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Write the sum using Σ-notation

1) $1+2+3+4+5+6 = \sum_{k=1}^{6} k$

k: 1 2 3 4 5 6

f(k): k k k k k k

2) $2+4+6+8+10+12 = \sum_{k=1}^{6} 2k$

k: 1 2 3 4 5 6

f(k): k+1 $\cancel{k+k}$ 2k 2k 2k ✓

All legit.

3) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16} = \sum_{k=1}^{5}(-1)^{k+1}\frac{1}{2^{k-1}} = \sum_{k=0}^{4}(-1)^{k}\frac{1}{2^{k}} = \sum_{k=-1}^{3}(-1)^{k+1}\left(\frac{1}{2^{k+1}}\right)$

k: 1 2 3 4 5

f(k): k $-\frac{1}{k}$ $\cancel{-\frac{1}{k}}$

$\frac{1}{k}$ $\cancel{\frac{1}{k}}$

Alternates $(-1)^{\text{something}}$

1st term is positive when k is odd.

Need even power of k-1 or k+1

k=1 k=2

$\frac{1}{1} = \frac{1}{2^0}$, $\frac{1}{2} = \frac{1}{2^1}$ Oh. I see. The power is k-1

k: 1 2 3 4 5

f(k): $\frac{1}{2^0}$ $\frac{1}{2^1}$ $\frac{1}{8} = \frac{1}{2^3}$ when k=4, 3 = k-1 ✓

$\frac{1}{2^{k-1}}$ $\frac{1}{2^{k-1}}$

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(4) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \cdots$ See #3

$$= \sum_{k=1}^{\infty} (-1)^{k+1}\left(\frac{1}{2^{k-1}}\right) \text{ OR } \sum_{k=0}^{\infty} (-1)^{k}\left(\frac{1}{2^{k}}\right) \text{ OR}$$

$$\sum_{k=1}^{\infty} (-1)^{k-1}\left(\frac{1}{2^{k-1}}\right)$$

Rewrite the series using new index j, as indicated.

(5) $$\sum_{i=4}^{13} (2i+1) = \sum_{j=1}$$

$$\sum_{j=1}^{13-3} \quad , \quad j = i-3, \text{ in general, so } i = j+3$$

$$\sum_{j=1}^{10} (2(j+3)+1) = \sum_{j=1}^{10} (2j+6+1) = \boxed{\sum_{j=1}^{10} (2j+7)}$$

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$\infty - 3 = \infty$!

(6) $\sum_{n=2}^{\infty} \frac{5^n e^{-5}}{n!} = \boxed{\sum_{j=5}^{\infty} \frac{5^{j-3} e^{-5}}{(j-3)!}}$

$n=2,\ j=5$

$j=n+3$, so $n=j-3$