121 §4.4

1 $\log_2(x) = 3$

$2^{\log_2(x)} = 2^3$

$\boxed{x = 2^3 = 8}$

$\boxed{x \in \{8\}}$

2 $\log_3(x) = 0$

$3^{\log_3(x)} = 3^0$

$x = 1$

$\boxed{x \in \{1\}}$

3 $\log(x+20) = 2$

$10^{\log(x+20)} = 10^2$

$x + 20 = 100$

$\boxed{x = 80}$

$x \in \{80\}$

4 $\log(x^2-15) = 1$

$10^{\log(x^2-15)} = 10^1$

$x^2 - 15 = 10$

$x^2 = 25$

$\boxed{x = \pm 5}$

$\boxed{x \in \{\pm 5\}}$

5 $-2 = \log_x(4)$

$x^{-2} = x^{\log_x(4)}$

$x^{-2} = 4$

$\frac{1}{x^2} = 4$

$1 = 4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \pm \frac{1}{2}$

$\boxed{x \in \{\frac{1}{2}\}}$

6 $\log_x(10) = 3$

$x^{\log_x(10)} = x^3$

$10 = x^3$

$x = \sqrt[3]{10}$

$\boxed{x \in \{\sqrt[3]{10}\}}$

121 §4.4

#5 Eq'ns involve more than one logarithm.

7 $\log_2(x+2)+\log_2(x-2)=5$

$\log_2((x^2-4))=5$

$2^{\log_2(x^2-4)}=2^5=32$

$x^2-4=32$

$x^2=36$

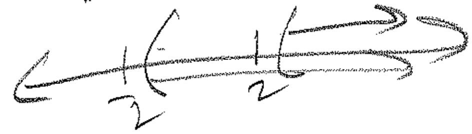
$x=\pm 6$, but $-6\notin \mathcal{D}$

$\boxed{x\in\{6\}}$

DOMAIN

$x+2>0$ and $x-2>0$

$x>-2$ and $x>2$



Need $x>2$

$(2,\infty)=\mathcal{D}$

$\mathcal{D}$: need $\frac{x-3}{2}>0$ and $\frac{x+2}{7}>0$

Need $x>3$

8 $\log\left(\frac{x-3}{2}\right)+\log\left(\frac{x+2}{7}\right)=0$

$\log\left(\frac{x^2-x-6}{14}\right)=0$

$10^{\log\left(\frac{x^2-x-6}{14}\right)}=10^0$

$\frac{x^2-x-6}{14}=1$

$x^2-x-6=14$

$x^2-x-20=0$

$(x-5)(x+4)=0$

$x=-4,5$

$-4\notin\mathcal{D}$

$x\in\{5\}$

121 §4.4

9

$$\log(x+1) - \log(x) = 3$$

$$\log\left(\frac{x+1}{x}\right) = 3$$

$$10^{\log\left(\frac{x+1}{x}\right)} = 10^3$$

$$\frac{x+1}{x} = 1000$$

$$x+1 = 1000x$$

$$-999x + 1 = 0$$

$$-999x = -1$$

$$\boxed{x = \frac{1}{999}}$$

10

$$\log_4(x) - \log_4(x+2) = 2$$

$\mathcal{D}$: $x>0$ and $x+2>0$

$x>0$ and $x>-2$

Need $x>0$

$$\log_4\left(\frac{x}{x+2}\right) = 2$$

$$4^{\log_4\left(\frac{x}{x+2}\right)} = 4^2$$

$$\frac{x}{x+2} = 16$$

$$x = 16x + 32$$

$$-15x = 32$$

$x = -\frac{32}{15}$, but $-\frac{32}{15} \notin \mathcal{D} = \{x \mid x>0\}$

121 §4.4

$\boxed{11}$ $\log_3(x) = \log_3(2) - \log_3(x-2)$ $\mathscr{D}$: Need

$\log_3(x) = \log_3\left(\frac{2}{x-2}\right)$

$x>0$ & $x-2>0$

$x>0$ & $x>2$

$x = \frac{2}{x-2}$ ($\log$ is 1-to-1) $\Longrightarrow$ $x>2$

$x(x-2) = x^2-2x = 2$

$x^2-2x-2=0$

$x^2-2x = 2$

$x^2-2x+1^2 = 2+1$

$(x-1)^2 = 3$

$x-1 = \pm\sqrt{3}$

$x = 1\pm\sqrt{3}$

$1-\sqrt{3} \notin \mathscr{D}$

$x \in \{1+\sqrt{3}\}$

$a=1, b=-2, c=-2$

$b^2-4ac = (-2)^2-4(1)(-2)$

$= 4+8 = 12$

$\Rightarrow \sqrt{12} = 2\sqrt{3}$

$x = \frac{2\pm2\sqrt{3}}{2} = 1\pm\sqrt{3}$

$1-\sqrt{3} \notin \mathscr{D}$ Need $x>2$

$x \in \{1+\sqrt{3}\}$

27

p. 121 § 4.4

12

$\log(4) + \log(x) = \log(5) - \log(x)$

$\log(4x) = \log\left(\frac{5}{x}\right)$ $\qquad \mathcal{D} = \{x \mid x > 0\}$

$10^{\log(4x)} = 10^{\log\left(\frac{5}{x}\right)}$

$4x = \frac{5}{x}$

$4x^2 = 5$

$x^2 = \frac{5}{4}$

$x = \pm\sqrt{\frac{5}{4}} = \pm\frac{\sqrt{5}}{2}$

$x \in \left\{\frac{\sqrt{5}}{2}\right\}$ $\quad \left(-\frac{\sqrt{5}}{2} \notin \mathcal{D}\right)$

13

$\log_3(x) + \log_3(1/x) = 0$ $\qquad \boxed{\mathcal{D} = \{x \mid x > 0\} = \text{Answer!}}$

$\log_3(1) = 0$

$3^{\log_3(1)} = 3^0$

$1 = 1$ Always true if $x \in \mathcal{D}$

#s 41–48 Solve. Round to 4 decimal places.

14

$6^x = 3^{x+1}$

$\boxed{M1}$ $\log_6(6^x) = \log_6(3^{x+1})$ $\qquad a = \log_6(3)$

$x = \log_6(3)\,(x+1)$

$x = a\,(x+1)$ $\qquad x(1-a) = a$

$x = ax + a$ $\qquad x = \frac{a}{1-a} = \frac{\log_6(3)}{1-\log_6(3)}$

$x - ax = a$ $\qquad \approx \boxed{1.5850}$

$= \frac{\frac{\ln(3)}{\ln(6)}}{1-\frac{\ln(3)}{\ln(6)}} \approx 1.584962501$

121 §4.4

Y1 $\boxed{M2}$

$$\log_3(6^x) = \log_3(3^{x+1})$$

$$\log_3(6)\,x = x+1 \qquad a = \log_3(6)$$

$$ax = x+1$$

$$ax - x = 1$$

$$x(a-1) = 1$$

$$x = \frac{1}{a-1} = \frac{1}{\log_3(6)-1} = \frac{1}{\frac{\ln 6}{\ln 3}-1} \approx 1.584962501$$

$$\approx \boxed{1.5850}$$

$$6^x = 3^{x+1}$$

$\boxed{M3}$

$$\ln(6^x) = \ln(3^{x+1})$$

$$\ln(6)\,x = \ln(3)(x+1) \qquad a = \ln 6,\ b = \ln 3$$

$$ax = b(x+1)$$

$$ax = bx + b$$

$$ax - bx = b$$

$$x(a-b) = b$$

$$x = \frac{b}{a-b} = \frac{\ln(3)}{\ln(6)-\ln(3)} \approx 1.584962501$$

$$\approx \boxed{1.5850}$$

This work (3 methods) says

that

$$\frac{\ln 3}{\ln 6 - \ln 1} = \frac{\frac{\ln 3}{\ln 6}}{1-\frac{\ln 3}{\ln 6}} = \frac{1}{\frac{\ln 6}{\ln 3}-1}$$

weird!

121 §4.4

#s 49-58 Same instructions

15

$$e^{-\ln(w)} = 3$$

$$e^{\ln(w^{-1})} = 3$$

$$w^{-1} = 3$$

$$\frac{1}{w} = 3$$

$$\boxed{\frac{1}{3} = w}$$

16

$$4(1.02)^x = 3(1.03)^x$$

$\boxed{M1}$ $(1.02)^x = \frac{3}{4}(1.03)^x$

$$\log_{1.02}\left((1.02)^x\right) = \log_{1.02}\left(\frac{3}{4}(1.03)^x\right)$$

$$x = \log_{1.02}\left(\frac{3}{4}\right) + \log_{1.02}\left((1.03)^x\right)$$

$$x = \log_{1.02}\left(\frac{3}{4}\right) + \log_{1.02}(1.03)\, x$$

$x = a + bx$, where $a = \log_{1.02}(3/4)$

$b = \log_{1.02}(1.03)$

$$x - bx = a$$

$$x(1-b) = a$$

$$x = \frac{a}{1-b} = \frac{\log_{1.02}(3/4)}{1-\log_{1.02}(1.03)}$$

$$= \frac{\frac{\ln(3/4)}{\ln(1.02)}}{1-\frac{\ln(1.03)}{\ln(1.02)}} \approx 29.48717854$$

$$\approx \boxed{29.4872}$$

$\boxed{M2}$ $\ln\left(4(1.02)^x\right) = \ln\left(3(1.03)^x\right)$

$$\ln 4 + \ln(1.02)\, x = \ln 3 + \ln(1.03)\, x$$

$a = \ln 4$, $b = \ln(1.02)$

$c = \ln 3$, $d = \ln(1.03)$

$$a + bx = c + dx$$

$$bx - dx = c - a$$

$$x(b-d) = c - a$$

$$x = \frac{c-a}{b-d}$$

$$= \frac{\ln 3 - \ln 4}{\ln(1.02) - \ln(1.03)}$$

$$\approx 29.48717854$$

$$\approx \boxed{29.4872}$$

121 §4.4

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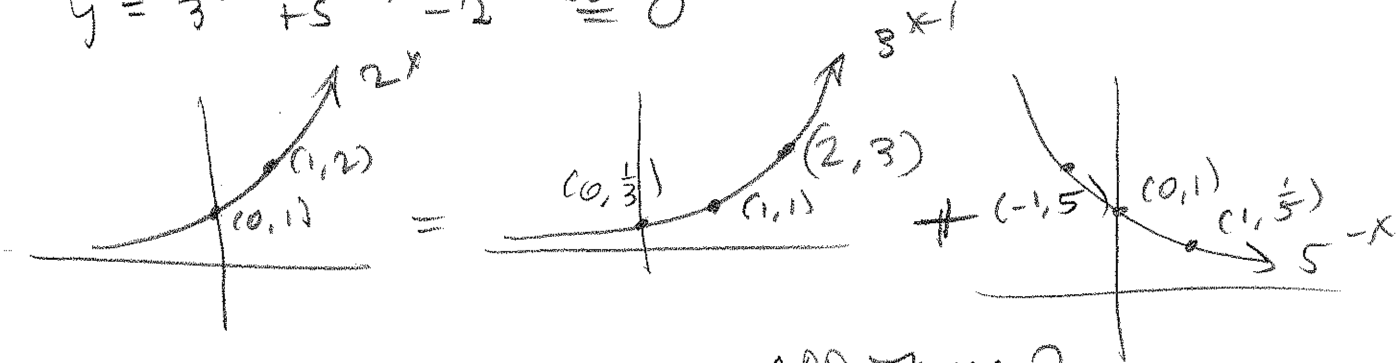
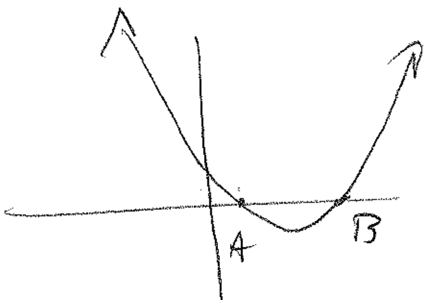
$2^x = 3^{x-1} + 5^{-x}$

$y = 3^{x-1} + 5^{-x} - 2^x$ SET 0



ADD These 2



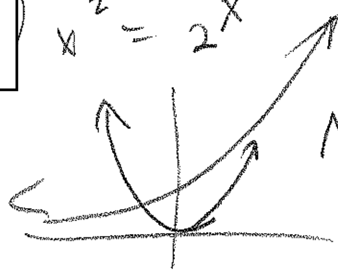
$A \approx (.19426166, 0)$

$B \approx (2.7046378, 0)$

$$\boxed{x \in \{.1943, 2.7046\}}$$

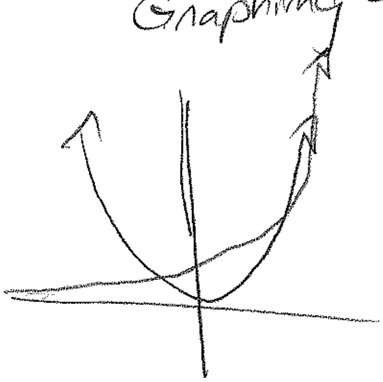
18

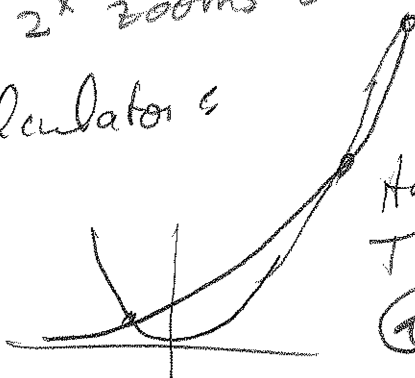
$x^2 = 2^x$



Not sure x^2 will catch 2^x before 2^x zooms off above it.

Graphing calculator:





Hard to draw. They're Both = 4 @ x=2

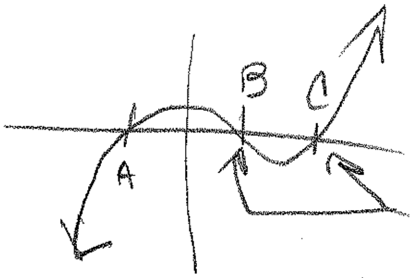
Have to zoom on relevant region to right of 2.

Graphs of 2^x & x^2 together.

Next Page: Graph $2^x - x^2$ & look for x-ints!

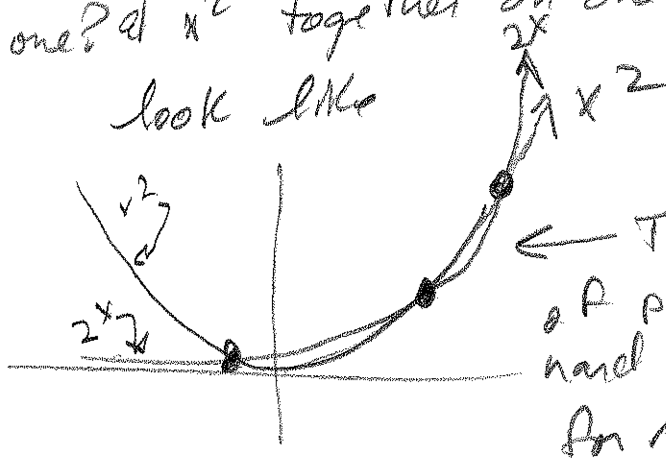
121 §4.4.

63 cnt'd $2^x - x^2 = f(x)$. Find zeros



The 2 crossings, here

say that the graphs of 2^x and x^2 together on one graph must look like



This part of picture is hard to draw for me.

$A \approx (-.7666647, 0)$

$B \approx (2, 0)$ The easy one?

$B = (2, 0)$

$C = (4, 0)$

$2^2 - 2^2 = 0$ ✓

$2^4 - 4^2 = 0$ ✓

$16 - 16 = 0$ ✓

$2^{-.7666647} - (-.7666647)^2 \approx -7.83648 \times 10^{-9}$

$= -\dfrac{7.83648}{1{,}000{,}000{,}000} \approx 0$ ✓

$\boxed{x \in \{-.7667, 2, 4\}}$

121 §4.4 #s

19 $\frac{1}{2}$-lif is 10,000 yrs. What's its (relative) decay rate?

$$Pe^{10000r} = \frac{1}{2}P$$

$$e^{10000r} = \frac{1}{2}$$

$$\ln\left(e^{10000r}\right) = \ln(1/2) = \ln 1 - \ln 2 = 0 - \ln 2$$
$$= -\ln 2$$

$$10000r = -\ln 2$$

$$r = -\frac{\ln 2}{10000} \approx -6.93147181 \times 10^{-5}$$

$$= \boxed{-.0000693147181}$$

20 15% of C-14 has decayed. How old? 15% gone $\longrightarrow$ 85% remains

C-14 has $\frac{1}{2}$-life of 5730 yrs

$$Pe^{5730r} = \frac{1}{2}P$$

$$e^{5730r} = \frac{1}{2}$$

$$5730r = \ln(1/2) = -\ln 2$$

$$\boxed{r = -\frac{\ln 2}{5730}}$$

So 15% gone $\longrightarrow$ $Pe^{rt} = .85P$

FINAL ANSWER

$$e^{rt} = .85$$

$$rt = \ln(.85) \longrightarrow t = \frac{\ln(.85)}{r} \approx 2344.652536$$

$$\approx \boxed{2345 \text{ yrs old}}$$

121 §4.4

21

79.3% of C-14 remains

$Pe^{rt} = .793 P$

use $r = -\frac{\ln 2}{5730}$ from previous.

$e^{rt} = .793$

$rt = \ln(.793)$

$t = \frac{\ln(.793)}{r} = \frac{\ln(.793)}{-\frac{\ln(2)}{5730}}$

≈ 1917.299422

$\approx \boxed{1917 \text{ yrs ago}}$

1951 was when they dated them.

$\therefore$ 1951 − 1917 = $\boxed{34 \text{ AD}}$