121 §5.2, 601

Solve the system of linear equations

5.2 way

$$\begin{aligned} x + y + z &= 6 \\ 2x - 2y - z &= -5 \\ 3x + y - z &= 2 \end{aligned}$$

$$\begin{array}{lrl} -2R1 & -2x - 2y - 2z &= -12 \\ R2 & 2x - 2y - z &= -5 \\ \hline & -4y - 3z &= -17 \end{array}$$

$$\begin{array}{ll} R1 & x + y + z = 6 \\ -2R1 + R2 & -4y - 3z = -17 \\ -3R1 + R3 & -2y - 4z = -16 \end{array}$$

$$\begin{array}{lrl} -3R1 & -3x - 3y - 3z &= -18 \\ R2 & 3x + y - z &= 2 \\ \hline & -2y - 4z &= -16 \end{array}$$

$$\begin{array}{ll} R1 & x + y + z = 6 \\ R2 & -4y - 3z = -17 \\ R2 - 2R3 & 5z = 15 \end{array}$$

$$\begin{array}{lrl} R2 & -4y - 3z &= -17 \\ -2R3 & 4y + 8z &= 32 \\ \hline & 5z &= 15 \end{array}$$

BACK-SUB:

① $5z = 15$

$z = 3$

② $-4y - 3(3) = -17$

$-4y - 9 = -17$

$-4y = -8$

$y = 2$

③ $x + 2 + 3 = 6$

$x + 5 = 6$

$x = 1$

$(x, y, z) = (1, 2, 3)$

Soln set: $(x, y, z) \in \{(1, 2, 3)\}$

The unknown

is in

this set, containing one point

121 S'02, 6.1

1

$$x + y + z = 6$$
$$2x - 2y - z = -5$$
$$3x + y - z = 2$$

GAUSSIAN ELIMINATION

$$\left[\begin{array}{ccc|c} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & \cdot & \cdot & \cdot \\ 0 & 1 & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & \cdot & \cdot & \cdot \\ 0 & 1 & \cdot & \cdot \\ 0 & 0 & 1 & A \end{array}\right]$$

$z = A$!

Back-sub

For x & y.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -2 & -1 & -5 \\ 3 & 1 & -1 & 2 \end{array}\right] \begin{array}{l} R1 \\ -2R1 + R2 \longrightarrow R2 \\ -3R1 + R3 \longrightarrow R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -4 & -3 & -17 \\ 0 & -2 & -4 & -16 \end{array}\right] \begin{array}{l} R1 \\ -\frac{1}{4}R2 \longrightarrow R2 \\ R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 3/4 & 17/4 \\ 0 & -2 & -4 & -16 \end{array}\right] \begin{array}{l} R1 \\ R2 \\ 2R2 + R3 \longrightarrow R3 \end{array}$$

$$(2)\left(\frac{3}{4}\right) - 4 = \frac{3}{2} - 4 = \frac{3}{2} - \frac{8}{2} = -\frac{5}{2}$$

$$(2)\left(\frac{17}{4}\right) - 16 = -\frac{17}{2} - 16$$
$$= \frac{17}{2} - \frac{32}{2} = -\frac{15}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 3/4 & 17/4 \\ 0 & 0 & -5/2 & -15/2 \end{array}\right] \begin{array}{l} R1 \\ R2 \\ -\frac{2}{5}R3 \to R3 \end{array}$$

$$\left(-\frac{2}{5}\right)\left(-\frac{15}{2}\right) = 3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 3/4 & 17/4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$z = 3$

$$y + \frac{3}{4}(3) = \frac{17}{4}$$
$$y + \frac{9}{4} = \frac{17}{4}$$
$$y = \frac{17-9}{4} = \frac{8}{4} = 2 = y$$

$$x + 2 + 3 = 6$$
$$x + 5 = 6$$
$$x = 1$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

121 §8.1, 6.1

(3)

2

$$4x - 2y + z = 13$$
$$3x - y + 2z = 13$$
$$x + 3y - 3z = -10$$

R3 $\quad x + 3y - 3z = -10$

R2 $\quad 3x - y + 2z = 13$

R1 $\quad 4x - 2y + z = 13$

$-3R1 \quad -3x - 9y + 9z = 30$

$R2 \quad 3x - y + 2z = 13$

$-10y + 11z = 43$

R1 $\quad x + 3y - 3z = -10$

$-3R1 + R2 \quad -10y + 11z = 43$

$-4R1 + R3 \quad -14y + 13z = 53$

$-4R1 \quad -4x - 12y + 12z = 40$

$R3 \quad 4x - 2y + z = 13$

$-14y + 13z = 53$

R1 $\quad x + 3y - 3z = -10$

R2 $\quad -10y + 11z = 43$

$-14R2 + 10R3 \quad -24z = -172$

$-14R2 \quad 140y - 154z = -602$

$10R3 \quad -140y + 130z = 530$

$-24z = -72$

$z = \frac{-72}{-24} = 3$

$z = 3$

$-10y + 11(3) = 43$

$-10y + 33 = 43$

$-10y = 10$

$y = -1$

$x + 3(-1) - 3(3) = -10$

$x - 3 - 9 = -10$

$x - 12 = -10$

$x = 2$

$\{(2, -1, 3)\}$

Same deal, with a matrix

$$\left[\begin{array}{ccc|c} 4 & -2 & 1 & 13 \\ 3 & -1 & 2 & 13 \\ 1 & 3 & -3 & -10 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -10 \\ 3 & -1 & 2 & 13 \\ 4 & -2 & 1 & 13 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -10 \\ 0 & -10 & 11 & 43 \\ 0 & -14 & 13 & 53 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -10 \\ 0 & 1 & -\frac{11}{10} & -\frac{43}{10} \\ 0 & -14 & 13 & 53 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -10 \\ 0 & 1 & -\frac{11}{10} & -\frac{43}{10} \\ 0 & 0 & -\frac{12}{5} & -\frac{36}{5} \end{array}\right]$$

$$(14)\left(-\frac{11}{10}\right) + 13$$
$$= -\frac{77}{5} + \frac{65}{5} = -\frac{12}{5}$$

$$(14)\left(-\frac{43}{10}\right) + 53 =$$
$$= \frac{-301}{5} + \frac{265}{5} = -\frac{36}{5}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -3 & -10 \\ 0 & 1 & -\frac{11}{10} & -\frac{43}{10} \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$z = 3$$

$$y - \frac{11}{10}(3) = -\frac{43}{10}$$
$$y - \frac{33}{10} = -\frac{43}{10}$$
$$y = \frac{33-43}{10} = -\frac{10}{10} = -1 = y$$

$$x + 3(-1) - 3(3) = -10$$
$$x - 3 - 9 = -10$$
$$x - 12 = -10$$
$$x = 2$$

$$(x, y, z) \in \{(2, -1, 3)\}$$

121 § 8.2, 6.1

(3) $x + 3y + 7z = 11$

$2x + 7y + 17z = 24$

$-x - 4y - 10z = -13$

$$\left[\begin{array}{ccc|c} 1 & 3 & 7 & 11 \\ 2 & 7 & 17 & 24 \\ -1 & -4 & -10 & -13 \end{array}\right] \begin{array}{l} R1 \\ -2R1 + R2 \\ R1 + R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 7 & 11 \\ 0 & 1 & 3 & 2 \\ 0 & -1 & -3 & -2 \end{array}\right] \begin{array}{l} R1 \\ R2 \\ R2 + R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 7 & 11 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + 3y + z = 11$

$y + 3z = 2 \rightarrow \boxed{y = -3z + 2}$ so

$x + 3y + z = 11$ means

$x + 3(-3z + 2) + 2 = 11$, etc.

See previous page for the big finish.

121 § 8.2, 6.1

(3) $x + 3y + 7z = 11$

$2x + 7y + 17z = 24$

$-x - 4y - 10z = -13$

$$\begin{array}{ll} -2R1 & -2x - 6y - 14z = -22 \\ R2 & 2x + 7y + 17z = 24 \\ \hline & y + 3z = 2 \end{array}$$

$$\begin{array}{ll} R1 & x + 3y + 7z = 11 \\ R3 & -x - 4y - 10z = -13 \\ \hline & -y - 3z = -2 \end{array}$$

$$\begin{array}{ll} R1 & x + 3y + 7z = 11 \\ -2R1 + R2 & y + 3z = 2 \\ R1 + R3 & -y - 3z = -2 \end{array}$$

$$\begin{array}{ll} R1 & x + 3y + 7z = 11 \\ R2 & y + 3z = 2 \\ R2 + R3 & 0 = 0 \end{array}$$

$y + 3z = 2$

$y = -3z + 2$

$x + 3y + 7z = 11$

$x + 3(-3z + 2) + 7z = 11$

$x - 9z + 6 + 7z = 11$

z is free!

$x + 6 - 2z = 11$

$x = 2z + 5$

Solution set: $\boxed{\{ (2z+5, -3z+2, z) \mid z \in \mathbb{R} \}}$

ONE FREE VARIABLE, z. This describes a straight line in space.

121 §5.2, 6.1

(4)

$2x-6y+4z=8$ $\quad \frac{1}{2}R1$

$3x-9y+6z=12$ $\quad R2$

$5x-15y+10z=20$ $\quad R3$

START HERE, on Matrix Version

$x-3y+2z=4$ $\quad R1$

$3x-9y+6z=12$

$5x-15y+10z=20$

$-3R1 \quad -3x+9y-6z=-12$

$R2 \quad 3x-9y+6z=12$

$0=0$

!

$-5R1 \quad -5x+15y-10z=-20$

$R3 \quad 5x-15y+10z=20$

$0=0$!

$R1 \quad 1x-3y+2z=4$

$-3R1+R2 \quad 0=0$

$-5R1+R3 \quad 0=0$

So, $x=+3y-2z+4$, and y & z are free!

2 degrees of freedom. 2-dimensional object.

The solution is a plane!

$$\{(3y-2z+4,\ y,\ z) \mid y,z \in \mathbb{R}\}$$

All points on the first plane.

121 S' 8.2, 6a1

4

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 3 & -9 & 6 & 12 \\ 5 & -15 & 10 & 20 \end{array}\right] \begin{array}{l} R1 \\ -3R1+R2 \\ -5R1+R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x - 3y + 2z = 4$, etc. See previous page.

121 §5.2, 6.1

(5)

$x - 3y + 2z = 4$

$3x - 9y + 6z = 12$

$5x - 15y + 10z = 21$

R1	$x - 3y + 2z = 4$
$-3R1 + R2$	$0 = 0$ $\longrightarrow$ No restriction
$-5R1 + R3$	$0 = 1$ $\longrightarrow$ FALSE !

We arrive at the conclusion that $0 = 1$ when we try to find points common to all three planes. Conclusion? **There is no solution.** The assumption that there WAS lead to something false / ridiculous.

121 §8.2, 6.1

(5)
$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 3 & -9 & 6 & 12 \\ 5 & -15 & 10 & 21 \end{array}\right] \begin{array}{l} R1 \\ -3R1+R2 \\ -5R1+R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \to \text{No restriction} \\ \to \text{No Way!} \end{array}$$

$0 = 1$!?! False $\therefore$ No Solution.

FACT: 1st two planes were same plane.
3rd plane was parallel to the 1st two.

121 §5.2, 6.1

(6) Find the coefficients a, b, c of a parabola $f(x) = ax^2+bx+c$ if you know that $(1,5)$, $(3,19)$ and $(-2,29)$ are all points on the parabola.

$f(1) = 5$

$a(1)^2 + b(1) + c = 5$

$a+b+c = 5$

$f(3) = 19$

$a(3)^2 + b(3) + c = 19$

$9a + 3b + c = 19$

$f(-2) = 29$

$a(-2)^2 + b(-2) + c = 29$

$4a - 2b + c = 29$

$a+b+c=5$

$9a+3b+c=19$

$4a-2b+c=29$

R1	$a+b+c=5$
$-9R1+R2$	$-6b-8c=-26$
$-4R1+R3$	$-6b-3c=9$

$$\begin{array}{rr} -9R1 & -9a-9b-9c=-45 \\ R2 & 9a+3b+c=19 \\ \hline & -6b-8c=-26 \end{array}$$

R1	$a+b+c=5$
R2	$-6b-3c=9$
	$5c=35$
$-R2+R3$	

$\boxed{c=7}$

$$\begin{array}{rr} -4R1 & -4a-4b-4c=-20 \\ & 4a-2b+c=29 \\ \hline & -6b-3c=9 \end{array}$$

$$\begin{array}{rr} -R2 & 6b+8c=26 \\ R3 & -6b-3c=9 \\ \hline & 5c=35 \end{array}$$

$c=7$

121 $\quad 882, 61$

$c = 7$

$-6b - 3c = 9$

$-6b - 3(7) = 9$

$-6b - 21 = 9$

$-6b = 30$

$b = -5$

$a + b + c = 5$

$a - 5 + 7 = 5$

$a + 2 = 5$

$a = 3$

$$\boxed{\therefore\ f(x) = 3x^2 - 5x + 7}$$

121 §8.2, 6.1

(6) Setup from previous

$$\begin{array}{cc} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 9 & 3 & 1 & 19 \\ 4 & -2 & 1 & 29 \end{array}\right] & \begin{array}{l} R1 \\ -9R1+R2 \\ -4R1+R3 \end{array} \end{array}$$

$$\begin{array}{cc} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -6 & -8 & -26 \\ 0 & -6 & -3 & 9 \end{array}\right] & \begin{array}{l} R1 \\ -R2 \\ -R2+R3 \end{array} \end{array}$$

$$\begin{array}{cc} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 6 & 8 & 26 \\ 0 & 0 & 5 & 35 \end{array}\right] & \begin{array}{l} R1 \\ R2 \\ R3 \end{array} \end{array}$$

$5c=35$

$c=7$

$6b+8c=26$

$6b+8(7)=26$

$6b+56=26$

$6b=-30$

$b=-5$

$a+b+c=5$

$a-5+7=5$

$a+2=5$

$a=3$

$(a,b,c)\in\{(3,-5,7)\}$ and $\boxed{f(x)=3x^2-5x+7}$