121 **3.4 Graphs of (factored) polynomial functions.**

Make a rough sketch of the graph

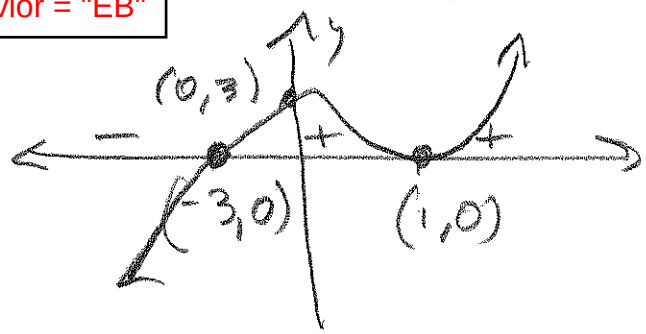
1

$f(x) = (x-1)^2(x+3)$

$f(0) = (-1)^2(3) = 3 \rightsquigarrow (0,3)$ *y*-intercept

EB: $(x-1)^2(x+3) = (x^2)(x) + \text{smaller} = x^3 + \text{smaller}$ EB ↗...↗

End Behavior = "EB"



$x=1, m=2$ Touch

$x=3, m=1$ Cross

"m" stands for "multiplicity"

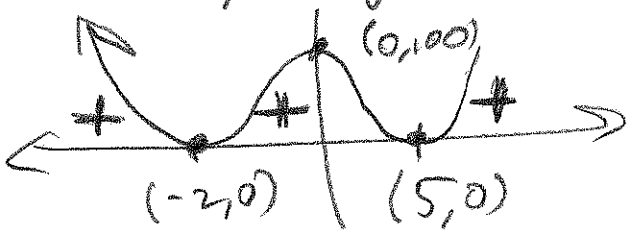
2

$f(x) = (x+2)^2(x-5)^2$

$f(0) = (2)^2(-5)^2 = 4(25) = 100 \rightsquigarrow (0,100)$

EB: $(x)^2(x)^2 = x^4$ ↗...↗

Throw out constants.
Analyze highest power(s)



$x=-2, m=2$

$x=5, m=2$

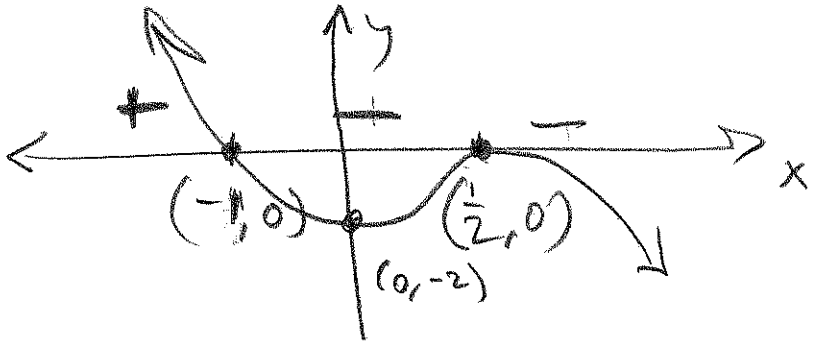
3.4 Graphs of (factored) polynomial functions.

3

$f(x) = -2(2x-1)^2(x+1)^3$

$f(0) = -2(-1)^2(1)^3 = -2 \rightsquigarrow (0,-2)$

E.B. $-2(2x)^2(x)^3 = -2(4x^2)(x^3) = -8x^5$



$x = \frac{1}{2}$ $m=2$ touch

$x=-1$ $m=3$ cross

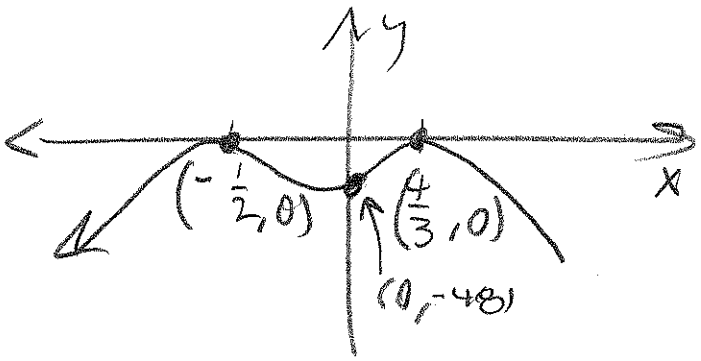
4

$f(x) = -3(3x-4)^2(2x+1)^4$

$f(0) = -3(-4)^2(1)^4 = -3(16) = -48 \rightsquigarrow (0,-48)$

EB: $-3(3x)^2(2x)^4 = -3(9x^2)(16x^4)$

$= -3(9)(16)x^6$



$x = \frac{4}{3}$ touch $m=2$

$x = -\frac{1}{2}$ touch $m=4$

3.4 Graphs of (factored) polynomial functions.

5

$f(x) = -x^3 - x^2 + 5x - 3$ $\pm 1, \pm 3$

$f(-x) = x^3 - x^2 - 5x - 3$

2 or 0 pos.

1 neg.

$-1 \mid -1 \quad -1 \quad 5 \quad -3$

$\quad\quad\quad 1 \quad 0$

$-1 \quad 0$ No

$-3 \mid -1 \quad -1 \quad 5 \quad -3$

$\quad\quad\quad 3 \quad -6 \quad 3$

$-1 \quad 2 \quad -1 \quad 0$

$(x+3)(-x^2+2x-1)$

$x = -3, m = 1$ cross

$-x^2 + 2x - 1 \overset{set}{=} 0$

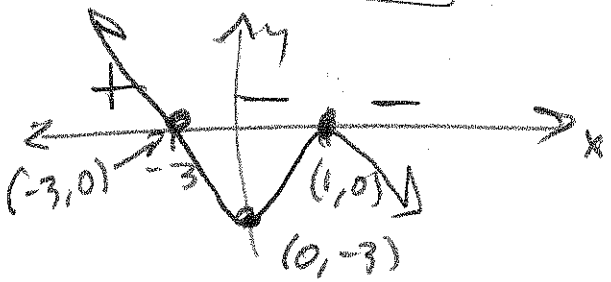
$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1, m = 2$ touch

$f(0) = -3 \rightarrow (0, -3)$

EBS $-x^3$ ↖ ... ↘



3.4 Graphs of (factored) polynomial functions.

6

$f(x) = x^3 - 10x^2 - 600x$

$= x(x^2 - 10x - 600)$

$b^2 - 4ac = (-10)^2 - 4(1)(-600)$

$= 100 + 2400 = 2500$

$\rightsquigarrow \sqrt{2500} = \sqrt{25 \cdot 100}$

$= \sqrt{25}\sqrt{100} = 5 \cdot 10 = 50$

It factors over the rationals

Sum	Factor
$-10 = -20 + 10$	-200
$= -40 + 30$	-1200
$= -30 + 20$	-600 ✓

$x^2 - 30x + 20x - 600$

$= x(x-30) + 20(x-30)$

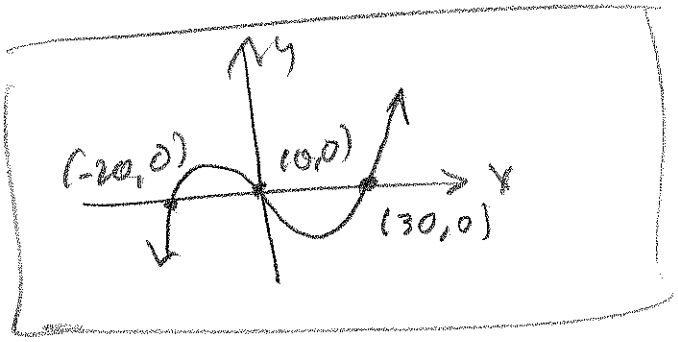
$= (x-30)(x+20)$ SET $= 0$

$\Rightarrow$ $x = -20, 30$

$x = 0$ from above

$P(x) = x(x+20)(x-30)$

EB: x^3 ↗



3.4 Graphs of (factored) polynomial functions.

Solve each inequality

7

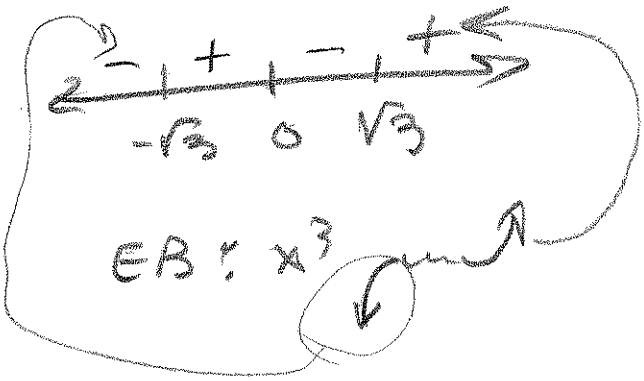
$x^3 - 3x > 0$

$x(x^2 - 3) > 0$

$x(x - \sqrt{3})(x + \sqrt{3}) > 0$

Want ">0", i.e. "+"

$x \in (-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$



These inequalities show up a LOT on tests. Easy to test for general skills with them. Easy to give you one polynomial in factored form and ask lots of questions that go quickly if you have a clue and are impossible if you don't.

8

$2x^2 - x^4 \leq 0$

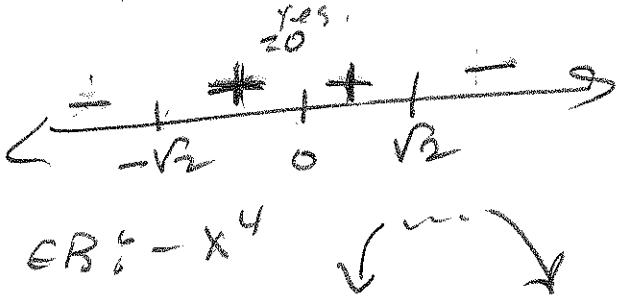
$x^2(2 - x^2) \leq 0$

$x^2(\sqrt{2} - x)(\sqrt{2} + x) \leq 0$

Want ≤ 0

$x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

(If it were "< 0", then $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.)



3.4 Graphs of (factored) polynomial functions.

9

$x^3+4x^2-x-4>0$

$x^2(x+4)-1(x+4)>0$

$(x+4)(x^2-1)>0$

$(x+4)(x-1)(x+1)>0$

EB x^3

Want >0 $\Rightarrow$

$x\in(-4,-1)\cup(1,\infty)$

10

$x^3-4x^2-20x+48\geq 0$

2	1	-4	-20	48
		2	-4	-48
	1	-2	-24	0

$x^2-2x-24=0$

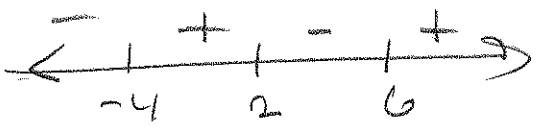
$x^2-2x+1^2=24+1$

$(x-1)^2=25$

$x-1=\pm 5$

$x=1\pm 5$ $\rightarrow 6$, -4

$x=-4,2,6$



EB: x^3

Want ≥ 0

$x\in[-4,2]\cup[6,\infty)$

3.4 Graphs of (factored) polynomial functions.

11

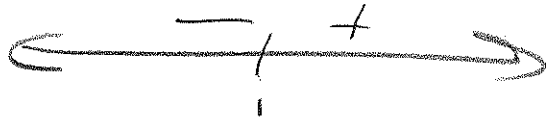
$x^3 - x^2 + x - 1 < 0$

$x^2(x-1) + 1(x-1) < 0$

$(x-1)(x^2+1) < 0$

x^2+1 has no real zeros,

so no role in the inequality. Just as it has no role in the graph.



Want < 0 :

$x \in (-\infty, 1)$

EB: x^3

12

$x^4 - 19x^2 + 90 \leq 0$

−3	1	−19	0	90	
		−3	66		
	1	−22			

oops! Missed placeholder for x^3:

−3	1	0	−19	0	90
		−3	9	30	−90
+3	1	−3	−10	30	
		3	0	−30	
	1	0	−10	0	

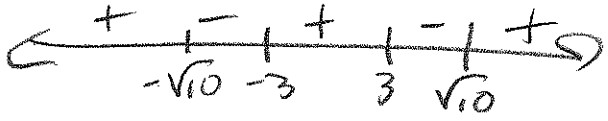
Want ≤ 0

$[-\sqrt{10}, -3] \cup [3, \sqrt{10}]$

$x^2 - 10 = 0$

$x^2 = 10$

$x = \pm\sqrt{10}$



EB: x^4