121 §2.1 #s 1–12, 18–27, 29, 30, 35–38, 41, 43, 45, 47–50*, 53, 54, 57, 60, 61, 65, 67, 69, 70, 73, 74, 77, 78, 85, 88, 89, 91, 93, 97, 99

* we've not covered graphing these guys yet.

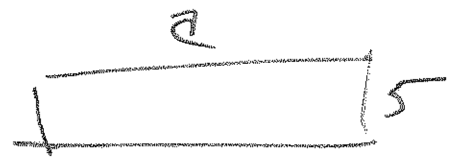
#s 9–18 Determine if a is a function of b, b is a function of a, or neither

9. a is the radius of any U.S. coin and b is the circumference.

$c = 2\pi a$ c is func of a Both

$a = \frac{c}{2\pi}$ a is func of c

10. a = length of any rectangle with width of 5 and b is its perimeter.



$b = 2a + 2(5)$ Both

$\Rightarrow a = \frac{b-10}{2}$

11. a is length of any paper money. b is its denomination.

a is always same a = f(b) is ok.

But b = f(a), you have \$1, \$5, \$10 all assigned to that same length, i.e.,

(a, 1), (a, 5), (a, 10) are all members of this relation. Not func!

But {(1, a), (5, a), (10, a)...} is a function (Horizontal Line)

121 §2.1 #s 12, 18-27

(12) a is the diameter of a U.S. coin and b is its value. Both.

~~All~~ Each diameter corresponds to different value, so there's no repetition of independent variable being associated with more than one independent variable.

(18) a is cost for mailing a 1st. class letter and b is the weight.

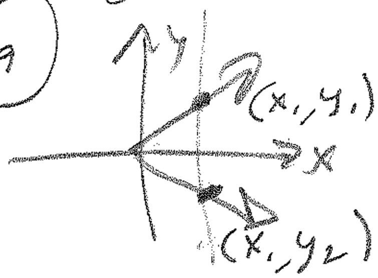
Heck, I'm not sure how this works!

Is a a sliding scale based on weight?

Book says a is func. of b. So, different weight $\Longrightarrow$ different cost, but I'd think that different cost $\Longrightarrow$ different weight, in that case.

#s 19-24 Use vertical line test to determine if y is func. of x.

(19)

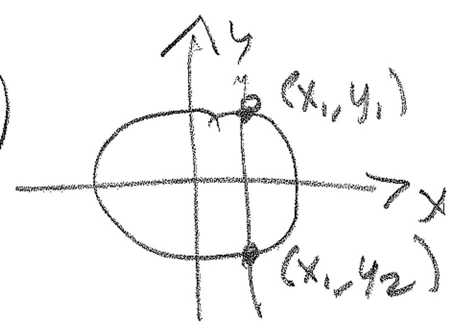


Nope

$x_1 = x_2$, but $y_1 \neq y_2$

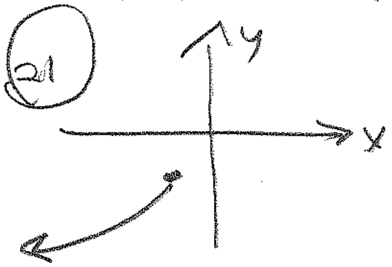
Not $y = f(x)$

(20)

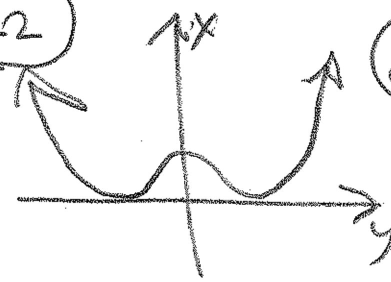


Nope.

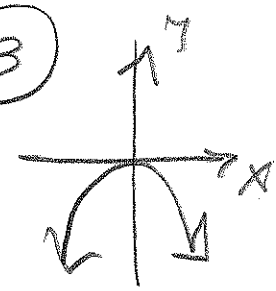
121 §2.1 #s 21–27, 29, 30, 35–38

21


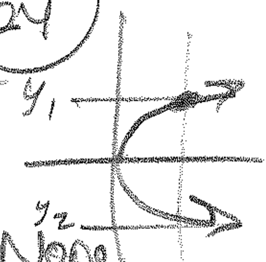
Yes.

22


Yes

23


Yes

24


Nope!

$x = x_1$

#s 25–32 Determine whether each relation is a function.

25 $\{(-1,-1), (2,2), (3,3)\}$ Yes

26 $\{(0.5, 7), (0, 7), (1, 7), (9, 7)\}$ Yes
It's OK for y-values to repeat.

27 $\{(25, 5), (25, -5), (0, 0)\}$ No

$25 \longmapsto 5$ and $25 \longmapsto -5$ can't have that.

29

x	y
3	6 *
4	9
3	12 *

No

30

x	y
1	6.98
5	5.98
9	6.98

Yes

#s 33–44 Determine whether equation defines y as a function of x.
(SOLVE FOR Y. SEE IF YOU GET ONE EXPRESSION ON RHS.)

33 $y = 3x - 8$ Yes

35 $x = 3y - 9$
$3y = x + 9$
$y = \frac{x+9}{3}$ Yes

36 $x = y^3$
$y = \sqrt[3]{x}$ Yes.

121 §2.1 #s 37-8, 41, 43, 45, 47-50, 53

37 $x^2 = y^2$

$y^2 = x^2$

$\sqrt{y^2} = \sqrt{x^2}$

$|y| = |x|$

$y = \pm x$

No. 2 solns!

38 $y^2 - x^2 = 9$

$y^2 = x^2 + 9$

$\sqrt{y^2} = \sqrt{x^2+9}$

$|y| = \sqrt{x^2+9}$

$y = \pm\sqrt{x^2+9}$

41 $y + 2 = |x|$

$y = |x| - 2$

Yes

43 $x = |2y|$

$2|y| = x$

$|y| = \frac{x}{2}$

$y = \pm \frac{x}{2}$

No.

#s 45-56

Determine $\mathcal{D}$ & R of each ~~function~~ relation. Variables represent real #s.

(Graphing Skills Preview!)

45 $\{(-3,1), (4,2), (-3,6), (5,6)\}$

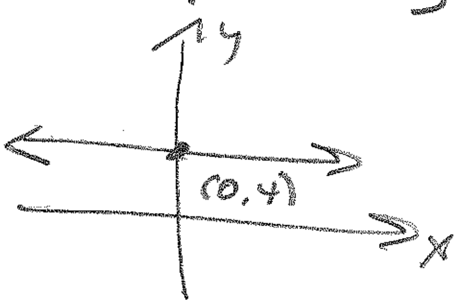
$\mathcal{D} = \{-3, 4, 5\}$ $\quad R = \{1, 2, 6\}$

47 $\{(x,y) \mid y = 4\}$

$\mathcal{D} = \mathbb{R} = (-\infty, \infty)$

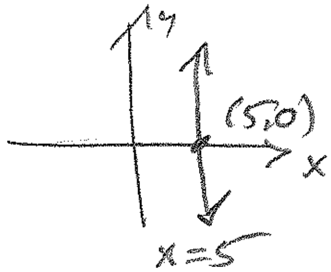
$R = \{4\}$

(Func)



121 §2.1 #s 49, 50 53, 54, 57, 60, 61, 65, 67, 69, 70

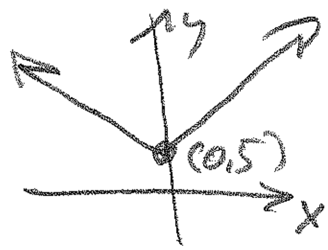
48 $\{(x, y) \mid x = 5\}$ $D = \{5\}$



$R = (-\infty, \infty)$

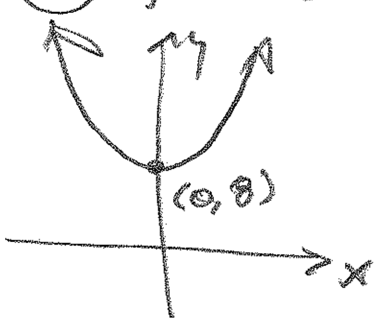
NOT
Func.

49 $y = |x| + 5$



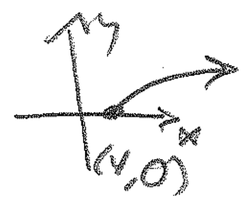
$D = (-\infty, \infty)$
$R = [5, \infty)$

50 $y = x^2 + 8$



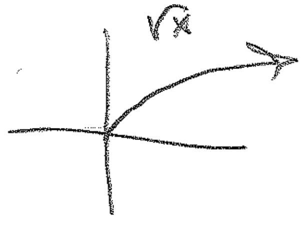
$D = (-\infty, \infty)$
$R = [8, \infty)$

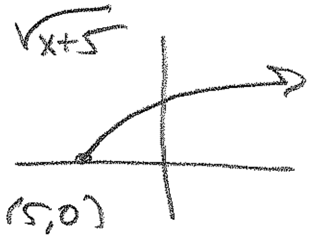
53 $y = \sqrt{x - 4}$

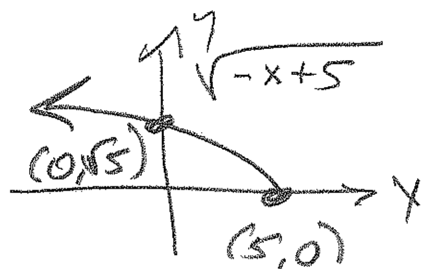


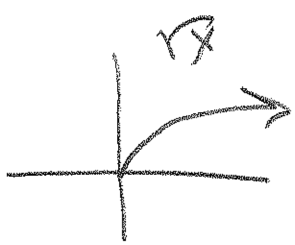
$D = [4, \infty)$
$R = [0, \infty)$

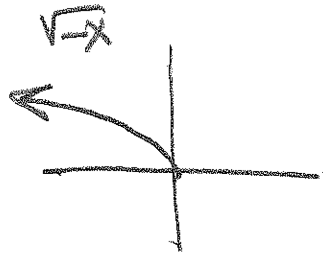
54 $y = \sqrt{5 - x} = \sqrt{-x + 5}$ OR $\sqrt{-(x - 5)}$

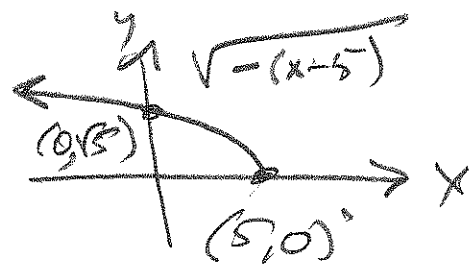












$D = (-\infty, 5]$

$R = [0, \infty)$

121 §2.1 #s 57, 60, 61, 65, 67, 69, 70, 73, 74, 77, 78

#s 57–66 $f = \{(2,6), (3,8), (4,5)\}$, $g = g(x) = 3x+5$

57 $f(2) = 6$ 58 $f(4) = 5$ 59 $g(2) = 3(2)+5 = 11$

60 $g(4) = 3(4)+5 = 12+5 = \boxed{17 = g(4)}$

61 x, if $f(x) = 8$, is $\boxed{x = 3}$

New notation

65 $f(4)+g(4) = 5 + 3(4)+5 = \boxed{22 = (f+g)(4)}$

#s 67–78 $f(x) = 3x^2 - x$ & $g(x) = 4x-2$

67 $f(a) = \boxed{3a^2 - a}$

69 $g(a+2) = 4(a+2)-2 = 4a+8-2 = \boxed{4a+6}$

70 $g(a-5) = 4(a-5)-2 = 4a-20-2 = \boxed{4a-22}$

73 $g(x+h) = 4(x+h)-2 = \boxed{4x+4h-2}$

74 $f(x+h) = 3(x+h)^2 - (x+h) = 3(x^2+2xh+h^2) - x - h$

$= \boxed{3x^2+6xh+3h^2-x-h} = f(x+h)$

77 $f(x+h) - f(x) = 3x^2+6xh+3h^2-x-h-(3x^2-x)$

$= 3x^2+6xh+3h^2-x-h-3x^2+x = \boxed{6xh+3h^2-h}$

121 § 2.1 #s 78, 85, 88, 89, 91, 93, 97, 99

78 $g(x+h) - g(x) = 4(x+h) - 2 - (4x-2)$

$= 4x + 4h - 2 - 4x + 2 = \boxed{4h}$

#s 85 - 100 Find the DIFFERENCE QUOTIENT

$\dfrac{f(x+h) - f(x)}{h}$ for each of the following

85 $f(x) = 4x \Longrightarrow \dfrac{f(x+h) - f(x)}{h} = \dfrac{4(x+h) - 4x}{h} = \dfrac{4x + 4h - 4x}{h}$

$= \dfrac{4h}{h} = \boxed{4}$

88 $f(x) = -2x + 3 \Longrightarrow \dfrac{f(x+h) - f(x)}{h} = \dfrac{-2(x+h) + 3 - (-2x+3)}{h}$

$= \dfrac{-2x - 2h + 3 - (-2x) - 3}{h} = \dfrac{-2x - 2h + 3 + 2x - 3}{h} = \dfrac{-2h}{h} = \boxed{-2}$

$y = -x^2 + x - 2$:

91 $y = "y(x)" = "y \text{ of } x" \Rightarrow \dfrac{y(x+h) - y(x)}{h}$

$= \dfrac{-(x+h)^2 + (x+h) - 2 - (-x^2 + x - 2)}{h}$

$= \dfrac{-(x^2 + 2xh + h^2) + x + h - 2 + x^2 - x + 2}{h}$

$= \dfrac{-x^2 - 2xh - h^2 + x + h - 2 + x^2 - x + 2}{h} = \dfrac{-2xh - h^2 + h}{h}$

$= \dfrac{h(-2x - h + 1)}{h} = \boxed{-2x - h + 1}$

121 §2.1 #s 93, 97, 99

93 $g(x) = 3\sqrt{x}$ $\longrightarrow$

$$\frac{g(x+h)-g(x)}{h} = \frac{3\sqrt{x+h} - 3\sqrt{x}}{h}.$$

There's little motivation, at THIS point, to go any farther, but the author is trying to point us at <u>calculus</u>, where we want h to be zero, eventually. So you play algebra tricks to get that naked 'h' out of the denominator.

The algebra trick is "rationalizing the NUMERATOR"!

In general:

$$\frac{\sqrt{a}-\sqrt{b}}{c} = \frac{\sqrt{a}-\sqrt{b}}{c} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}{c(\sqrt{a}+\sqrt{b})} = \frac{\sqrt{a}^2-\sqrt{b}^2}{c(\sqrt{a}+\sqrt{b})} = \frac{a-b}{c(\sqrt{a}+\sqrt{b})}$$

In particular:

$(3\sqrt{x+h})^2 =$

$$\frac{3\sqrt{x+h}-3\sqrt{x}}{h} \cdot \frac{3\sqrt{x+h}+3\sqrt{x}}{3\sqrt{x+h}+3\sqrt{x}} = \frac{9(x+h)-9x}{h(3\sqrt{x+h}+3\sqrt{x})}$$

$$= \frac{9x+9h-9x}{h(3\sqrt{x+h}+3\sqrt{x})} = \frac{9h}{3h(\sqrt{x+h}+\sqrt{x})} = \boxed{\frac{3}{\sqrt{x+h}+\sqrt{x}}}$$

Now, $h \rightarrow 0$ is no hardship! (Calculus!)

121 §2.1 #s 97, 99

97 $g(x) = \frac{1}{x} \Longrightarrow \frac{g(x+h) - g(x)}{h}$

$$= \underbrace{\frac{\frac{1}{x+h} - \frac{1}{x}}{h}}_{\text{Divide by } h} \quad \text{OR} \quad \underbrace{\frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right)}_{\text{multiply by } \frac{1}{h}}$$

Same thing! 2^{nd} way's easier for this.

$$= \frac{1}{h}\left[\frac{1}{x+h}\cdot\frac{x}{x} - \frac{1}{x}\cdot\frac{x+h}{x+h}\right] = \frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right]$$

$$= \frac{1}{h}\left[\frac{x-x-h}{x(x+h)}\right] = \frac{1}{h}\left[\frac{-h}{x(x+h)}\right] = \boxed{\frac{-1}{x(x+h)}}$$

99 $g(x) = \frac{3}{x+2} \Longrightarrow \frac{g(x+h) - g(x)}{h}$

$$= \frac{1}{h}\left[\frac{3}{x+h+2} - \frac{3}{x+2}\right] = \frac{1}{h}\left[\frac{3(x+2) - 3(x+h+2)}{(x+2)(x+h+2)}\right]$$

$$= \frac{1}{h}\left[\frac{3x+6-3x-3h-6}{(x+2)(x+h+2)}\right] = \frac{1}{h}\left[\frac{-3h}{(x+2)(x+h+2)}\right]$$

$$= \boxed{\frac{-3}{(x+2)(x+h+2)}}$$

Let $h \to 0$ and #s 85-99 are the 1^{st} week of Calculus I!