121 §3.1 #s 1-9, 11-19 ODDS, 75-90

#s 9-19 Show vertex & intercepts

Complete the square to write in the form

$a(x-h)^2 + k$

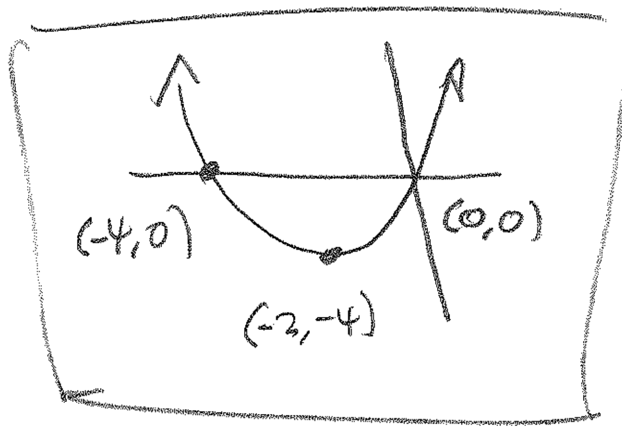
Check vertex with $(h,k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

State intervals of increase & decrease

(9) $f(x) = x^2+4x$

$= x^2+4x+2^2-2^2$

$= \boxed{(x+2)^2-4}$ $\overset{SET}{=} 0$



$(x+2)^2 = 4$

$x+2 = \pm 2$

$x = -2 \pm 2$ → $0 \rightsquigarrow (0,0)$; → $-4 \rightsquigarrow (-4,0)$

$f(0) = 0 \rightsquigarrow (0,0)$

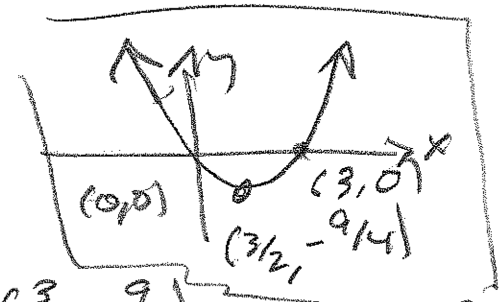
$\boxed{\text{Check:}}$ $-\frac{b}{2a} = -\frac{4}{2(1)} = -2 = -\frac{b}{2a} = h$ ✓

$f\left(-\frac{b}{2a}\right) = (-2)^2+4(-2) = 4-8 = -4 = k$ ✓

$\boxed{\begin{array}{l}\text{Inc: } (-2,\infty) \\ \text{Dec: } (-\infty,-2)\end{array}}$

121 §3.1 #s 11–19 ODDS, 75–90

(11) $y = x^2 - 3x$



$= x^2 - 3x + \left(\frac{3}{2}\right)^2 - \frac{9}{4}$

$= \boxed{\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}}$ $\qquad (h,k) = \left(\frac{3}{2}, -\frac{9}{4}\right)$

SET = 0 $\Longrightarrow$

$\left(x - \frac{3}{2}\right)^2 = \frac{9}{4}$ $\qquad$ Inc: $\left(\frac{3}{2}, \infty\right)$

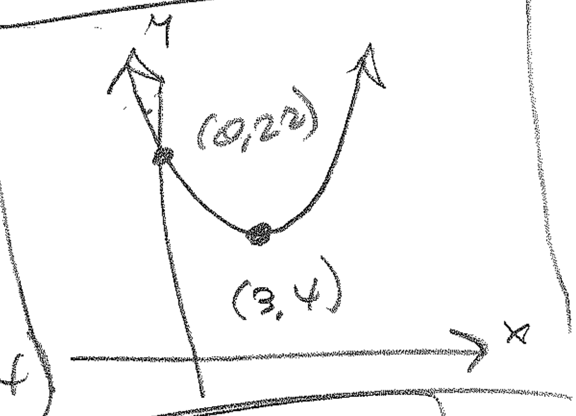
$x - \frac{3}{2} = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$ $\qquad$ Dec: $\left(-\infty, \frac{3}{2}\right)$

$x = \frac{3}{2} \pm \frac{3}{2} \nearrow \frac{6}{2} = 3$, $\searrow 0$

$\boxed{\text{Check:}}$ $-\frac{b}{2a} = -\frac{-3}{2(1)} = \frac{3}{2} = h$ ✓

$f\left(-\frac{b}{2a}\right) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) = \frac{9}{4} - \frac{9}{2} = -\frac{9}{4} = k$ ✓

(13) $y = 2x^2 - 12x + 22$



$= 2(x^2 - 6x \qquad) + 22$

$= 2(x^2 - 6x + 3^2) + 22 - 2(9)$

$= \boxed{2(x-3)^2 + 4}$ $\qquad (h,k) = (3, 4)$

$\boxed{\begin{array}{l}\text{Inc: } (3, \infty) \\ \text{Dec: } (-\infty, 3)\end{array}}$

Check: $a = 2, b = -12, c = 22$

$-\frac{b}{2a} = -\frac{-12}{2(2)} = \frac{12}{4} = 3 = h$ ✓

$f\left(-\frac{b}{2a}\right) = f(3) = 2(3)^2 - 12(3) + 22 = 18 - 36 + 22$

$= 40 - 36 = 4 = k$ ✓

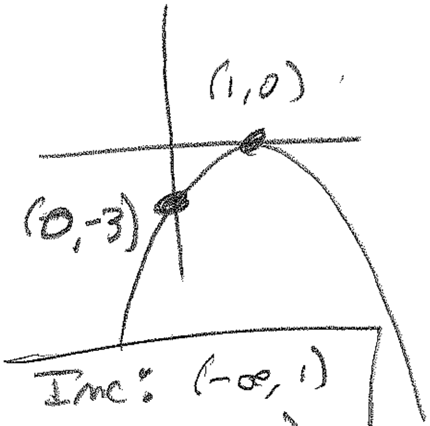
121 §3.1 #s 13–19, 75–90

(15) $y = -3x^2 + 6x - 3$

$= -3(x^2 - 2x \quad) - 3$

$= -3(x^2 - 2x + 1^2) - 3 + 3(1)$

$= \boxed{-3(x-1)^2}$ $\quad (h,k) = (1,0)$



Inc: $(-\infty, 1)$
Dec: $(1, \infty)$

$\underset{=}{\text{SET}}\ 0 \Longrightarrow$

$-3(x-1)^2 = 0$

$(x-1)^2 = 0$

$x - 1 = 0$

$x = 1$

Check $\quad a = -3,\ b = 6,\ c = -3$

$-\frac{b}{2a} = -\frac{6}{2(-3)} = +1 = h$ ✓

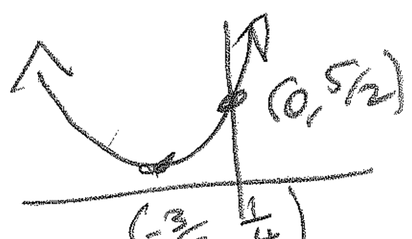
$f(-\frac{b}{2a}) = -3(1)^2 + 6(1) - 3$

$= -3 + 6 - 3 = 0 = k$ ✓

(17) $y = x^2 + 3x + \frac{5}{2}$

$= x^2 + 3x + \left(\frac{3}{2}\right)^2 + \frac{5}{2} - \frac{9}{4}$

$= \boxed{\left(x + \frac{3}{2}\right)^2 + \frac{1}{4}}$ $\quad (h,k) = \left(-\frac{3}{2}, \frac{1}{4}\right)$



Inc: $\left(-\frac{3}{2}, \infty\right)$
Dec: $\left(-\infty, -\frac{3}{2}\right)$

$\underset{=}{\text{SET}}\ 0 \Longrightarrow$

$\left(x + \frac{3}{2}\right)^2 + \frac{1}{4} = 0$

$\left(x + \frac{3}{2}\right)^2 = -\frac{1}{4}$ No real solutions!

No x-intercepts in graph!

Check: $a = 1,\ b = 3,\ c = \frac{5}{2}$

$-\frac{b}{2a} = -\frac{3}{2(1)} = -\frac{3}{2} = h$ ✓

$f(-\frac{b}{2a}) = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) + \frac{5}{2}$

$= \frac{9}{4} - \frac{9}{2} + \frac{5}{2} = -\frac{9}{4} + \frac{10}{4} = \frac{1}{4} = k$ ✓

'21 §3.1 #s 19, 75–90

19 $f(x) = -2x^2 + 3x - 1$

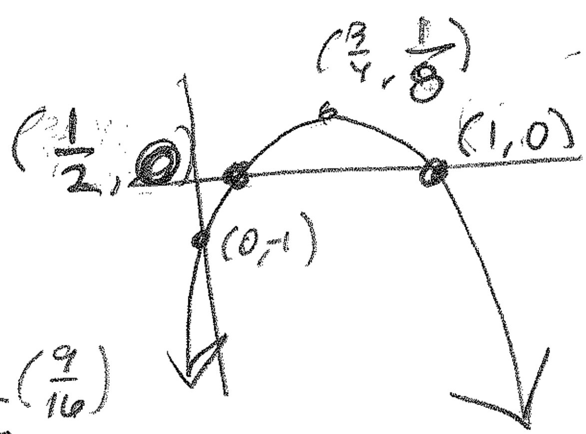
$= -2\left(x^2 - \frac{3}{2}x \qquad\right) - 1$

$= -2\left(x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2\right) - 1 + 2\left(\frac{9}{16}\right)$

$\frac{-8+9}{8} = \frac{1}{8}$

$\boxed{= -2\left(x - \frac{3}{4}\right)^2 + \frac{1}{8}}$

$\boxed{(h,k) = \left(\frac{3}{4}, \quad\right)}$



SET $= 0 \Rightarrow -2\left(x - \frac{3}{4}\right)^2 = -\frac{1}{8}$

$\left(x - \frac{3}{4}\right)^2 = \frac{1}{16}$

$x - \frac{3}{4} = \pm \frac{1}{4}$

$x = \frac{3}{4} \pm \frac{1}{4}$ → $\frac{4}{4} = 1$, $\frac{2}{4} = \frac{1}{2}$

$\boxed{\text{Inc: } \left(-\infty, \frac{3}{4}\right) \quad \text{Dec: } \left(\frac{3}{4}, \infty\right)}$

$\boxed{\text{Check}}$ $a = -2,\ b = 3,\ c = -1$ ✓

$-\frac{b}{2a} = -\frac{3}{2(-2)} = \frac{3}{4} = h$

$f\left(-\frac{b}{2a}\right) = -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) - 1$

$= -2\left(\frac{9}{16}\right) + \frac{9}{4} - 1$

$= -\frac{9}{8} + \frac{9}{4} - 1$ ✓

$= \frac{-9 + 18 - 8}{8} = \frac{1}{8} = k$

#s 75–90 Show sign pattern
Show graph with x-intercepts
Answers in interval notation.

121 §3.1 #s 75-90

(75) $a^2+20 \le 8a$

$a^2-8a+20 \le 0$

$x^2-8x+20 \le 0$

$a=1,\ b=-8,\ c=20$

$b^2-4ac = (-8)^2-4(1)(20)$

$= 64-80 = -16$

$x^2-8x \quad = -20$

$x^2-8x+4^2 = -20+16$

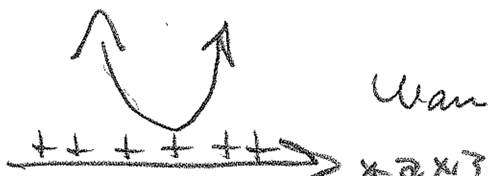
$(x-4)^2 = -4$

$x-4 = \pm 2i$

$x = 4 \pm 2i$ Not real

No x-intercepts!

No real zeros
opens up } must be above x-axis!



Want ≤ 0

Never negative!

Never ≤ 0!

No Solution! → $\boxed{\varnothing}$ = Empty set.

(76) $6t \le t^2+25$

$0 \le t^2-6t+25$

$t^2-6t+25 \ge 0$

$t^2-6t+25=0$

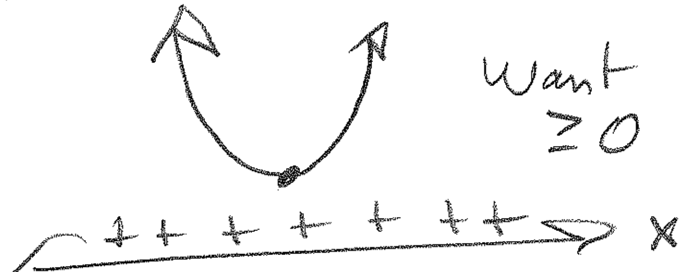
$t^2-6t \quad = -25$

$t^2-6t+3^2 = -25+9$

$(t-3)^2 = -16$

t

No real sol'n
opens up } must be above x-axis, always.



Want ≥ 0

Always ≥ 0

$\boxed{(-\infty, \infty)}$

121 §3.1 #s 77-90

77 $-2w^2 + 5w < 6$

$-2w^2 + 5w - 6 < 0$

$a = -2,\ b = 5,\ c = -6$

$b^2 - 4ac = 5^2 - 4(-2)(-6)$

$= 25 - 48$

$= -23$

No real zeros!

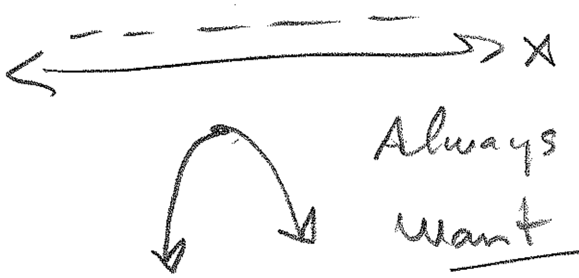
$-2(w^2 + \frac{5}{2}w \quad) - 6 < 0$

$-2(w^2 + \frac{5}{2}w + (\frac{5}{4})^2) - 6 + 2(\frac{25}{16}) < 0$

$-2(w + \frac{5}{4})^2 - 6 + \frac{25}{8} < 0$ $\qquad \frac{-48 + 25}{8} = \frac{-23}{8}$

$-2(w + \frac{5}{4})^2 - \frac{23}{8} < 0$

Opens down, vertex below x-axis



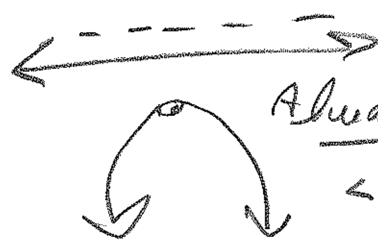
Always < 0

Want < 0

so $\boxed{(-\infty, \infty)}$

No real zeros

opens down



Always < 0

78 $-3z^2 - 5 > 2z$

$-3z^2 - 2z - 5 > 0$

$3z^2 + 2z + 5 < 0$ This is the one I solve

$3(z^2 + \frac{2}{3}z \quad) = -5$

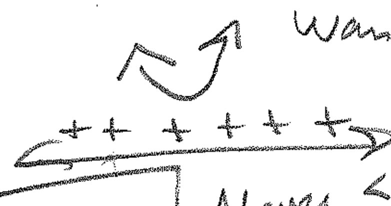
$3(z^2 + \frac{2}{3}z + (\frac{1}{3})^2) = -5 + 3(\frac{1}{3})^2$ $\qquad \frac{-15 + 1}{3} = -\frac{14}{3}$

$3(z + \frac{1}{3})^2 = -\frac{14}{3}$

No real soln

opens up

Want < 0



Want < 0

Never < 0

$\emptyset$

121 §3.1 #s 79–90

78 Re-Do Different Way

$-3z^2 - 5 > 2z$

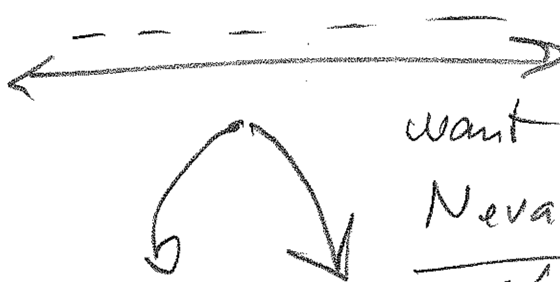
$-3z^2 - 2z - 5 > 0$

$a = -3,\ b = -2,\ c = -5$

$b^2 - 4ac = (-2)^2 - 4(-3)(-5)$

$= 4 - 60 = -56$

No real zeros
opens down
want > 0



want > 0
Nevah!

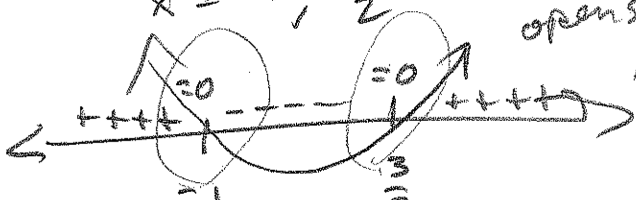
$\boxed{\varnothing}$

79 $2x^2 - x - 3 < 0$

$(2x - 3)(x + 1) < 0$

$x = -1,\ \frac{3}{2}$

opens up
want < 0

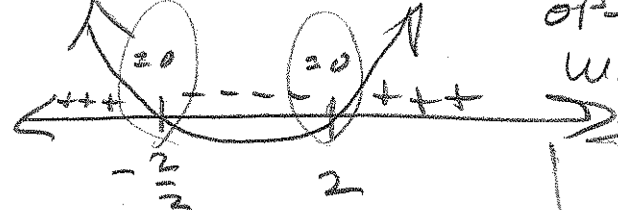


$\boxed{\left(-1, \frac{3}{2}\right)}$

80 $3x^2 - 4x - 4 \le 0$

$(3x + 2)(x - 2) \le 0$

opens up
want ≤ 0



$\boxed{\left[-\frac{2}{3}, 2\right]}$

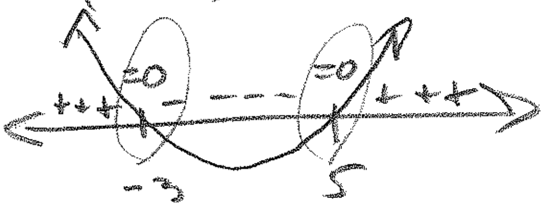
121 §3.1 #s 81–90

81 $2x + 15 < x^2$

$-x^2 + 2x + 15 < 0$ Meh

$x^2 - 2x - 15 < 0$ Easier

$(x-5)(x+3) < 0$



opens up

Want < 0

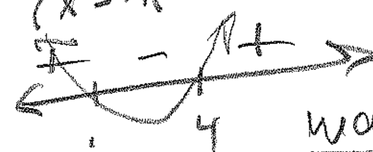
$\boxed{(-3, 5)}$

82 $5x - x^2 < 4$

$-x^2 + 5x - 4 < 0$

$x^2 - 5x + 4 > 0$

$(x-1)(x-4) > 0$

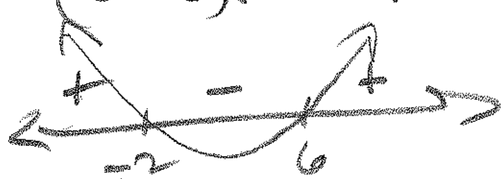


Want > 0

$\boxed{(-\infty, 1) \cup (4, \infty)}$

83 $w^2 - 4w - 12 \geq 0$

$(w-6)(w+2) \geq 0$

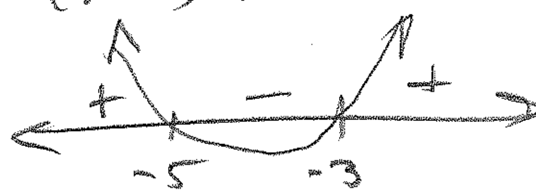


Want ≥ 0

$\boxed{(-\infty, -2] \cup [6, \infty)}$

84 $y^2 + 8y + 15 \leq 0$

$(y+3)(y+5) \leq 0$



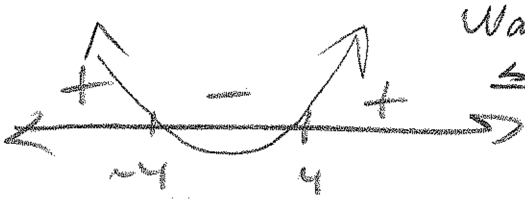
Want ≤ 0

$\boxed{[-5, -3]}$

85 $t^2 \leq 16$

$t^2 - 16 \leq 0$

$(t-4)(t+4) \leq 0$



Want ≤ 0

$\boxed{[-4, 4]}$

86 $36 \leq h^2$

$0 \leq h^2 - 36$

$h^2 - 36 \geq 0$

$(h-6)(h+6) \geq 0$

Want ≥ 0

$\boxed{(-\infty, -6] \cup [6, \infty)}$

121 §3.1 #s 87 – 90

87 $a^2+6a+9 \le 0$

$(a+3)^2 \le 0$

opens up
want ≤ 0

-3

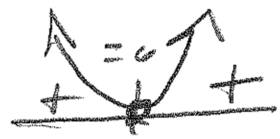
only get ≤ 0 @
$x=-3$, where it $=0$

$\boxed{\{-3\}}$

88 $c^2+4 \le 4c$

$c^2-4c+4 \le 0$

$(c-2)^2 \le 0$



opens up
want ≤ 0

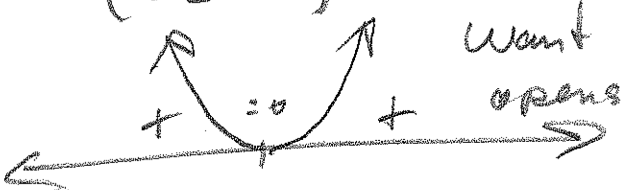
2

$\boxed{\{2\}}$

89 $4z^2-12z+9>0$

$(2z-3)^2>0$

want >0
opens up



$\frac{3}{2}$

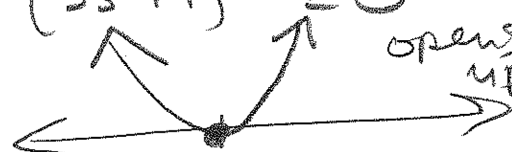
$(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

$z=\frac{3}{2}$ is where
$4z^2-12z+9=0$.
want only ">0".

90 $9s^2+6s+1 \ge 0$

$(3s+1)^2 \ge 0$

opens up



want ≥ 0

$-\frac{1}{3}$

$\boxed{(-\infty, \infty)}$