Last time we discussed 5.2, 6.1

Book makes a big fuss about

1. 1's on the diagonal
2. 0's above & below the diagonal (GAUSS-JORDAN)

Meh. Get 0's below the main diagonal & we're good. GAUSS

5.2

$$\begin{aligned} x + y + z &= 2 \\ 3x - y - 5z &= 7 \\ 2x + y + z &= 11 \end{aligned}$$

6.1

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 3 & -1 & -5 & 7 \\ 2 & 1 & 1 & 11 \end{array}\right]$$

Same exact moves. Different Notation

Nov 17-7:49 AM

232 coins

pennies, nickels, dimes

How many coins of each type?

§5.2 #71

For fun.

Let x = the # of pennies

y = " " " nickels

z = " " " dimes

Total is \$10.36

$x + y + z = 10.36$ No! This is MONEY! NOT THE # of coins!

$.01x + .05y + .10z = 10.36 \rightsquigarrow x + 5y + 10z = 1{,}036$

\$ = \$

# of pennies = dimes plus nickels

$x = y + z \longrightarrow x - y - z = 0$

232 coins

$x + y + z = 232$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 232 \\ 1 & 5 & 10 & 1036 \end{array}\right]$$

$-R1+R2, -R1+R3$

$-3R2+R3, \frac{1}{2}R2$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 2 & 2 & 232 \\ 0 & 6 & 11 & 1036 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & 1 & 116 \\ 0 & 0 & 5 & 340 \end{array}\right]$$

$$\begin{array}{r} 1036 \\ -\ 696 \\ \hline 340 \end{array}$$

$5z = 340$

$\boxed{z = 68}$

$y + 68 = 116$

$\boxed{y = 48}$

$x - 48 - 68 = 0$

$x - 116 = 0$

$\boxed{x = 116}$

§ 8.2, 8.3 Geometric Sums

$\boxed{E}$ Pop. growth during period of immigration

Exponential!

Geometric!

ANNUITY

Putting away money every month to save for school

Each deposit grows exponentially (compound interest). But you keep throwing more deposits on top.

How an annuity grows

R = pmts/deposits

r = Apr

m = periods/yr

$\Big\}$ $i = \frac{r}{m}$ = interest rate per period.

Each deposit is like a savings account.

Future Value = $A = R(1+i)^{*}$ $*$ = some power

n payments at the end of each month.

Interest compounded monthly.

No interest 'til it's been in there for a month.

Nov 17-8:35 AM

1st payment draws interest for n-1 months.
At the end, it grows to:

$R(1+i)^{n-1}$

2nd pmt:

$R(1+i)^{n-2}$

⋮

Next to last:

$R(1+i)^{1}$

Last pmt:

R

$$A = P\left(1+\frac{r}{m}\right)^{mt} = P(1+i)^{n}$$

It all adds up to

$$\underbrace{\underset{k=1}{R} + \underset{k=2}{R(1+i)} + \underset{k=3}{R(1+i)^2} + \cdots + \underset{k=n-1}{R(1+i)^{n-2}} + \underset{k=n}{R(1+i)^{n-1}}}_{n \text{ of them}}$$

0 1 2 . . . . n-1

1st term, $k=1$, R
2nd .. $k=2$, $R(1+i)^1$
3rd .. $k=3$, $R(1+i)^2$ $= R(1+i)^{k-1} =$ kth term in the sum.

$$\sum_{k=1}^{n} R(1+i)^{k-1}$$

Geometric Growth Example.

$$\sum_{K=1}^{5} 2K = 2(1)+2(2)+2(3)+2(4)+2(5)$$
$$= 2+4+6+8+10$$
$$= 30$$

$$\boxed{\sum_{K=1}^{4}(3K^2-2K)} = (3(1)^2-2(1))+(3(2)^2-2(2))+(3(3)^2-2(3))+(3(4)^2-2(4))$$

$$=(3-2)+8+21+40 = 1+29+40 = 70$$

1+3+5+7+9+11 in Σ-notation ?

$2K$

$K=1$	1	$=K$, $=K^2$, $2K-1$
$K=2$	3	~~$=K?$~~, $\neq K^2$, $2(2)-1=3$ ✓
$K=3$	5	$2(3)-1=5$ ✓
$K=4$	7	
$K=5$	9	
$K=6$	11	$2(6)-1=11$! Sweet.

$2K+1$

$$\sum_{K=1}^{6}(2K-1)$$

Geometric Sums

$$\sum_{K=1}^{n} a r^{K-1}$$

1st term = a

r = common ratio

Our example: $a = R = 1^{st}$ Term

$r = (1+i)$ = Common Ratio