121 §2.5 Inverse Functions!

§2.5 #s 1–6, 7–31 ODDS, 38–40, 43, 45, 51a, g, 53–59 ODDS
69–87 odds

I like #s 61–68, but don't like the time & care needed to get the graphs just right.

#s 7–12 Determine if the function is 1-to-1.

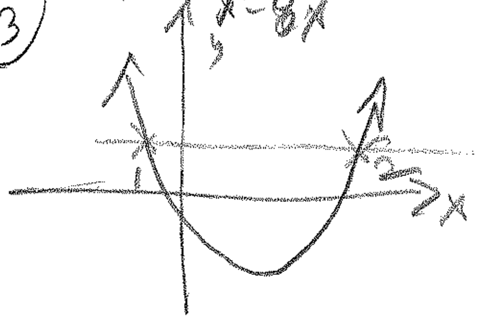
(7) {(3,3), (5,5), (9,9)} Yes

(9) {(-1,1), (1,1), (-2,4), (2,4)} No

(11) {(1,99), (2,98), (3,97), (4,96), (5,99)} No

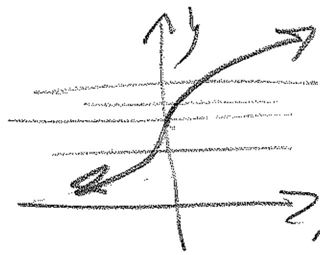
#s 13–17 use horizontal line test to determine whether each function is 1-to-1.

(13) $x^2 - 8x$



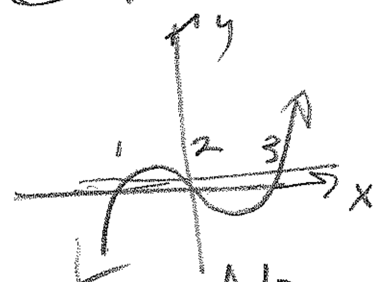
No

(15) $\sqrt[3]{x} + 7$



Yes

(17) $y = x^3 - 2x$



No

§2.5 #s 19–31 ODDS, 38–40, 43, 45, 51 ag, 53–59 ODDS
69–87 ODDS

#s 19–21 Determine if function is 1-to-1.

(19) $f(x) = 2x - 3$

$2x_1 - 3 = 2x_2 - 3$

$2x_1 = 2x_2$

$x_1 = x_2$

Yes

(21) $g(x) = \frac{1-x}{x-5}$

$\frac{1-x_1}{x_1-5} = \frac{1-x_2}{x_2-5}$

$(1-x_1)(x_2-5) = (1-x_2)(x_1-5)$

$x_2 - 5 - x_1x_2 + 5x_1 = x_1 - 5 - x_2x_1 + 5x_2$

$x_2 + 5x_1 = x_1 + 5x_2$

$4x_1 = 4x_2$

$x_1 = x_2$ Yes

(23) $p(x) = |x+1|$

$|x_1+1| = |x_2+1|$

$x_1 + 1 = x_2 + 1$ OR $x_1 + 1 = -(x_2+1)$

$x_1 = x_2$ OR $x_1 + 1 = -x_2 - 1$

OR $x_1 = -x_2 - 2$

→ Two possible vals of x_1!

No

(25) $w(x) = x^2 + 3$

$x_1^2 + 3 = x_2^2 + 3$

$x_1^2 = x_2^2$

$\sqrt{x_1^2} = \sqrt{x_2^2}$

$|x_1| = |x_2|$

$x_1 = \pm x_2$

No

121 §2.5 #s 27-31 ODDS, 38-40, 43, 45, 51 a-g, 53-59 ODDS, 69-87 ODDS

(27) $k(x) = \sqrt[3]{x+9}$

#s 29-36 Determine if the function is invertible. If it is, then find the inverse

$\sqrt[3]{x_1+9} = \sqrt[3]{x_2+9}$

$\left(\sqrt[3]{x_1+9}\right)^3 = \left(\sqrt[3]{x_2+9}\right)^3$

$x_1+9 = x_2+9$

$x_1 = x_2$

Yes

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(29) $\{(9,3),(2,2)\} = f$ (Yes)

$\Rightarrow f^{-1} = \{(3,9),(2,2)\}$

(31) $\{(-1,0),(1,0),(5,0)\}$ (No)

$y=0$ is the image of $x=-1,1,5$. Not 1-to-1!

#s 38-40 Determine whether each function is invertible & explain your answer.

(38) f pairs universal product code (UPC) with a price.

$f = \{(x,y) \mid x = \text{UPC}, y = \text{price}\}$

Not 1-to-1, because different products can have the same price.

121 §2.5 #s 39, 40, 43, 45, 51a9, 53-59 ODDS, 69-87 odds

(39) f pairs the # of days ~~with~~ since birth with your age, in years.

$f = \{(x, y) \mid x = \text{age in days}, y = \text{age in years}\}$

Not invertible, because 365, 366, 367, ... are all paired with y = 1 !

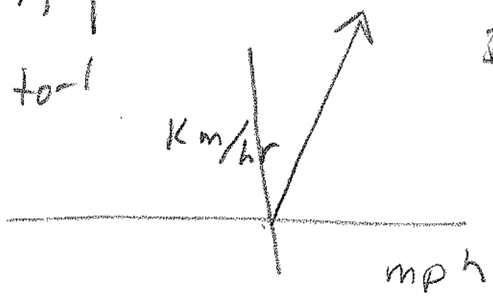
(40) f pairs your speed in mph with your speed in Km per hour.

$f = \{(x, y) \mid x = \text{speed in mph}, y = \text{speed in Km/hr}\}$

It's just a line!

Yes. 1-to-1

1-to-1



#s 43-46 ∀ f, find f^{-1}, $f^{-1}(5)$, $(f^{-1} \circ f)(2)$

(43) $f = \{(2,1), (3,5)\}$ ⟹

$f^{-1} = \{(1,2), (5,3)\}$, $f^{-1}(5) = 3$, $(f^{-1} \circ f)(2) = f^{-1}(1) = 2$

(45) $f = \{(-3,-1), (0,5), (2,-7)\}$ ⟹

$f^{-1} = \{(-1,-3), (5,0), (2,-7)\}$, $f^{-1}(5) = 0$, $(f^{-1} \circ f)(2) = 2$

121 §2.5 #s 51 a g, 53-59 ODDS, 69-87 ODDS

(51) Find the inverse of each function by reversing a composition.

(a) $f(x) = 5x + 1$

① TIMES 5 — ① Subtract 1 — $f^{-1}(x) = \frac{x-1}{5}$

② ADD 1 — ② Divide by 5

(g) $f(x) = \sqrt[3]{x} - 9$

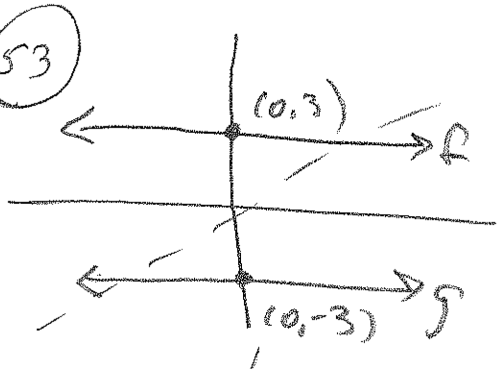
① cube root — ① Add 9

② subtract 9 — ② cube

$$(x+9)^3 = f^{-1}(x)$$

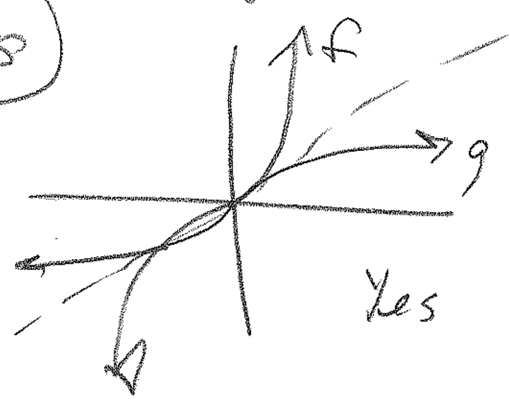
#s 53-56 Determine if f and g are inverses

(53)



No!

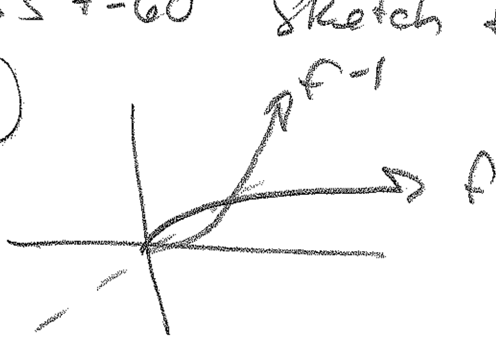
(55)



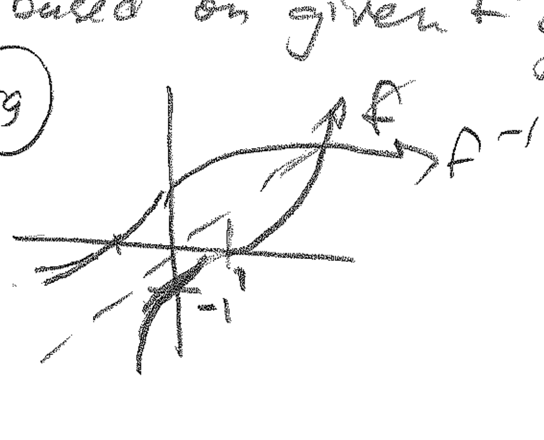
Yes

#s 57-60 Sketch f^{-1} based on given f graph.

(57)



(59)



12' §2.5 #s 69–87 odds

#s 69–82 Find f^{-1} by swith-and-solve.

$\mathcal{D} = [3, \infty)$

$\mathcal{R} = [2, \infty)$

(69) $f(x) = 3x - 7$

$3y - 7 = x$

$3y = x + 7$

$y = \boxed{\frac{x+7}{3} = f^{-1}(x)}$

(71) $f(x) = \sqrt{x-3} + 2$

$\sqrt{y-3} + 2 = x$

$\sqrt{y-3} = x - 2$

$y - 3 = (x-2)^2$

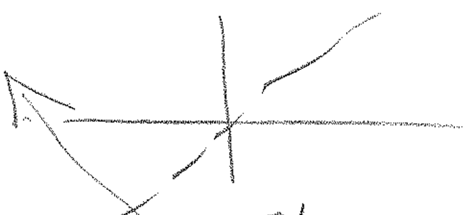
$\boxed{y = (x-2)^2 + 3 \quad (x \geq 2) \quad = f^{-1}(x)}$

(73) $f(x) = -x - 9$

$-y - 9 = x$

$-y = x + 9$

$y = -x - 9 = f^{-1}(x)$?



Oh ok.
It's $\perp$ to

$-x - 9 \quad y = x$!

(75) $f(x) = \frac{x+3}{x-5}$

$\frac{y+3}{y-5} = x$

$\frac{y+3}{y-5} = \frac{x(y-5)}{y-5}$

$y + 3 = x(y-5)$

$y + 3 = xy - 5x$

$y - xy = -5x - 3$

$y(1-x) = -5x - 3$

$y = \frac{-5x-3}{1-x} = \boxed{\frac{5x+3}{x-1} = f^{-1}(x)}$

'21 §2.5 #s 77–87 odds

(77) $f(x) = -\frac{1}{x}$

$-\frac{1}{y} = x$

$-1 = xy$

$xy = -1$

$y = \boxed{-\frac{1}{x} = f^{-1}(x)}$

(79) $f(x) = \sqrt[3]{x-9} + 5$

$\sqrt[3]{y-9} + 5 = x$

$\sqrt[3]{y-9} = (x-5)$

$y - 9 = (x-5)^3$

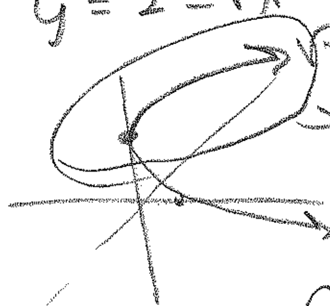
$y = \boxed{(x-5)^3 + 9 = f^{-1}(x)}$

(81) $f(x) = (x-2)^2$ for $x \geq 2$

$(y-2)^2 = x$

$y - 2 = \pm\sqrt{x}$

$y = 2 \pm \sqrt{x}$



$\sqrt{x} + 2$

$\to 2 + \sqrt{x}$ is reflection of $y = (x-2)^2$ about line $y = x$

$-\sqrt{x} + 2$

Also, from $\mathcal{D}(f) = [2, \infty)$ and $\mathcal{R}(f) = [0, \infty)$ &

we know $\mathcal{D}(f^{-1}) = [0, \infty)$ &

$\mathcal{R}(f^{-1}) = [2, \infty)$ &

$\boxed{f^{-1}(x) = 2 + \sqrt{x}}$ has proper $\mathcal{D}$ & $\mathcal{R}$!

#s 83–90 Find $f(g(x))$ & $g(f(x))$ & determine if f & g are inverses

121 §2.5 #s 83–87 odds

(83) $f(x) = 4x+4,\ g(x) = .25x-1$

$f(g(x)) = 4g(x)+4 = 4(.25x-1)+4 = x-4+4 = x$

$g(f(x)) = .25f(x)-1 = .25(4x+4)-1 = x+1-1 = x$ (Yes)

(85) $f(x) = x^2+1,\ g(x) = \sqrt{x-1}$

$f(g(x)) = (g(x))^2+1 = (\sqrt{x-1})^2+1 = x-1+1 = x$ (No)

$g(f(x)) = \sqrt{f(x)-1} = \sqrt{x^2+1-1} = \sqrt{x^2} = |x|$

Note In this one example, a restriction of the domain of $f(x)$ to $x \in [0, \infty)$ and then $|x| = x$ & they ARE inverses!

(87) $f(x) = \frac{1}{x}+3,\ g(x) = \frac{1}{x-3}$

$f(g(x)) = \frac{1}{g(x)}+3 = \frac{1}{\left(\frac{1}{x-3}\right)}+3 = \frac{x-3}{1}+3 = x$

Yes

$g(f(x)) = \frac{1}{f(x)-3} = \frac{1}{(\frac{1}{x}+3)-3} = \frac{1}{\frac{1}{x}} = \frac{x}{1} = x$