121 §4.2

(1) Inverse of an exponential is a logarithm

(2) Base 10 is common

(3) Base e is natural

(4) $f(x)=\log_a(x)$ is increasing if $a>1$

and decreasing if $a<1$ (Need $a>0$, too)

(5) y-axis is vertical asymptote for $f(x)=\log_a(x)$

(6) The domain of $f(x)=\log_a(x)$ is $(0,\infty)$

(7) The logarithmic family of funcs is all

functions of the form $y=b\log_a(x-h)+k$

(8) The one-to-one property says that

$\log_a(x)=\log_b(y)$ implies that $x=y$.

Solve for the "?"

9 $2^{?}=64=2^6 \Rightarrow \boxed{?=6}$

10 $3^{?}=\frac{1}{81}=\frac{1}{3^4}=3^{-4} \Rightarrow \boxed{?=-4}$

$$\begin{array}{l} 2\lfloor 64 \\ 2\lfloor 32 \\ 2\lfloor 16 \\ 2\lfloor 8 \\ 2\lfloor 4 \\ \quad 2 \end{array} \qquad \begin{array}{l} 3\lfloor 81 \\ 3\lfloor 27 \\ 3\lfloor 9 \\ \quad 3 \end{array}$$

~~#s~~ 17-32 Evaluate

11 $\log(10)=1$

12 $\ln(0)$ $\not\exists$

13 $\ln(e)=1$

If you're not automatically factoring integers into primes in these situations or not doing it *fast*, then this is a prealgebra skill that you need to practice, practice, practice! And you're not alone! No big deal. Just get fast at it.

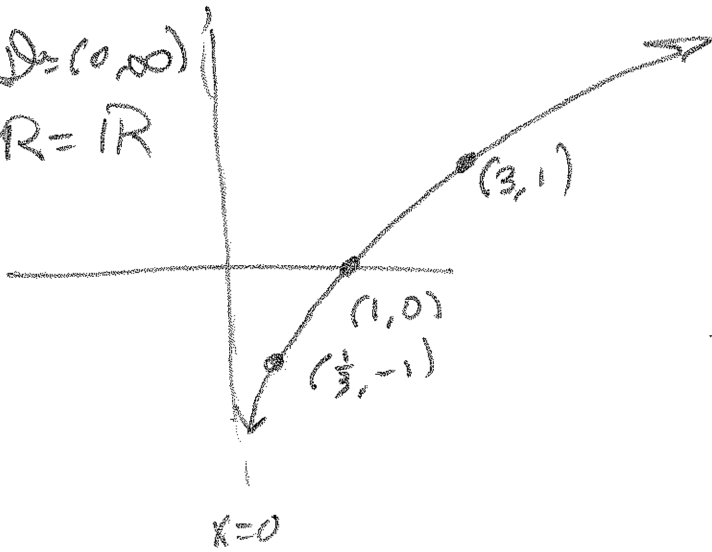
121 §4.2

Sketch.

14) $y = \log_3(x)$

$D = (0, \infty)$

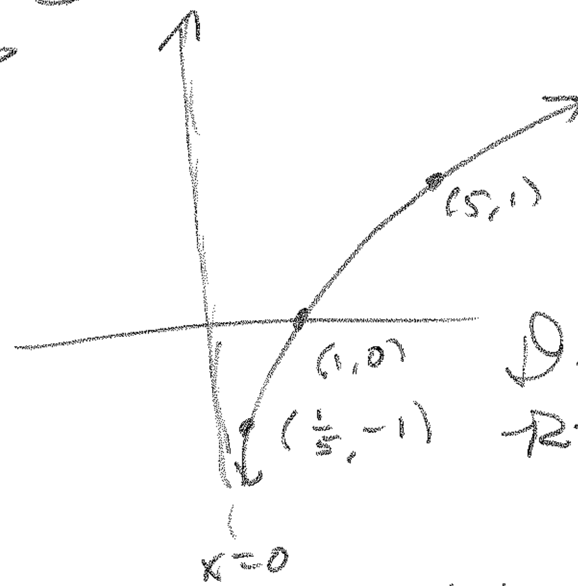
$R = \mathbb{R}$



15) $f(x) = \log_5(x)$

$D = (0, \infty)$

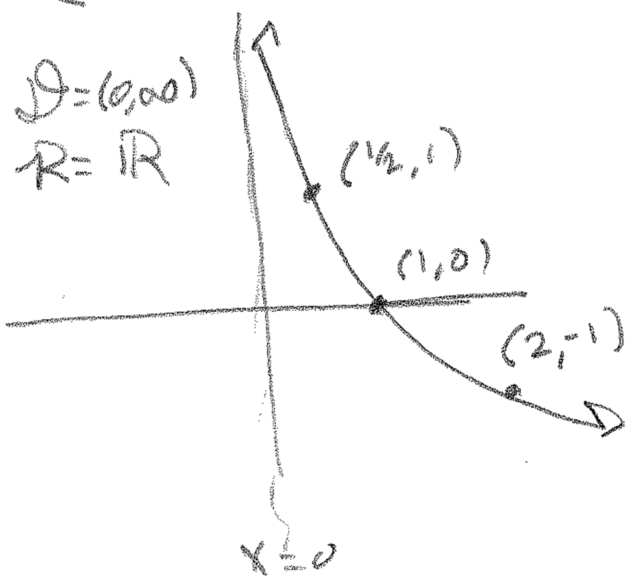
$R = \mathbb{R}$



16) $y = \log_{1/2}(x)$

$D = (0, \infty)$

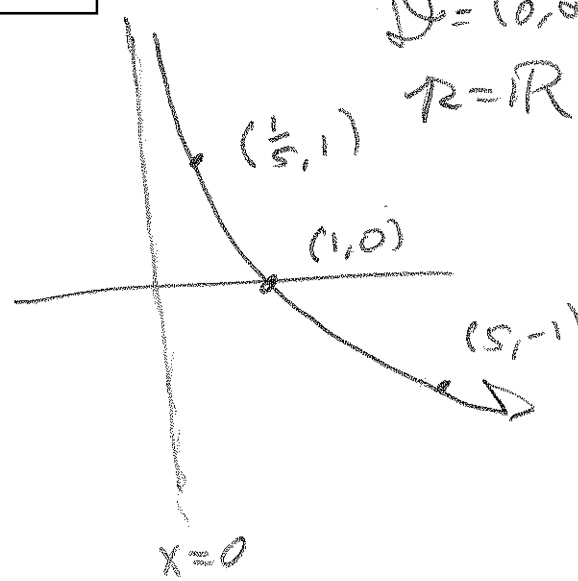
$R = \mathbb{R}$



17) $h(x) = \log_{1/5}(x)$

$D = (0, \infty)$

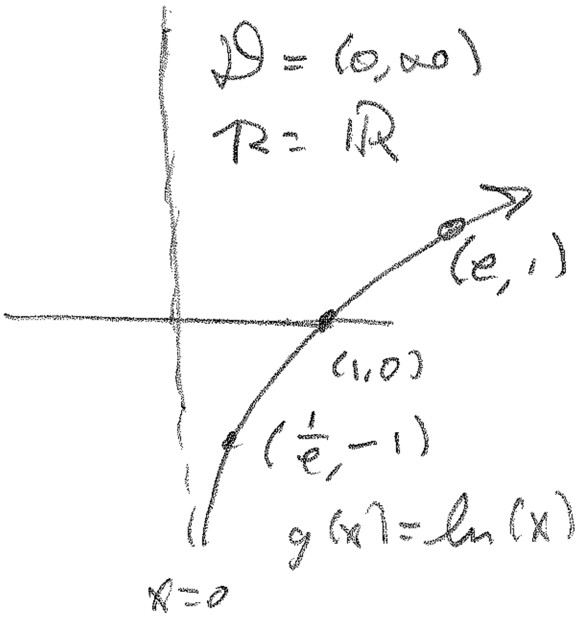
$R = \mathbb{R}$

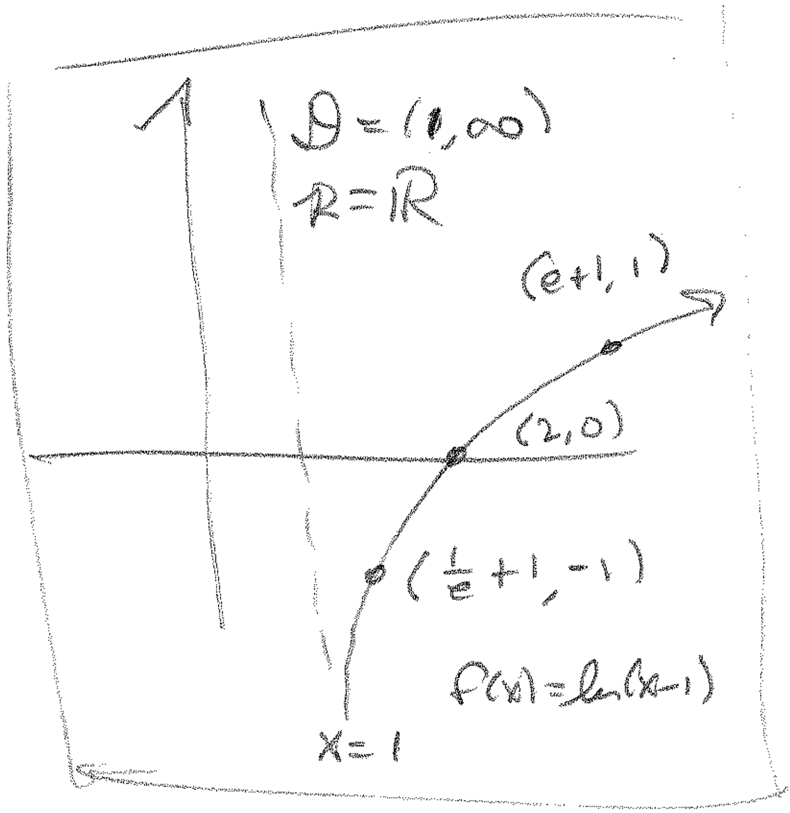


121 §4.2

18

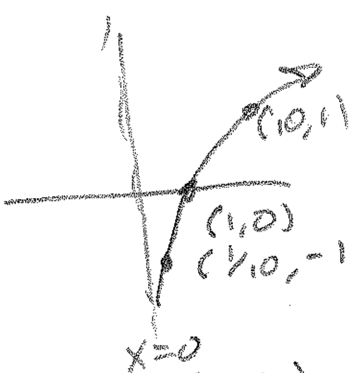
$f(x) = \ln(x-1)$

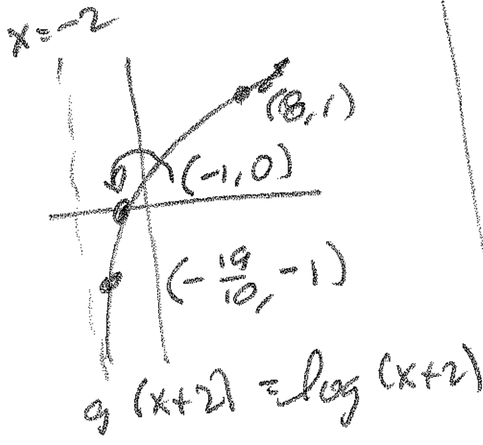


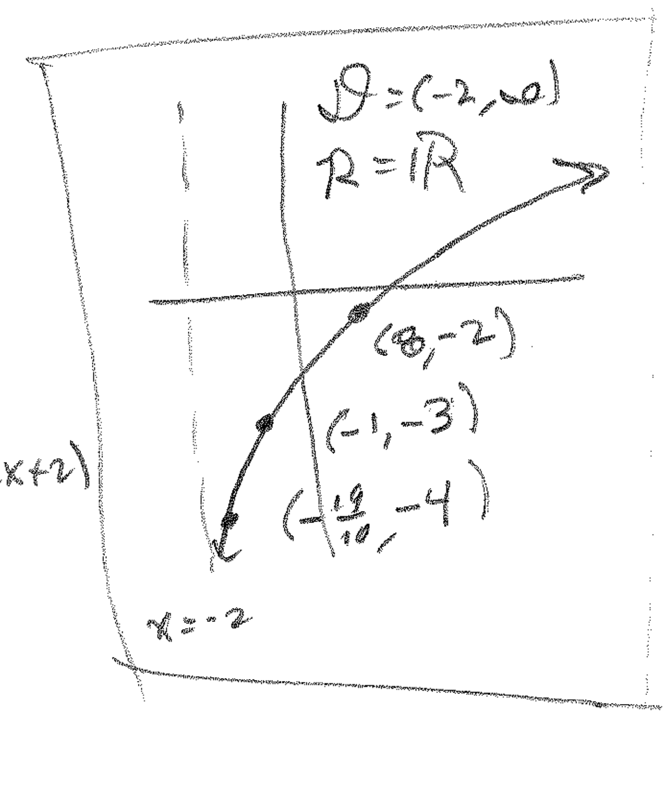


19

$f(x) = \log(x+2) - 3$







$$\frac{1}{10} - 2 = \frac{1}{10} - \frac{20}{10} = -\frac{19}{10}$$

121 §4.2

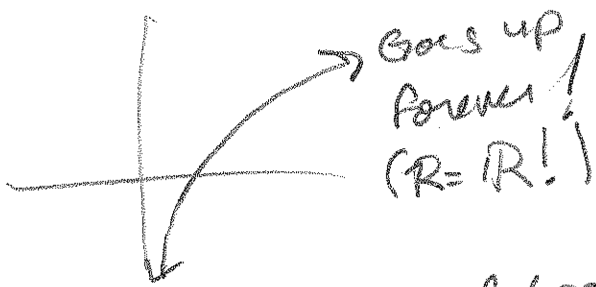
Use graph / table to find the limit

20

$\lim_{x\to\infty} \log_3(x) = \infty$

Easiest way to get these is to know what the picture looks like. It can take a long time of feeding humongous numbers to a logarithm, before you're convinced that it grows without bound.

The other discussion, below, is *way* advanced, compared to 121, but some students will glom on to it in a hurry and some will greatly benefit from seeing this kind of "Anything you can BIG I can Big BIGGER!"



$\log_3(5000) = \frac{\ln(5000)}{\ln(3)} \approx 7.753$

$\log_3(10^6) = 6\log_3(10) = 6\frac{\ln 10}{\ln 3} \approx 12.5754$

ADVANCED APPROACH:
PROVE It grows without bound this way:

Let M be BIG. Want to see if $\log_3(x)$ can be made bigger:

$\log_3(x) > M$?

$3^{\log_3(x)} > 3^M$?

$x > 3^M$

To make $\log_3(x) > 1000000000$

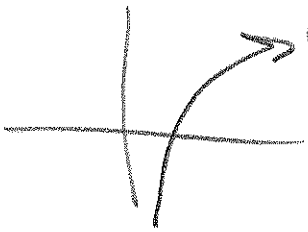
Take $x > 3^{1000000000}$

It takes a while, but it eventually is bigger than 1000000000!

121 §4.2

21

$\lim_{x \to \infty} \log(x) = \infty$



Goes up forever is the visual

The FORMAL way is to say "Give me a big number and I can make log(x) bigger!

Want $\log(x) > BIG$

Want $10^{\log(x)} > 10^{BIG}$

Want $x > 10^{BIG}$

Now prove it by reversing the want steps:

This is a formal proof. Bonus Only.

Let M be given.

Let $x > 10^M$. Then

$\log(x) > \log(10^M) = M \log(10) = M$

This is a Bonus:

"PROVE $\lim_{x \to \infty} \log(x) = \infty$.