.05)^{2-1} + 2(1.05)^{3-1} + 2(1.05)^{4-1} +$$

$$+2(1.05)^{5-1} + 2(1.05)^{6-1} + 2(1.05)^{7-1} + 2(1.05)^{8-1}$$

$$+2(1.05)^{9-1} + 2(1.05)^{10-1} + 2(1.05)^{11-1} + 2(1.05)^{12-1}$$

$$\approx 31.8343$$

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$$\sum_{k=0}^{7} 200(1.01)^k = \sum_{k=1}^{8} 200(1.01)^{k-1}$$

Rectifying it to the form $\sum_{k=1}^{n} ar^{k-1}$

$$= 200\left(\frac{(1.01)^8-1}{1.01-1}\right) = 200\left(\frac{(1.01)^8-1}{.01}\right) \approx 1657.134113$$

$$\approx 1657.1341$$

$$200 + 200(1.01) + 200(1.01)^2 + 200(1.01)^3 + 200(1.01)^4$$

$$+200(1.01)^5 + 200(1.01)^6 + 200(1.01)^7$$

$$\approx 1657.1341$$

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write in Σ-notation

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$3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27}$

$a = 3, \quad r = -\frac{1}{3}$

$\frac{1}{27} = 3\left(-\frac{1}{3}\right)^{n-1}$

$\frac{1}{81} = \left(-\frac{1}{3}\right)^{n-1}$

$\frac{1}{3^4} = \left(-\frac{1}{3}\right)^{n-1} = (-1)^{n-1}\left(\frac{1}{3}\right)^{n-1} = (-1)^{n-1}\left(\frac{1}{3^{n-1}}\right)$

$4 = n-1 \Longrightarrow 5 = n$

$$\boxed{\sum_{k=1}^{5} 3\left(-\frac{1}{3}\right)^{k-1}}$$

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$.6 + .06 + .006 + \cdots$

$a = .6, \quad r = \frac{1}{10}$

$$\boxed{\sum_{k=1}^{\infty} (.6)\left(\frac{1}{10}\right)^{k-1}}$$