201 §4.3

$\log(xy) = \log(x) + \log(y)$

① Log of the product is <u>sum</u> of the logs

② Log of the <u>quotient</u> is the difference of the logs

$\log(x/y) = \log(x) - \log(y)$

③ log of the power is that power times the log

$$\log(a^x) = x\log(a) = \log(a)\,x$$

④ <u>Change-of-base</u> formula says $\log_b x = \dfrac{\log_a x}{\log_a b}$

Simplify

⑤ $e^{\ln(\sqrt{y})} = \boxed{\sqrt{y}}$

⑥ $10^{\log(3x+1)} = \boxed{3x+1}$

⑦ $\log(10^{y+1}) = \boxed{y+1}$

⑧ $\ln(e^{2K}) = \boxed{2K}$

⑨ $7^{\log_7(999)} = \boxed{999}$

⑩ $\log_4(2^{300}) = \log_4(2^{2\cdot 150}) = \log_4((2^2)^{150})$

$= \log_4(4^{150}) = \boxed{150}$

121 §4.3

Write as a single log.

11 $\ln(x^8) - \ln(x^3) = \ln\left(\frac{x^8}{x^3}\right) = \boxed{\ln(x^5)}$

Rewrite as sum/diff of logs

$\log_3(xy) = \log_3(x) + \log_3(y)$

Rewrite as sum/diff of multiples of logs

12 $\log(3\sqrt{x}) = \log(3x^{\frac{1}{2}}) = \log(3) + \log(x^{\frac{1}{2}})$

$= \boxed{\log(3) + \frac{1}{2}\log(x)}$

13 $\log(3\cdot 2^{x-1}) = \log(3) + \log(2^{x-1})$

$= \boxed{\log(3) + (x-1)\log(2)}$

14 $\ln\left(\frac{\sqrt[3]{xy}}{t^{4/3}}\right) = \ln\left(\frac{(xy)^{\frac{1}{3}}}{t^{\frac{4}{3}}}\right) = \ln\left(\frac{x^{\frac{1}{3}}y^{\frac{1}{3}}}{t^{4/3}}\right)$

$= \ln(x^{1/3}) + \ln(y^{1/3}) - \ln(t^{4/3})$

$= \boxed{\frac{1}{3}\ln(x) + \frac{1}{3}\ln(y) - \frac{4}{3}\ln(t)}$

121 §4.3

15 $\ln\left(\frac{6\sqrt{x-1}}{5x^3}\right) = \ln\left(\frac{6(x-1)^{\frac{1}{2}}}{5x^3}\right)$

$= \ln 6 + \ln\left((x-1)^{\frac{1}{2}}\right) - \ln(5) - \ln(x^3)$

$= \boxed{\ln(6) + \frac{1}{2}\ln(x-1) - \ln(5) - 3\ln(x)}$

Re-write as a single log.

16 $\log_2(5) + 3\log_2(x)$

$= \log_2(5) + \log_2(x^3)$

$= \boxed{\log_2(5x^3)}$

17 $3\log_4(x^2) - 4\log_4(x^{-3}) + 2\log_4(x)$

$= \log_4\left((x^2)^3\right) - \log_4\left((x^{-3})^4\right) + \log_4(x^2)$

$= \log_4(x^6) - \log_4(x^{-12}) + \log_4(x^2)$

$= \log_4\left(\frac{x^6 x^2}{x^{-12}}\right) = \log_4(x^8 x^{12}) = \boxed{\log_4(x^{20})}$

121 §4.8

Find each log to 4 decimal places

18 $\log_4(9) = \frac{\ln(9)}{\ln(4)} \approx 1.584962501 \approx \boxed{1.5850}$

RECALL we had a $\frac{\ln(3)}{\ln(2)}$ earlier

Why the same #?

$$\frac{\ln(9)}{\ln(4)} = \frac{\ln(3^2)}{\ln(2^2)} = \frac{2\ln(3)}{2\ln(2)} = \frac{\ln(3)}{\ln(2)} \; !$$

19 $\log_{1/2}(12) = \frac{\ln(12)}{\ln(1/2)} \approx -3.584962501$

$\approx \boxed{-3.5850}$

Solve round to 4 decimal places.

20 $(1.0001)^{365t} = 3.5$

$\ln\left((1.0001)^{365t}\right) = \ln(3.5)$

$\ln(1.0001)(365)t = \ln(3.5)$

$t = \frac{\ln(3.5)}{365\ln(1.0001)} \approx 34.32398919$

$\approx \boxed{34.3240}$

121 §4.3

21

$$(1+r)^3 = 2.3$$
$$\sqrt[3]{(1+r)^3} = \sqrt[3]{2.3}$$
$$1+r = \sqrt[3]{2.3}$$
$$r = \sqrt[3]{2.3} - 1$$
$$\approx 1.320006122$$
$$\approx \boxed{1.3200}$$

$$(1+r)^3 = 2.3$$
$$((1+r)^3)^{\frac{1}{3}} = 2.3^{\frac{1}{3}}$$
$$1+r = (2.3)^{\frac{1}{3}}$$
$$r = (2.3)^{\frac{1}{3}} - 1$$

22

1960 → \$.49

2009 → \$4.59

Find annual growth rate.

Madison said compound annually.

I said compound continuosly.

Book likes Madison more.

Let t = # of years after 1960.

Then $P(1+\frac{r}{m})^{mt} = .49(1+r)^t$ m = 1 Madison

Now $2009 - 1960 = 49 = t$ →

$.49(1+r)^{49} = 4.59$ solve for r

$$(1+r)^{49} = \frac{4.59}{.49}$$
$$\sqrt[49]{(1+r)^{49}} = \sqrt[49]{\frac{4.59}{.49}}$$
$$1+r = \sqrt[49]{\frac{4.59}{.49}}$$
$$r = \sqrt[49]{\frac{4.59}{.49}} - 1$$
$$\approx .046716145$$
$$\approx .0467 \text{ OR } \boxed{4.67\%}$$

121 $\S$4.4

23

Compounded Continuously version

$$Pe^{rt} = .49e^{rt}$$

$$.49e^{49r} = 5.49$$

Should be 4.59, instead of 5.49. That explains my question about the final answer, here.
That will make the final answer come out smaller, as it SHOULD be.

$$e^{49r} = \frac{5.49}{.49}$$

$$\ln\left(e^{49r}\right) = \ln\left(\frac{5.49}{.49}\right)$$

$$49r = \ln\left(\frac{5.49}{.49}\right)$$

$$r = \frac{\ln\left(\frac{5.49}{.49}\right)}{49} \approx .049317988$$

$$\approx .0493$$

OR

$\boxed{4.93\%}$

Note it takes HIGHER rate compounded continuously ?!

Something must be wrong. Continuous compounding grows faster with same r. So why does it need a BIGGER r?

Can somebody help with this?

See above. Scott Corliss spotted the mistake I made, here.