121 §3.2

Find Quotient & Remainder
ORDINARY DIVISION

1

$$s^2-5 \overline{)\, s^4+0s^3-3s^2+6}$$

$$-(s^4 \qquad -5s^2)$$

$$2s^2+6$$

$$-(2s^2-10)$$

$$16$$

s^2+2 = quotient

16 = remainder

Two ways to interpret this:

$$\frac{s^4-3s^2+6}{s^2-2} = s^2+2+\frac{16}{s^2-2} \quad \text{or}$$

$$s^4-3s^2+6 = (s^2-2)(s^2+2)+16$$

2

~~4~~ 5 11 − 22 SYNTHETIC DIVISION for quotient & remainder

x^2+4x+1, $x-2$

$$x^2+4x+1 = (x+6)(x-2)+13$$

-2 | 1 4 1

2 12

1 6 13

quotient: ~~x~~ $x+6$

remainder: 13

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3 $-x^3+x^2-4x+9$, $x+3$

-3 | -1 1 -4 9
$\quad\quad\ \ 3$ -6 6
$\quad\quad -1$ 2 -2 3

x^2-2x+2 = quotient
3 = remainder

4 $4x^3-5x+2$, $x-\frac{1}{2}$

$\frac{1}{2}$ | 4 0 -5 2
$\quad\quad\ \ 2$ 1 -2
$\quad\quad 4$ 2 -4 0

quotient: $4x^2+2x-4$
remainder: 0

5 $2a^3-3a^2+4a+3$, $a+\frac{1}{2}$

$-\frac{1}{2}$ | 2 -3 4 3
$\quad\quad\ \ -1$ 2 -3
$\quad\quad 2$ -4 6 0

quotient: $2x^2-4x+6$
remainder: 0

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$f(x) = x^5 - 1$, $g(x) = x^3 + 4x^2 + 8$, $h(x) = 2x^4 + x^3 - x^2 + 3x + 3$

Find function vals by synthetic Division

6 $f(1)$

$1 \mid 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1$

$\quad\quad\; 1 \quad 1 \quad 1 \quad 1 \quad 1$

$\quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad | \quad 0 = f(1)$

7 $g(-\frac{1}{2})$

$64 - 9 = 55$

$-\frac{1}{2} \mid 1 \quad -4 \quad 0 \quad 8$

$\quad\quad\; -\frac{1}{2} \quad \frac{9}{4} \quad -\frac{9}{8}$

$\quad 1 \quad -\frac{9}{2} \quad \frac{9}{4} \quad | \quad \frac{55}{8} = g(-\frac{1}{2})$

Determine if ~~it is~~ the binomial is a factor. If it is, then factor completely.

8 $x+3$, $x^3 + 4x^2 + x - 6$

$-3 \mid 1 \quad 4 \quad 1 \quad -6$

$\quad\quad\; -3 \quad -3 \quad 6$

$\quad 1 \quad 1 \quad -2 \quad 0$ ← Yes!

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = 1, -2$

$\Rightarrow (x+3)(x-1)(x+2)$

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9

$x-4,\quad x^3+4x^2-17x-60$

$$\begin{array}{r|rrrr} 4 & 1 & 4 & -17 & -60 \\ & & 4 & 32 & 60 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$ Yes!

$x^2+8x+15=0$

$(x+3)(x+5)=0$

$x\in\{-5,-3\}$

$(x-4)(x+5)(x+3)$

Determine if # given is a zero of the given polynomial.

10

3, $P(x)=2x^3-5x^2-4x+3$

$$\begin{array}{r|rrrr} 3 & 2 & -5 & -4 & 3 \\ & & 6 & 3 & -3 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$ Yes

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$-2,\ g(d)=d^3+2d^2+3d+1$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 3 & 1 \\ & & -2 & 0 & -6 \\ \hline & 1 & 0 & 3 & -5 \end{array}$$ No

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Find all possible rational zeros

12 $f(x) = x^3 - 9x^2 + 26x - 24$

$p: 24$ $\quad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$q: 1$

13 $h(x) = x^3 - x^2 - 7x + 15$

$p: 15$ $\quad \pm 1, \pm 3, \pm 5, \pm 15$

$q: 1$

#s 55–78 Find all real and nonreal zeros

14 $P(x) = x^3 - 9x^2 + 26x - 24$

+ $p: 24$ $\quad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$q: 1$

1 | 1 −9 26 −24
1 −8 18
1 −8 18 NO

−1 | 1 −9 26 −24
−1 10 −36
1 −10 36 NO

2 | 1 −9 26 −24
2 −14 24
1 −7 12 0 Yes!

$x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x = 3, 4$

$x \in \{2, 3, 4\}$

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15 $h(x) = x^3 - x^2 - 7x + 15$

$p: 15 \quad \pm 1, \pm 3, \pm 5, \pm 15$

$q: 1$

1| 1 −1 −7 15
1 0 −7
1 0 −7 NO

−1| 1 −1 −7 15
−1 2 5
1 −2 −5 NO

3| 1 −1 −7 15
3 6 −3
1 2 −1 NO

−3| 1 −1 −7 15
−3 12 −15
1 −4 5 0 Yes!

$x^2 - 4x + 5 = 0$

$a = 1, b = -4, c = 5$

$b^2 - 4ac = (-4)^2 - 4(1)(5)$

$= 16 - 20$

$= -4$

$\sqrt{-4} = 2i$

$x = \frac{4 \pm 2i}{2} = 2 \pm i$

$x \in \{-3, 2 \pm i\}$

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16 $m(x) = x^3 + 4x^2 + 4x + 3$

$p: 3 \quad \pm 1, \pm 3$

$q: 1$

1 | 1 4 4 3
1 5 9
1 5 9 No

-1 | 1 4 4 3
-1 -3 -1
1 3 1 No

3 | 1 4 4 3
3 21 75
1 7 25 No

-3 | 1 4 4 3
-3 -3 -3
1 1 1 0 Yes

$x = \frac{-1 \pm i\sqrt{3}}{2(1)} = \frac{-1 \pm i\sqrt{3}}{2}$

$x \in \{-3, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\}$

$x^2 + x + 1 = 0$

$a=1, b=1, c=1$

$b^2 - 4ac = 1^2 - 4(1)(1)$

$= 1 - 4$

$= -3$

$\sqrt{-3} = i\sqrt{3}$

17 $M(t) = 18t^3 - 21t^2 + 10t - 2$

$p: 2$

$q: 18$

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{1}{9}, \pm \frac{1}{18}$

$\pm 2, \cancel{\pm \frac{2}{2}}, \pm \frac{2}{3}, \pm \frac{2}{6}, \cancel{\pm \frac{2}{9}}$

$\frac{1}{2}$ | 18 -21 10 -2
9 -6 2
18 -12 4 0 Yes!

$18x^2 - 12x + 4 = 0$

$9x^2 - 6x + 2 = 0$

$a=9, b=-6, c=2$

$b^2 - 4ac = (-6)^2 - 4(9)(2)$

$= 36 - 72 = -36$

$\sqrt{-36} = 6i$

$x = \frac{6 \pm 6i}{2(18)} = \frac{6(1 \pm i)}{36}$

$= \frac{1 \pm i}{6}$

$x \in \{\frac{1}{2}, \frac{1 \pm i}{6}\}$

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18 $V(x) = x^4 + 2x^3 - x^2 - 4x - 2$

$p: 2$
$q: 1$
$\pm 1, \pm 2$

-1 | 1 2 −1 −4 −2
−1 −1 2 2
1 1 −2 −2 0 Yes

-1 | 1 1 −2 −2 0 Yes
−1 0 2
1 0 −2 0 Yes

$x = -1, m = 2$

$x^2 - 2 = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$

$x \in \{-1, \pm\sqrt{2}\}$

19 $f(x) = (x^2 - 4x + 1)(x^3 - 9x^2 + 23x - 15)$
#1 #2

#1 $x^2 - 4x + 1 = 0$
$x^2 - 4x + 2^2 = -1 + 4$
$(x-2)^2 = 3$
$x - 2 = \pm\sqrt{3}$
$x = 2 \pm \sqrt{3}$

#2 $p: 15$ $\pm 1, \pm 3, \pm 5, \pm 15$
$q: 1$

$x = 3$

3 | 1 −9 23 −15
3 −18 15
1 −6 5 0

$x^2 - 6x + 5 = 0$
$(x-5)(x-1) = 0$
$x = 1, 5$

$x \in \{1, 3, 5, 2 \pm \sqrt{3}\}$

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use division to write the rational expression in the form: quotient + $\frac{\text{remainder}}{\text{division}}$

20 $\frac{2x+1}{x-2} = f(x)$

$2 \mid 2 \quad 1$
$\quad\quad 4$
$2 \quad 5$
$c \quad r$

$\Rightarrow \boxed{f(x) = 2 + \frac{5}{x-2}}$

21 $\frac{a^2-3a+5}{a-3} = f(a)$

$3 \mid 1 \quad -3 \quad 5$
$\quad\quad\quad 3 \quad 0$
$1 \quad 0 \quad 5$
$a' \quad c \quad r$

$\Rightarrow f(a) = \boxed{a + \frac{5}{a-3}}$

22 $f(c) = \frac{c^2-3c-4}{c^2-4}$

$$\begin{array}{r} 1 \quad r-3c \\ c^2-4 \overline{)\, c^2-3c-4} \\ -(c^2 \quad -4) \\ \hline -3c \end{array}$$

$\Rightarrow \boxed{f(c) = 1 - \frac{3c}{c^2-4}}$