121 §3.3

$i^2 = -1$

1 $(x-3i)(x+3i) = x^2 - (3i)^2 = x^2 - 3^2 i^2$

$(a-b)(a+b) = a^2 - b^2$

$= \boxed{x^2+9}$

2 $(x-(1+\sqrt{2}))(x-(1-\sqrt{2}))$

$= x^2 - (1-\sqrt{2})x - (1+\sqrt{2})x + (1+\sqrt{2})(1-\sqrt{2})$

$= x^2 - x + \sqrt{2}x - x - \sqrt{2}x + 1^2 - (\sqrt{2})^2$

$= x^2 - 2x + 1 - 2 = \boxed{x^2 - 2x - 1}$

3 $(x-(3+2i))(x-(3-2i))$

M1 $= x^2 - (3-2i)x - (3+2i)x + (3+2i)(3-2i)$

$= x^2 - 3x + 2ix - 3x - 2ix + 3^2 - (2i)^2$

$= x^2 - 6x + 9 - 4i^2$

$= x^2 - 6x + 9 + 4$

$= \boxed{x^2 - 6x + 13}$

M2 $(x-3-2i)(x-3+2i)$

$= x^2 - 3x + 2ix - 3x + 9 - 6i - 2ix + 6i - 4i^2$

$= x^2 - 6x + 9 + 4$

$= \boxed{x^2 - 6x + 13}$

(121) $\S 3.3$

Find polynomial eq'ns with real coefficients that has the given roots

4 $-3, 5$: $\boxed{(x+3)(x-5)=0}$

5 $0, i\sqrt{3}$: $\boxed{x(x-i\sqrt{3})(x-(-i\sqrt{3}))=0}$

CPT

"CPT" stands for Conjugate Pairs Theorem.

6 $3, 1-i$: $\boxed{(x-3)(x-(1-i))(x-(1+i))=0}$

7 $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$: $\boxed{(x-\frac{1}{2})(x-\frac{1}{3})(x-\frac{1}{4})=0}$

201 §3.3.

Descartes's rule of signs only.

8 $f(x) = x^3+5x^2+7x+1 = 0$

0 sign changes $\boxed{\text{0 positive roots}}$

$f(-x) = \underbrace{-x^3+5x^2}_{1}\underbrace{-7x}_{2}\underbrace{+1}_{3}$

$\boxed{\text{3 or 1 negative roots}}$

9 $f(y) = y^4+5y^2+7$

$\boxed{\text{0 positive}}$

$f(-y) = y^4+5y^2+7$

$\boxed{\text{0 negative}}$

(4 nonreal.)

10 $x^5+x^3+5x=0$

0 positive

This one seems like a contradiction, but $x = 0$ is neither positive nor negative!

$f(-x) = -x^5-x^3-5x$

$\boxed{\text{0 negative}}$

(4 nonreal!!)

$x=0$ is the only real root

For fun:

$u=y^2$

$u^2+5u+7=0$

$u^2+5u = -7$

$u^2+5u+\left(\frac{5}{2}\right)^2 = -7+\frac{25}{4}$

$\left(u+\frac{5}{2}\right)^2 = -\frac{3}{4}$

$u+\frac{5}{2} = \pm\sqrt{-\frac{3}{4}} = \pm\frac{i\sqrt{3}}{2}$

$u = \frac{-5\pm i\sqrt{3}}{2} = y^2$

To find y, we'd have to have some notion of

$\pm\sqrt{\frac{-5\pm i\sqrt{3}}{2}}$, which we don't, because we're not taking Complex Analysis.

121 §3.3.

Right here is where there was going to be a theorem on bounds on real roots question. We'll spend minimal time on it. Not a time-saver, typically, on a time-controlled test.
All I want to say about it is
*If you're checking a positive number out and the bottom row of the synthetic division is all positive numbers, you don't have to look for any bigger, positive zeros (to the right); and, if you're checking a negative number out and the bottom row of the synthetic division is alternating signs, you don't have to look for anything **more** negative (nothing to the left).*

Use thms on roots to find all real & imaginary roots

11 $x^3-4x^2-7x+10=0$

p's: 10

q's: 1

$\frac{p}{q}$'s: $\pm1, \pm2, \pm5, \pm10$

Descartes's: 2 or 0 pos.

$f(-x)=-x^3-4x^2+7x+10$

one neg.

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -7 & 10 \\ & & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$x^2-3x-10=0$

$(x-5)(x+2)=0$ *

$x=-2,\ x=5$

$\boxed{\text{zeros: } x=-2, 1, 5 \quad \text{each multiplicity} = 1}$

* you can always clobber the quadratic w/ formula or completing the square.

$\therefore \boxed{f(x)=(x-1)(x+2)(x-5)}$
see sketch @ end

201 §3.3

12 $f(x) = x^4 + 2x^3 - 7x^2 + 2x - 8 \overset{\text{set}}{=} 0$

p's: 8 $\quad \frac{p}{q}$'s: $\pm 1, \pm 2, \pm 4, \pm 8$

q's: 1

Descarte's r.

3 or 1 pos.

$f(-x) = \underbrace{x^4 - 2x^3}_{1} - \underbrace{7x^2 - 2x}_{2} \underbrace{- 8}_{3}$

3 or 1 neg.

$$\begin{array}{r|rrrrr} 2 & 1 & 2 & -7 & 2 & -8 \\ & & 2 & 8 & 2 & 8 \\ \hline -4 & 1 & 4 & 1 & 4 & 0 \\ & & -4 & 0 & -4 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1} = \pm i$

$\boxed{\text{zeros: } 2, -4, \pm i \quad m = 1\ \forall.}$

$\boxed{f(x) = (x-2)(x+4)(x-i)(x+i)}$ see sketch @ end

13 $6x^3 + 25x^2 - 24x + 5 = 0$

p's: 5 $\quad \frac{p}{q}$'s: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$

q's: 6 $\quad \pm 5, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{6}$

$\boxed{\text{zeros: } \frac{1}{3}, \frac{1}{2}, -5 \quad m = 1\ \forall.}$

$f(x) = 6(x-5)(x-\frac{1}{3})(x-\frac{1}{2})$

$\boxed{= (x+5)(3x-1)(2x-1)}$ for style pts.

$$\begin{array}{r|rrrr} -5 & 6 & 25 & -24 & 5 \\ & & -30 & 25 & -5 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

$6x^2 - 5x + 1 = 0$

$a = 6, b = -5, c = 1$

$b^2 - 4ac = (-5)^2 - 4(6)(1)$

$= 25 - 24 = 1 \rightsquigarrow \sqrt{1} = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{5 \pm 1}{2(6)} = \frac{5 \pm 1}{12}$ $\quad \frac{6}{12} = \frac{1}{2}$, $\quad \frac{4}{12} = \frac{1}{3}$

'21 § 3.3

14 $x^4+2x^3-3x^2-4x+4=f(x) \overset{set}{=} 0$

p's: 4 $\quad \frac{p}{q}$'s: ±1, ±2, ±4

q's: 1

Descartes: 2 or 0 pos

$f(-x)=x^4-2x^3-3x^2+4x+4$

2 or 0 neg.

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -3 & -4 & 4 \\ & & 1 & 3 & 0 & -4 \\ \hline 1 & 1 & 3 & 0 & -4 & 0 \\ & & 1 & 4 & 4 & \\ \hline & 1 & 4 & 4 & 0 & \end{array}$$

$(x-1)(x^3+3x^2-4)$

$(x-1)^2(x^2+4x+4)$

$x^2+4x+4=0$

$(x+2)^2=0$

$x+2=\pm 0$

$x=-2,\ m=2$

$\boxed{(x-1)^2(x+2)^2}$

see sketch @ end.

$\boxed{\text{zeros: } x=-1,\ x=2 \quad m=2\ \forall.}$

121 §3.3

15 $x^5+3x^3+2x = f(x) \overset{\text{set}}{=} 0$

Takes some doing!

$x(x^4+3x^2+2) = 0$

$\boxed{x=0}$ OR $x^4+3x^2+2=0$

Our work will show that $\pm1, \pm2$ from Rational Zeros Theorem is no help.

What, then? RECOGNIZE

x^4+3x^2+2 is "Quadratic in form."

Let $u=x^2$, then

$u^2+3u+2=0$

$(u+2)(u+1)=0 \Rightarrow$

$u=-2$ OR $u=-1$

$x^2=-2$

$x=\pm\sqrt{-2} = \boxed{\pm i\sqrt{2}}$

$x^2=-1$

$x=\pm\sqrt{-1}$

$=\boxed{\pm i}$

Zeros:

$x = 0, \pm i\sqrt{2}, \pm i$

$f(x)$ factors as $x\cdot(x-i)(x+i)(x-i\sqrt{2})(x+i\sqrt{2})$

See Sketches

Notice that the nonreal zeros have no expression in the graph. The graph is real. The nonreal zeros? Well, they ain't!

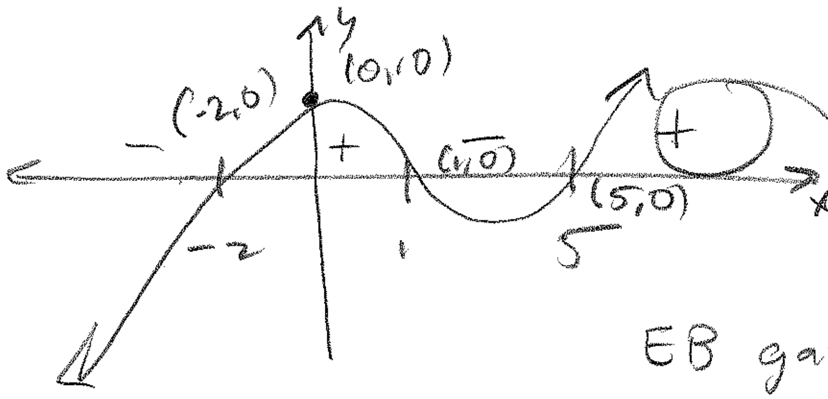
121 §3.3 Sketches

Sketch for #11

$f(x) = (x-1)(x+2)(x-5) = x^3-4x^2-7x+10$

$f(0) = 10 \rightsquigarrow (0,10)$

E.B.: x^3 $\swarrow \cdots \nearrow$



3 rough sketch

EB gave me. The rest from alternating signs

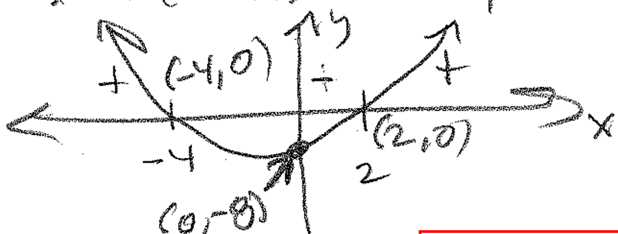
Sketch for #12

$f(x) = x^4+2x^3-7x^2+2x-8 = (x-2)(x+4)(x-i)(x+i)$

$x = \pm i$ has no expression in the graph

$f(0) = -8 \rightsquigarrow (0,-8)$

x^4 $\nwarrow \cdots \nearrow$

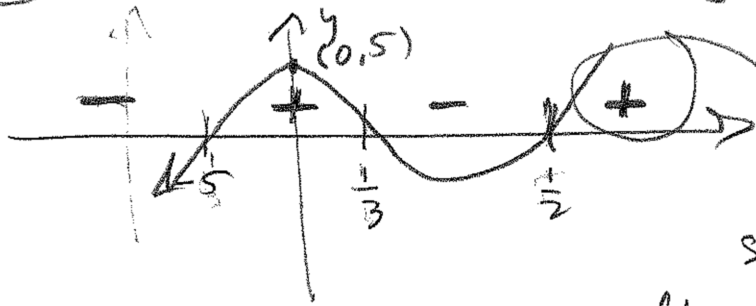


Notice that ONLY THE REAL ZEROS CORRESPOND TO X-INTERCEPTS IN THIS *REAL* GRAPH. NON-REAL ZEROS ARE OF THEORETICAL INTEREST IN ALGEBRA AND YOU'LL SEE THEM AGAIN IN DIFFERENTIAL EQUATIONS AND UPPER DIVISION APPLIED MATH.

121 §3.3 IV Ending

Sketch for #13

$f(x) = 6x^3 + 25x^2 - 24x + 5 = 6(x-5)(x-\frac{1}{3})(x-\frac{1}{2})$



$f(0) = 5 \rightsquigarrow (0,5)$

E.B. $6x^3$

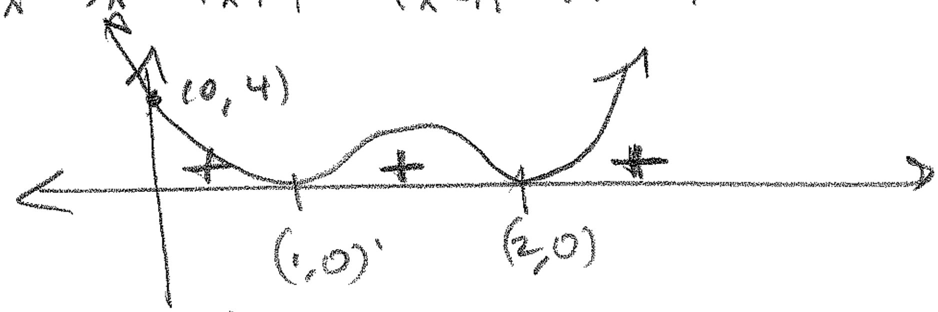
signs alternate because all multiplicities are 1 and 1 is odd.

Sketch for #14

$f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4 = (x-1)^2(x+2)^2$

$f(0) = 4 \rightsquigarrow (0,4)$

x^4: ↖ … ↗



Never crosses x-axis,

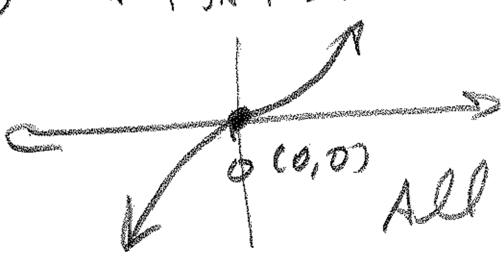
$x=1$, $m=2$ Touch (not cross)

$x=2$, $m=2$ Touch

Sketch for #15

$x^5 + 3x^3 + 2x = x(x-i)(x+i)(x-i\sqrt{2})(x+i\sqrt{2})$

EB: x^5 ↙ … ↗



All we know is it crosses x-axis @ $x=0$ & its end behavior