121 §2.4 #s 1–6, 7–15 ODDS, 16–19, 23–27 ODDS,
31–33, 35–51 ODDS, 55–60, 72, 77–79,
85, 91, 92,

#s 7–18 $f(x) = x-3$, $g(x) = x^2 - x$. Find and simplify* the following.

* On tests, I often say "DO NOT SIMPLIFY" just to hit the one concept under a time control.

(7) $(f+g)(2) = f(2)+g(2) = (2-3) + (2^2-2)$

$= -1 + 2 = \boxed{1}$

(9) $(f-g)(-2) = f(-2) - g(-2) = (-2-3) - ((-2)^2 - (-2))$

$= -5 - (6) = \boxed{-11}$

(11) $(f \cdot g)(-1)$ is NOT to be confused with $(f \circ g)(-1)$.

$f \cdot g$ multiply ($g(x)$ TIMES $f(x)$)

$f \circ g$ compose (Feed $g(x)$ to $f(x)$)

$(f \cdot g)(-1)$ can be written $(fg)(-1)$

$= f(-1)g(-1) = ((-1)-3)((-1)^2 - (-1)) = (-4)(2) = \boxed{-8}$

(2) §2.4 #s 13-15 ODDS, 16-17, 25-[illegible] ODDS, [illegible],

35-51 ODDS, 55-60, 72, 77-79

(13) $(f/g)(4) = \left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{4-3}{4^2-4} = \boxed{\frac{1}{12}}$

(15) $(f+g)(a) = f(a)+g(a) = a-3+(a^2-a)$

$= a-3+a^2-a = \boxed{a^2-3}$

(16) $(f-g)(b) = f(b)-g(b) = b-3-(b^2-b)$

$= b-3-b^2+b = \boxed{-b^2+2b-3}$

(17) $(f\cdot g)(a) = f(a)g(a) = (a-3)(a^2-a)$

$= a^3-a^2-3a^2+3a = \boxed{a^3-4a^2+3a}$

(18) $(f/g)(b) = \left(\frac{f}{g}\right)(b) = \frac{f(b)}{g(b)} = \boxed{\frac{b-3}{b^2-b}}$

#s 19—26 Let

$f = \{(-3,1), (0,4), (2,0)\}$, $g = \{(-3,2), (1,2), (2,6), (4,0)\}$

and $h = \{(2,4), (1,0)\}$.

Find each of the following functions and state the domain of each.

§2.4 #'s 19, 23-27 ODDS, 31-33, 35-51 ODDS
55-60, 72, 77-79

$\mathcal{D}(f+g) = \mathcal{D}(f-g) = \underbrace{\mathcal{D}(f \cdot g)}_{\mathcal{D}(fg)} = \mathcal{D}(f) \cap \mathcal{D}(g)$

Intersection!

$\mathcal{D}\left(\frac{f}{g}\right) = \mathcal{D}(f) \cap \mathcal{D}(g) \cap \{x \mid g(x) \neq 0\}$

Also need $g(x) \neq 0$, downstairs

(19) $f+g = \{(-3, 1+2), (2, 0+6)\} = \boxed{\{(-3,3), (2,6)\}}$

$\boxed{\mathcal{D}(f+g) = \{-3, 2\}}$ (Did this first!)

(23) $f \cdot g = \{(-3, (1)(2)), (2, (0)(6))\} = \boxed{\{(-3,2), (2,0)\}}$

$\boxed{\mathcal{D} = \{-3, 2\}}$

(25) $\frac{g}{f} = \{(-3, \frac{2}{1})\} = \{(-3, 2)\}$

$\mathcal{D} = \{-3, 2\} \cap \{x \mid f(x) \neq 0\} = \{-3, 2\} \cap \{-3, 0\}$

$\boxed{\mathcal{D}\left(\frac{g}{f}\right) = \{-3\}}$

121 §2.4 #s 27, 31-33, 35-51 ODDS, 55-60, 72, 77-79

~~27~~ #s 27-34 Let $f(x) = \sqrt{x}$, $g(x) = x - 4$, $h(x) = \frac{1}{x-2}$

Find an equation defining each function, and state the domain of the function

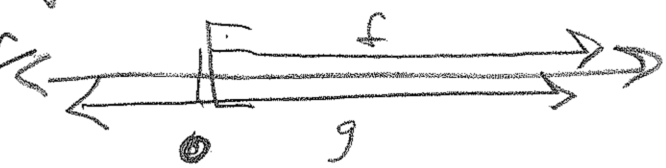
$\mathcal{D}(f) = [0, \infty)$, $\mathcal{D}(g) = (-\infty, \infty)$, $\mathcal{D}(h) = \mathbb{R} \setminus \{2\}$
$= (-\infty, 2) \cup (2, \infty)$

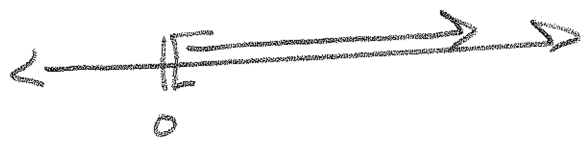
27 $f+g = (f+g)(x) = f(x) + g(x)$

$= \boxed{\sqrt{x} + x - 4}$

$\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g) = \boxed{[0, \infty)}$

Scratch





Stop here, on test!

31 $g \cdot h = (g \cdot h)(x) = g(x)\, h(x) = \boxed{(x-4)\left(\frac{1}{x-2}\right)} = \boxed{\frac{x-4}{x-2}}$

$\mathcal{D}(g \cdot h) = \mathcal{D}(g) \cap \mathcal{D}(h) = \mathbb{R} \setminus \{2\} = \boxed{(-\infty, 2) \cup (2, \infty)}$

Scratch

$= (-\infty, 2) \cup (2, \infty)$

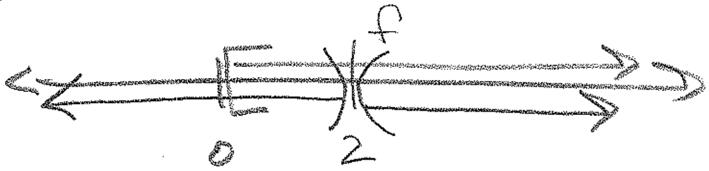
121 § 2.4 #s 32, 33, 35-51 ODDS, 55-60, 72, 77-79

(32) $f \cdot h = (f \cdot h)(x) = f(x)\,h(x)$

$= (\sqrt{x})\left(\frac{1}{x-2}\right) = \boxed{\frac{\sqrt{x}}{x-2}}$

$\mathcal{D}(f \cdot h) = \mathcal{D}(f) \cap \mathcal{D}(h) = \boxed{[0,2) \cup (2,\infty)}$

Scratch



"AND"

$= [0,2) \cup (2,\infty)$

(33) $f/g = \frac{f}{g} = \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{\sqrt{x}}{x-3}}$

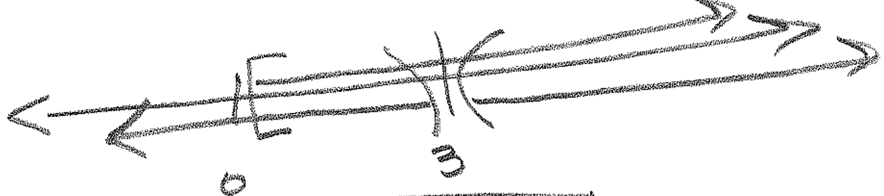
$\mathcal{D}(f/g) = \underbrace{\mathcal{D}(f) \cap \mathcal{D}(g)}_{[0,\infty)} \cap \{x \mid g(x) \neq 0\}$

$[0,\infty)$ by previous work. So now we need to intersect that with $\{x \mid g(x) \neq 0\}$

$g(x) = x - 3 \neq 0$

$x \neq 3$

$x = 3$ is bad.



$= \boxed{[0,3) \cup (3,\infty)}$

121 §2.4 #s 35-51 ODDS, 55-60, 72, 77, 79

#s 35-40 NOW we're doing composition!

$f \circ g$ open circle. Feed g to f, g is inside f.

Let $f = \{(-3,1), (0,4), (2,0)\}$

$g = \{(-3,2), (1,2), (2,6), (4,0)\}$

$h = \{(2,4), (1,0)\}$ Find the function and give its domain!

$\mathcal{D}(f \circ g) = \mathcal{D}(f(g(x)))$

$= \{x \mid \underbrace{x \in \mathcal{D}(g)}_{\text{g must digest x}} \text{ and } \underbrace{g(x) \in \mathcal{D}(f)}_{\text{f must digest the resulting g(x).}}\}$

$\mathcal{D}(f) = \{-3, 0, 2\}$, $\mathcal{D}(g) = \{-2, 1, 2, 4\}$, $\mathcal{D}(h) = \{2, 1\}$

35 $\boxed{f \circ g = \{0, 4\},\ \mathcal{D}(f \circ g) = \{-3, 1, 4\}}$

Work below is how I brute-forced it.

$f(g(-3)) = f(2) = 0$

$f(g(1)) = f(2) = 0$

$f(g(2)) = f(6)$ $\nexists$ $g(2) \notin \mathcal{D}(f)$! So,

$2 \notin \mathcal{D}(f \circ g)$!

$f(g(4)) = f(0) = 4$

121 §2.4 #s 37-51 ODDS, 55-60, 72, 77, 79

(37) $\boxed{f \circ h = \{(1,4)\}}$ $\boxed{\mathcal{D}(f \circ h) = \{1\}}$

$f(h(2)) = f(4) = \nexists$ $2 \notin \mathcal{D}$

$f(h(1)) = f(0) = 4$

(39) $\boxed{h \circ g = \{(-3,4), (1,4)\}, \mathcal{D}(h \circ g) = \{-3, 1\}}$

$h(g(-3)) = h(2) = 4$

$h(g(1)) = h(2) = 4$

$h(g(2)) = h(6)$ $\nexists$ $2 \notin \mathcal{D}$

$h(g(4)) = h(0)$ $\nexists$ $4 \notin \mathcal{D}$

$\mathcal{D}(h \circ g)$ always lives inside $\mathcal{D}(g)$. Just a matter of seeing when $g(x)$ can or can't be digested by h.

#s 41-54 Let $f(x) = 3x - 1$, $g(x) = x^2 + 1$, $h(x) = \frac{x+1}{3}$.

Evaluate each expression. Round all approximate answers to 3 decimal places.

In some ways, these are easier than the previous stretch of exercizes, because it's a RULE for each function, rather than just a set of isolated points in the plane.

121 §2.4 #s 41-51 ODDS, 55-60, 72, ~~77, 79~~

41 $f(g(-1)) = f(2) = 3(2) - 1 = \boxed{5}$

$g(-1) = (-1)^2 + 1 = 2$

43 $(f \circ h)(5) = f(h(5)) = f(2) = \boxed{5}$

$h(5) = \frac{5+1}{3} = \frac{6}{3} = 2$

45 $(f \circ g)(4.39) = f(g(4.39)) =$

(scratch $g(4.39) = (4.39)^2 + 1 = 19.2721$)

$$\begin{array}{r} 4.39 \\ \times\ 4.39 \\ \hline 3951 \\ 13170 \\ 175600 \\ \hline 19.2721 \\ \times\ 3 \\ \hline 57.8163 \end{array}$$

$= f(19.2721) = 3(19.2721) - 1$

$= 57.8163 - 1 = 56.8163 \approx \boxed{56.816}$

Don't round 'til final step.

47 $(g \circ h \circ f)(2) = g(h(f(2)))$

$= g(h(5)) = g(2) = 2^2 + 1 = \boxed{5}$

$f(2) = 3(2) - 1 = 5$

$h(5) = \frac{5+1}{3} = 2$

49 $(f \circ g \circ h)(2) = f(g(h(2))) = f(g(1)) = f(2) = \boxed{5}$

$h(2) = \frac{2+1}{3} = 1$ $\qquad$ $f(2) = 3(2) - 1 = 5$

$g(1) = 1^2 + 1 = 2$

121 §2.4 #s 51, 55-60, 72, ~~77~~, 79

51 $(f \circ h)(a) = f(h(a)) = f\left(\frac{a+1}{3}\right) = \boxed{a}$

$f\left(\frac{a+1}{3}\right) = 3\left(\frac{a+1}{3}\right) - 1 = a+1-1 = a$

~~55~~ #s 55-66 Let $f(x) = x-2$, $g(x) = \sqrt{x}$, $h(x) = \frac{1}{x}$

Find an ~~equation~~ EXPRESSION for each function, and state its domain.

SETUP FOR DOMAINS

$\mathcal{D}(f) = \mathbb{R}$; $\mathcal{D}(g) = [0, \infty)$

$\mathcal{D}(h) = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$

55 $f \circ g = (f \circ g)(x) = f(g(x)) = g(x) - 2 = \sqrt{x} - 2$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid \underbrace{x \in [0, \infty)}_{x \geq 0, \text{ i.e.}} \text{ and } \sqrt{x} \in \mathbb{R}\}$

$= \{x \mid x \geq 0\} = \boxed{[0, \infty)}$

56 $g \circ f = g(f(x)) = \boxed{\sqrt{x-2}}$

$\mathcal{D}(g \circ f) = \{x \mid x \in \mathcal{D}(f) \text{ and } f(x) \in \mathcal{D}(g)\}$

$= \{x \mid \underbrace{x \in \mathbb{R}}_{\text{No restriction}} \text{ and } \underbrace{x-2 \in [0, \infty)}_{\substack{x-2 \geq 0 \\ x \geq 2}}\}$

$= \{x \mid x \geq 2\} = \boxed{[2, \infty)}$

'21 §2.4 #s 57-60, 72, 77-79

57 $f \circ h = f(h(x)) = f\left(\frac{1}{x}\right) = \boxed{\frac{1}{x} - 2}$

$$\mathcal{D}(f(h(x))) = \{x \mid x \in \mathcal{D}(h) \text{ and } h(x) \in \mathcal{D}(f)\}$$
$$= \{x \mid x \neq 0 \text{ and } \underbrace{h(x) \in \mathbb{R}}_{\text{No restriction}}$$
$$= \{x \mid x \neq 0\} = \boxed{(-\infty, 0) \cup (0, \infty)} = \mathbb{R} \setminus \{0\}$$

58 $h \circ f = h(f(x)) = h(x-2) = \frac{1}{x-2}$

$$\mathcal{D}(h(f(x))) = \{x \mid x \in \mathcal{D}(f) \text{ and } f(x) \in \mathcal{D}(h)\}$$
$$= \{x \mid \underbrace{x \in \mathbb{R}}_{\text{No restriction}} \text{ and } x - 2 \neq 0\}$$
$$= \{x \mid x \neq 2\} = \boxed{(-\infty, 2) \cup (2, \infty)}$$

59 $h \circ g = h(g(x)) = \frac{1}{g(x)} = \boxed{\frac{1}{\sqrt{x}}}$

$$\mathcal{D}(h(g(x))) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(h)\}$$
$$= \{x \mid x \geq 0 \text{ and } \underbrace{\sqrt{x} \neq 0}_{x \neq 0}\}$$
$$= \{x \mid \underbrace{x > 0}_{x \geq 0 \text{ \& } x \neq 0 \iff x > 0}\} = \boxed{(0, \infty)}$$

121 §2.4 #s 60, 72, 77–79

60 $g \circ h = g(h(x)) = \sqrt{h(x)} = \boxed{\sqrt{\frac{1}{x}}}$

or think of it as... $= g\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}}$

$\mathcal{D}(g(h(x))) = \{x \mid x \in \mathcal{D}(h) \text{ and } h(x) \in \mathcal{D}(g)\}$

$= \{x \mid x \neq 0 \text{ and } \frac{1}{x} \geq 0\}$

$= \{x \mid \frac{1}{x} > 0\} = \{x \mid x > 0\}$

$= \boxed{(0, \infty)}$

#s 67–76

Write the function as a composition of

$f(x) = |x|$, $g(x) = x - 7$, and $h(x) = x^2$

72 Outside-in $|\ x\ | = f(\quad)$

(or "last thing done to x" approach

$|x - 7|$ $f(g(\quad))$ Maybe not as Natural.

$|x^2 - 7|$ $f(g(h(x)))$

Building from inside-out:

$$x \longrightarrow \underset{h(x)}{x^2} \longrightarrow \underset{g(h(x))}{x^2 - 7} \longrightarrow \underset{f(g(h(x)))}{|x^2 - 7|}$$

$= \boxed{f \circ g \circ h}$

121 §2.4 #s 77-79

#s 77-84 Find functions $h(x)$, $g(x)$ $\ni$

$f = h \circ g$

outside, last thing done → h; Inside, 1st → g

(77) $f(x) = x^3 - 2$

Pathological
$h(x) = x$
$g(x) = x^3 - 2$
} works, but not for points!

Standard

$$\boxed{\begin{array}{l} h(x) = x - 2 \\ g(x) = x^3 \end{array}} \longrightarrow f(x) = h(g(x))$$

$$x \to x^3 \longrightarrow x^3 - 2$$

-2 is outside the cube. $x-2$ happens later

(78) $f(x) = (x-2)^3$

$$x \to x - 2 \longrightarrow (x-2)^3$$

$x-2$ is inside the cube. $x-2$ happens 1st

$h(x)$ = outside/last = x^3
$g(x)$ = inside/first = $x-2$

$\therefore f(x) = h(g(x))$

(79) $f(x) = \sqrt{x+5}$

$g(x)$ = inside = $x+5$
$h(x)$ = outside = $\sqrt{x}$

$\therefore f(x) = h(g(x))$

Again, a smart-aleck could say $g(x) = x$, $h(x) = \sqrt{x+5}$, but wouldn't earn any points!