121 §1,6

#s1–10 Find all real and imaginary solutions. Check your work!

(1) $x^4+3x^3-4x-12=0$

$x^3(x+3)-4(x+3)=0$

$x^3A-4A = A(x^3-4)$

$=(x+3)(x^2-4)$

$=(x+3)(x-2)(x+2)\stackrel{?}{=}0$

$\boxed{x\in\{-3,-2,2\}}$

Graphing Calculator's a good check. So is synthetic division for this one

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -4 & -12 \\ & & -3 & 0 & 12 \\ \hline & 1 & 0 & -4 & 0\checkmark \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & 12 \\ \hline & 1 & 5 & 6 & 0\checkmark \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -4 & -12 \\ & & -2 & -2 & 12 \\ \hline & 1 & 1 & -6 & 0\checkmark \end{array}$$

(6) $b^3+20b=9b^2$

$b^3-9b^2+20b=0$

$b(b^2-9b+20)=0$ *

$b(b-4)(b-5)=0$

$b\in\{0,4,5\}$

$b^2-9b+20$

$x^2-9x+20$

$a=1, b=-9, c=20$

$b^2-4ac=(-9)^2-4(1)(20)$

$=81-80=1$

$x=\dfrac{9\pm1}{2}$ → $\frac{10}{2}=5$, $\frac{8}{2}=4$

$(x-4)(x-5)$

$7\overline{)161}$
23

* $b(b^2-9b+20)=0$ →

$b=0$ OR $b^2-9b+20=0$

solve separately

121 §1.6

(9) $a^4-16=0$

$(a^2)^2-4^2=0$

$(a^2-4)(a^2+4)=0$

$(a-2)(a+2)(a^2+4)=0$

$a^2+4=0$

$a^2=-4$

$a=\pm\sqrt{-4}=\pm 2i$

$a\in\{\pm 2, \pm 2i\}$

or $\{-2, 2, -2i, 2i\}$

Check:

$a^4-16 = a^4+0a^3+0a^2+0a-16$

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 0 & 0 & -16 \\ & & -2 & 4 & -8 & 16 \\ \hline & 1 & -2 & 4 & -8 & 0 \checkmark \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 0 & 0 & -16 \\ & & 2 & 4 & 8 & 16 \\ \hline & 1 & 2 & 4 & 8 & 0 \checkmark \end{array}$$

$$\begin{array}{r|rrrrr} 2i & 1 & 0 & 0 & 0 & -16 \\ & & 2i & -4 & -8i & 16 \\ \hline & 1 & 2i & -4 & -8i & 0 \checkmark \end{array}$$

$(2i)(2i)=4i^2=-4$

$(-8i)(2i)=-16i^2=16$

$$\begin{array}{r|rrrrr} -2i & 1 & 0 & 0 & 0 & -16 \\ & & -2i & -4 & 8i & 16 \\ \hline & 1 & -2i & -4 & 8i & 0 \checkmark \end{array}$$

121 §1.6

12 $\sqrt{x-1} = x-7$

$(\sqrt{x-1})^2 = (x-7)^2$

$x-1 = x^2-14x+49$

$x^2-15x+50=0$

$(x-10)(x-5)=0$

$x \in \{5, 10\} \longrightarrow \boxed{x \in \{10\}}$

$\sqrt{5-1} = 5-7$?

No

$x \neq 5$

$\sqrt{10-1} = 10-7$?

$\sqrt{9} = 3$ ✓

18 $\dfrac{1}{p} - \dfrac{2}{\sqrt{9p+1}} = 0$ LCD $= p\sqrt{9p+1}$

5|65
13

$\dfrac{1}{p}\cdot\dfrac{\sqrt{9p+1}}{\sqrt{9p+1}} - \dfrac{2}{\sqrt{9p+1}}\cdot\dfrac{p}{p} = 0 = \dfrac{0}{LCD}$

$\dfrac{\sqrt{9p+1}-2p}{LCD} = \dfrac{0}{LCD}$

$\sqrt{9p+1} - 2p = 0$

$\sqrt{9p+1} = 2p$

$9p+1 = (2p)^2 = 4p^2$

$4p^2-9p-1=0$

$a=4,\ b=-9,\ c=-1$

$b^2-4ac = 81+16 = 97$

$x = \dfrac{9 \pm \sqrt{97}}{2(4)} = \dfrac{9\pm\sqrt{97}}{8}$

Tough to check by hand!

$\dfrac{1}{\frac{9+\sqrt{97}}{8}} - \dfrac{2}{\sqrt{9\left(\frac{9+\sqrt{97}}{8}\right)}} = \dfrac{8}{9+\sqrt{97}} - \dfrac{2\sqrt{8}}{\sqrt{81+9\sqrt{97}}}$

= Wow! Time for graphing utility

$\dfrac{9\pm\sqrt{97}}{8} \approx$ 2.356107, -0.106107

$p \approx 2.356107 \rightarrow$
2.11×10^{-8} which is close to zero

$p \approx -0.106107 \rightarrow -18.8489$ Not close to zero

$\boxed{x \in \left\{\dfrac{9+\sqrt{97}}{8}\right\}}$

121 §1.6

19 $\sqrt{x^2-2x-15} = 3$

$x^2-2x-15 = 9$

$x^2-2x-24 = 0$

$(x-6)(x+4) = 0$

$\boxed{x \in \{-4, 6\}}$

$\sqrt{(-4)^2-2(-4)-15}$

$= \sqrt{16+8-15}$

$= \sqrt{9} = 3$ ✓

$\sqrt{6^2-2(6)-15}$

$= \sqrt{36-12-15}$

$= \sqrt{9} = 3$ ✓

27 $\sqrt{x+40} - \sqrt{x} = 4$

$\sqrt{x+40} = \sqrt{x} + 4$

$x+40 = (\sqrt{x}+4)^2 = \sqrt{x}^2 + 2(\sqrt{x})(4) + 4^2 = x + 8\sqrt{x} + 16$

$x+40 = x + 8\sqrt{x} + 16$

$x + 8\sqrt{x} + 16 = x + 40$

$8\sqrt{x} = 24$

$\sqrt{x} = 3$

$x = 9$

$\boxed{x \in \{9\}}$

$\sqrt{9+40} - \sqrt{9}$

$= \sqrt{49} - 3 = 7 - 3 = 4$

#23 Solution shows more detail

121 §1.6

(23) $\sqrt{x} + \sqrt{x-36} = 2$

$\sqrt{x-36} = 2 - \sqrt{x}$

By inspection, this can't happen

$x - 36 = 4 - 4\sqrt{x} + x$

$x \geq 36$ is needed.

$x \geq 36$ makes RHS negative

$-36 = 4 - 4\sqrt{x}$

LHS is never negative.

Impossible.

$-40 = -4\sqrt{x}$

$\sqrt{x} = 10$

CHECK:

$x = 100$

$\sqrt{100} + \sqrt{100-36} = 2$?

$10 + 8 = 2$? Nope

$\boxed{\varnothing}$

(23) $\sqrt{x+4} + \sqrt{x-1} = 5$

$\sqrt{x+4} = 5 - \sqrt{x-1}$

$(\sqrt{x+4})^2 = (5 - \sqrt{x-1})^2$

$x + 4 = 25 - 2(5)\sqrt{x-1} + (\sqrt{x-1})^2$

$x + 4 = 25 - 10\sqrt{x-1} + x - 1$

$\sqrt{5+4} + \sqrt{5-1} \stackrel{?}{=} 5$

$4 = 24 - 10\sqrt{x-1} = 4$

$\sqrt{9} + \sqrt{4} = 5$

$-10\sqrt{x-1} = -20$

$3 + 2 = 5$ ✓

$\sqrt{x-1} = 2$ (square both)

$x - 1 = 4$

$x \in \{5\}$

$x = 5$

121 §1.6

27 $x^{2/3} = 2$

$$\left(x^{2/3}\right)^{3/2} = 2^{3/2}$$

$$x = 2^{3/2} = \left(2^3\right)^{\frac{1}{2}} = \sqrt{8} = 2\sqrt{2}$$

Seems legit, but misses a solution.

It's the $\frac{1}{2}$ power that's messing with us.

$x^{2/3} = 2$ BETTER METHOD

$\left(x^{2/3}\right)^3 = 2^3 = 8$

$x^2 = 8$

$x = \pm\sqrt{8} = \pm 2\sqrt{2}$

$\boxed{x \in \{\pm 2\sqrt{2}\}}$

$\sqrt{x^2} = \sqrt{8}$

$|x| =$

$\left(2\sqrt{2}\right)^{2/3} = 2$?

$(4 \cdot 2)^{1/3} = 2$?

$8^{1/3} = 2$?

$2 = 2$ ✓

$(-2\sqrt{2})$ works, also

2 | 8

2 | 4

2

$2\sqrt{2}$

121 §1.6

(29) $w^{-4/3} = 16$

$$\begin{array}{r} 256 \\ \times\ 16 \\ \hline 1536 \\ 2560 \\ \hline 4096 \end{array}$$

$\dfrac{1}{w^{4/3}} = 16$

$1 = 16w^{4/3}$

$16w^{4/3} = 1$

$(16w^{4/3})^3 = 1^3$

$(16^3)(w^{4/3})^3 = 1$

$4096w^4 = 1$

$w^4 = \dfrac{1}{4096}$

$$\begin{array}{r|l} 2 & 4096 \\ 2 & 2048 \\ 2 & 1024 \\ 2 & 512 \\ 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

$4096 = 2^{12}$

$(4096)^{\frac14} = \sqrt[4]{4096}$

$= (2^{12})^{\frac14} = 2^{\frac{12}{4}} = 2^3 = 8$

$\sqrt[4]{w^4} = \sqrt[4]{\dfrac{1}{4096}}$ or $(w^4)^{\frac14} = \left(\dfrac{1}{4096}\right)^{\frac14}$

$w = \pm \dfrac18$

$w \in \{\pm \frac18\}$

Check

$\left(\frac18\right)^{-\frac43} = (8)^{\frac43} = \left(8^{\frac13}\right)^4 = 2^4 = 16.$

$\left(-\frac18\right)^{-\frac43} = (-8)^{\frac43} = \left((-8)^{\frac13}\right)^4 = (-2)^4 = 16$

121 §1.6 #s 28, 31, 34, 35, 36, 39, 41, 43, 47, 53, 55, 59, 63, 65, 67, 69, 73

(28) $x^{2/3} = \frac{1}{2}$

$\left(x^{\frac{2}{3}}\right)^3 = \left(\frac{1}{2}\right)^3$

$x^2 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$x = \pm\sqrt{\frac{1}{8}} = \pm\frac{\sqrt{1}}{\sqrt{8}} = \pm\frac{1}{2\sqrt{2}} = \pm\frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{2}}{4}$

$\boxed{x \in \left\{\pm\frac{\sqrt{2}}{4}\right\}}$

Check: $\left(\frac{\sqrt{2}}{4}\right)^{2/3} = \left(\left(\frac{\sqrt{2}}{4}\right)^2\right)^{\frac{1}{3}}$

$= \left(\frac{2}{16}\right)^{\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1^{1/3}}{8^{1/3}} = \frac{1}{2}$ ✓

$-\frac{\sqrt{2}}{4}$ also checks

(31) $w^{-\frac{3}{2}} = 27$

$27^2 = 27 \cdot 27 = 3^3 \cdot 3^3 = 3^{3+3} = 3^6$

$\left(w^{-\frac{3}{2}}\right)^2 = 27^2$

$w^{-3} = 27^2 = 3^6$

$\frac{1}{w^3} = 3^6$

$w^3 = \frac{1}{3^6}$

$w = \left(\frac{1}{3^6}\right)^{\frac{1}{3}} = \frac{1^{\frac{1}{3}}}{(3^6)^{\frac{1}{3}}} = \frac{1}{3^{\frac{1}{2}}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Check: $\left(\frac{\sqrt{3}}{3}\right)^{-3/2} = \left(\frac{3}{\sqrt{3}}\right)^{\frac{3}{2}}$

$= \left(\frac{3^3}{(\sqrt{3})^3}\right)^{\frac{1}{2}} = \left(\frac{27}{3\sqrt{3}}\right)^{\frac{1}{2}} = \left(\frac{9}{\sqrt{3}}\right)^{\frac{1}{2}} =$

= Not working.

Here's the boo-boo.

121 §1.6 #s 31, 34, 35, 36, 39, 41, 43, 47, 53, 55, 59, 63, 65, 67, 69, 73

31

$$w^{-\frac{3}{2}} = 27$$

$$\frac{1}{w^{\frac{3}{2}}} = 27$$

$$w^{\frac{3}{2}} = \frac{1}{27}$$

$$\left(w^{\frac{3}{2}}\right)^2 = \left(\frac{1}{27}\right)^2$$

$$w^3 = \left(\frac{1}{3^3}\right)^2 = \frac{1}{3^6} = 3^{-6}$$

$$\left(w^3\right)^{\frac{1}{3}} = \left(3^{-6}\right)^{\frac{1}{3}}$$

$$w = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$\boxed{w \in \left\{\frac{1}{9}\right\}}$$

Check: $\left(\frac{1}{9}\right)^{-\frac{3}{2}} = \left(\left(\frac{1}{9}\right)^{-\frac{1}{2}}\right)^3$

$= \left(\left(\frac{9}{1}\right)^{\frac{1}{2}}\right)^3 = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$ ✓

34

$$(x-2)^{-\frac{1}{2}} = \frac{1}{2}$$

$(6-2)^{-\frac{1}{2}} = \frac{1}{(6-2)^{1/2}} = \frac{1}{4^{1/2}} = \frac{1}{2}$ ✓

$$(x-2)^{\frac{1}{2}} = 2$$

$$x-2 = 2^2$$

$$x = 2+4 = 6$$

$$\boxed{x \in \{6\}}$$

121 §1.6 #s 35, 36, 39, 41, 43, 47, 53, 55, 59, 63, 65, 67, 69, 73

35 #s 35–52 Find all real and imaginary solutions to each eq'n.

35 $x^4 - 12x^2 + 27 = 0$

$\Rightarrow u^2 - 12u + 27 = 0$, where $u = x^2$

$(u-9)(u-3) = 0$

$u = 9 \qquad u = 3$

$x^2 = 9 \qquad x^2 = 3$

$x = \pm 3 \qquad x = \pm\sqrt{3}$

$\cancel{9^4 - 12(9)^2 + 27}$

$\cancel{= 9^2(9^2 - 12) + 27}$

$\cancel{= 81(81 - 12) + 27}$

$\cancel{= 81(}$

$3^4 - 12(3^2) + 27 =$

$3^4 - 3^3 \cdot 4 + 3^3 =$

$3^3(3 - 4 + 1) = 27(0) = 0$ ✓

same for -3

$(\sqrt{3})^4 - 12(\sqrt{3})^2 + 27 =$

$= 9 - 36 + 27 = 36 - 36 = 0$ ✓

same for $(-\sqrt{3})$.

$$\boxed{x \in \{\pm 3, \pm\sqrt{3}\}}$$

$= \{-3, -\sqrt{3}, \sqrt{3}, 3\}$ if listed left-to-right on # line.

121 §1.6 #s 36, 39, 41, 43, 47, 53, 55, 59, 63, 65, 67, 69, 73

36 $x^4+10=7x^2$

$x^4-7x^2+10=0$

$u^2-7u+10=0$

$(u-5)(u-2)=0$

$u=5 \quad u=2$

$x^2=5 \quad x^2=2$

$x=\pm\sqrt{5},\ x=\pm\sqrt{2}$

$\boxed{x\in\{\pm\sqrt{5},\ \pm\sqrt{2}\}}$

$(\sqrt{5})^4+10=7(\sqrt{5})^2$?

$25+10=35$ ✓

39 $x^4-81=0$

$u^2-81=0$

$(u-9)(u+9)=0 \longrightarrow (x^2-9)(x^2+9)=0$

$(x-3)(x+3)(x-3i)(x+3i)=0$

etc.

$u=9 \quad u=-9$

$x^2=9 \quad x^2=-9$

$x=\pm3 \quad x=\pm\sqrt{-9}=\pm3i$

$x\in\{\pm3,\pm3i\}$

$3^4-81=0$ ✓

$(3i)^4-81=(3^4)(i)^4=81(i^2)^2=81(-1)^2=81$ ✓

121 §1.6 #s 41, 43, 47, 53, 55, 59, 63, 65, 67, 69, 73

41 $\left(\frac{2x-3}{5}\right)^2 + 2\left(\frac{2x-3}{5}\right) = 8$

$u^2 + 2u = 8$

$u^2 + 2u - 8 = 0$

$(u+4)(u-2) = 0$

$u = -4 \qquad u = 2$

$\frac{2x-3}{5} = -4 \qquad \frac{2x-3}{5} = 2$

$2x - 3 = -20 \qquad 2x - 3 = 10$

$2x = -17 \qquad 2x = 13$

$x = -\frac{17}{2} \qquad x = \frac{13}{2}$

$\boxed{x \in \left\{-\frac{17}{2}, \frac{13}{2}\right\}}$

$\left(\frac{2\left(\frac{-17}{2}\right) - 3}{5}\right)^2 + 2\left(\frac{2\left(\frac{-17}{2}\right) - 3}{5}\right)$

$= \left(\frac{-17-3}{5}\right)^2 + 2\left(\frac{-17-3}{5}\right)$

$= \left(\frac{-20}{5}\right)^2 + 2\left(\frac{-20}{5}\right)$

$= (-4)^2 - 8 = 16 - 8 = 8$ ✓

~~$= \frac{196 + 140}{25} \neq 8$ Nope.~~

43 $\frac{1}{(5x-1)^2} + \frac{1}{5x-1} - 12 = 0$

$u^2 + u - 12 = 0$

$(u+4)(u-3) = 0$

$u = -4 \quad u = 3$

$\frac{1}{5x-1} = -4 \qquad \frac{1}{5x-1} = 3$

$1 = -20x + 4 \qquad 1 = 15x - 3$

$20x = 3 \qquad 15x = -4$

$x = \frac{20}{3} \qquad x = -\frac{4}{15}$

Nope! Upside-Down!

$\frac{1}{\left(5\left(\frac{20}{3}\right) - 1\right)^2} + \frac{1}{5\left(\frac{20}{3}\right) - 1} - 12$

$= \frac{1}{\left(\frac{100}{3} - \frac{3}{3}\right)^2} + \frac{1}{\frac{100-3}{3}} - 12$

$= \frac{1}{\left(\frac{97}{3}\right)^2} + \frac{1}{\frac{97}{3}} - 12$

$= \frac{9}{97^2} + \frac{3}{97} \cdot \frac{97}{97} - 12$

$= \frac{9 + 291}{97^2} - 12$

121 §1.6 #s 43, 47, 53, 55, 59, 63, 65, 67, 69, 73

43

$20x = 3$

$x = \frac{3}{20}$

$$\frac{1}{\left(5\left(\frac{3}{20}\right)-1\right)^2} + \frac{1}{5\left(\frac{3}{20}\right)-1} - 12$$

$$= \left(\frac{1}{\frac{3}{4}-\frac{4}{4}}\right)^2 + \frac{1}{\frac{3}{4}-\frac{4}{4}} - 12$$

$$= \frac{1}{\left(-\frac{1}{4}\right)^2} + \frac{1}{-\frac{1}{4}} - 12$$

$$= \frac{1}{\left(\frac{1}{16}\right)} - 4 - 12 = 16 - 4 = 12 \checkmark$$

47

$x - 4\sqrt{x} + 3 = 0$

$u^2 - 4u + 3 = 0 \quad (u = \sqrt{x})$

$(u-3)(u-1) = 0$

$u = 3 \qquad u = 1$

$\sqrt{x} = 3 \qquad \sqrt{x} = 1$

$x = 9 \qquad x = 1$

$\boxed{x \in \{1, 9\}}$

$1 - 4 + 3 = 0 \checkmark$

$9 - 12 + 3 = 0 \checkmark$

53

#s 53–64 Solve each absolute value equation

$|A| = B \Rightarrow A = \pm B$

$|A| = |B| \Rightarrow A = \pm B$, also!

121 §11.6 #s 53, 55, 59, 63, 65, 67, 69, 73

53 $|w^2-4| = 3$

$w^2-4 = 3$ $\qquad$ $w^2-4 = -3$

$w^2 = 7$ $\qquad$ $w^2 = 1$

$w = \pm\sqrt{7}$ $\qquad$ $w = \pm 1$

$\boxed{w \in \{\pm 1, \pm\sqrt{7}\}}$

$|1^2-4| = |-3| = 3$ ✓

$|(-1)^2-4| = |1-4| = 3$

$|(\sqrt{7})^2-4| = |7-4| = 3$ ✓

$|(-\sqrt{7})^2-4| = |7-4| = 3$ ✓

55 $|v^2-3v| = 5v$

$v^2-3v = 5v$ $\qquad$ $v^2-3v = -5v$

$v^2-8v = 0$ $\qquad$ $v^2+2v = 0$

$v(v-8) = 0$ $\qquad$ $v(v+2) = 0$

$v = 0, 8$ $\qquad$ $v = 0, -2$

$\boxed{v \in \{\cancel{-2}, 0, 8\}}$

$|2^2-3(2)| = |4-6| = -2$

$= 5(2)$? No

$\boxed{v=-2 \text{ extraneous}}$

59 $|x+5| = |2x+1|$

$x+5 = 2x+1$ $\qquad$ $x+5 = -(2x+1)$

$-x = -4$ $\qquad$ $x+5 = -2x-1$

$x = 4$ $\qquad$ $3x = -6$

$x = -2$

$\boxed{x \in \{-2, 4\}}$

$|4+5| = |2(4)+1|$?

$9 = 9$ ✓

$|-2+5| = |2(-2)+1|$?

$|3| = |-3|$ ✓

121 §1.6 #s 63, 65, 67, 69, 73

63 $|x-4| = |x-2|$

$x-4 = x-2$ OR $x-4 = -x+2$

$0 = 2$ $\quad$ $2x = 6$

$x = 3$

$\boxed{x \in \{3\}}$

$|3-4| = |-1| = 1$

$|3-2| = |1| = 1$ ✓

65 #s 65–84 Solve eq'ns. Find imaginary solutions, when possible.

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

65 $\sqrt{16x+1} - \sqrt{6x+13} = -1$

$\sqrt{16x+1} = \sqrt{6x+13} - 1$

$16x+1 = 6x+13 - 2\sqrt{6x+13}(1) + 1^2$

$16x+1 = 6x+14 - 2\sqrt{6x+13}$

$+2\sqrt{6x+13} - 16x - 1 = -16x - 1 + 2\sqrt{6x+13}$

$+2\sqrt{6x+13} = -10x + 13$

$\boxed{x \in \{\frac{1}{2}, \cancel{\frac{117}{50}}\}}$

$2\sqrt{6x+13} = -10x+13$

$4(6x+13) = 100x^2 - 2(10x)(13) + 169$

$24x + 52 = 100x^2 - 260x + 169$

$100x^2 - 284x + 117$

$a=100, b=-284, c=117$

$169 - 52 = 117$

$2 \,|\, 33856$, $2 \,|\, 16928$, $2 \,|\, 8464$, $2 \,|\, 4232$, $2 \,|\, 2116$, $2 \,|\, 1058$, $23 \,|\, 529$, 23

$2^6 \cdot 23^2$

$b^2-4ac = 284^2 - 4(100)(117)$

$= 80656 - 46800$

$= 33856$

$\sqrt{b^2-4ac} = \sqrt{2^6 \cdot 23^2}$

$= 2^3 \cdot 23^1 = 8(23) = 184$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{284 \pm 184}{2(100)}$

$\frac{468}{200} = \frac{117}{50}$

$\frac{100}{200} = \frac{1}{2}$

The $\frac{117}{50}$ doesn't check. $\frac{1}{2}$ Does? $\sqrt{8+1} - \sqrt{16}$

$= \sqrt{9} - 4 = 3 - 4 = -1$

121 §1.6 #s 67, 69, 73

67 $v^6 - 64 = 0$

$v^6 = (v^2)^3$ OR $(v^3)^2$

Difference of cubes approach

Difference of squares approach

$(v^2)^3 - 64 = 0$

$(v^2)^3 - 4^3 = 0$ Diff of cubes

$(v^2 - 4)((v^2)^2 + 4v^2 + 4^2) = 0$

$(v-2)(v^4 - 4v^2 + 16) = 0$

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

$v - 2 = 0$ OR $v^4 - 4v^2 + 16 = 0$

$v = 2$

$u^2 - 4u + 16 = 0$

$u^2 - 4u = -16$

$u^2 - 4u + 2^2 = -16 + 4$

$(u-2)^2 = -12$

$u - 2 = \pm\sqrt{-12} = \pm 2i\sqrt{3}$

$u = 2 \pm 2i\sqrt{3}$

$v^2 = 2 \pm 2i\sqrt{3}$ & we don't know enough about complex #s to evaluate $\pm\sqrt{2 \pm 2i\sqrt{3}}$

121 §1.6 # s 67, 69, 73

67 Diff of squares approach

$v^6 - 64 = 0$ $\qquad u = v^3$

$(v^3)^2 - 8^2 = 0$ $\qquad u^2 - 8^2$

$(v^3 - 8)(v^3 + 8) = 0$ $\qquad (u-8)(u+8)$

$(v-2)(v^2+2v+4)(v+2)(v^2-2v+4)$

$v = \pm 2$, $\quad v^2+2v+4=0$ OR $v^2-2v+4=0$

$v^2+2v = -4$ $\qquad v^2-2v = -4$

$v^2+2v+1^2 = -4+1$ $\qquad v^2-2v+1^2 = -4+1^2$

$(v+1)^2 = -3$ $\qquad (v-1)^2 = -3$

$v+1 = \pm\sqrt{3}\,i$ $\qquad v-1 = \pm\sqrt{3}\,i$

$v = -1 \pm \sqrt{3}\,i$ $\qquad v = 1 \pm \sqrt{3}\,i$

$\boxed{v \in \{\pm 2, -1 \pm i\sqrt{3}, 1 \pm i\sqrt{3}\}}$

THESE ARE NOT IMAGINARY SOL'NS. THEY'RE COMPLEX.

I put "i" in front of the radical, some times, so it ~~never~~ won't look like it's an "i" INSIDE the radical, which we don't yet know how to handle (MAT 122)

121 §1.6 #s 69, 73

69 $(7x^2-12)^{\frac{1}{4}} = x$

$7x^2-12 = x^4$

$x^4-7x^2+12=0$

$u^2-7u+12=0$

$(u-3)(u-4)=0$

$u=3 \qquad u=4$

$x^2=3 \qquad x^2=4$

$x=\pm\sqrt{3} \qquad x=\pm 2$

$\cancel{\boxed{x \in \{\pm\sqrt{3}, \pm 2\}}}$

$(7(\sqrt{3})^2-12)^{\frac{1}{4}} = (21-12)^{\frac{1}{4}}$

$= 9^{\frac{1}{4}} = (3^2)^{\frac{1}{4}} = 3^{\frac{1}{2}} = \sqrt{3}$ ✓

But $3^{\frac{1}{2}} \neq -\sqrt{3}$ so $-\sqrt{3}$ is out

Same deal for $x=-2$

~~Final~~ ans:

$\boxed{x \in \{\sqrt{3}, 2\}}$

73 $\left(\frac{x-2}{3}\right)^2 - 2\left(\frac{x-2}{3}\right) + 10 = 0$

$u^2-2u+10=0$

$u^2-2u = -10$

$u^2-2u+1^2 = -10+1$

$(u-1)^2=-9$

$u-1 = \pm 3i$

$u = \frac{x-2}{3} = 1 \pm 3i$

$x-2 = 3 \pm 9i$

$x = 5 \pm 9i$

Really painful to check <u>this</u>.

$\left(\frac{5+9i-2}{3}\right)^2 - 2\left(\frac{5+9i-2}{3}\right)+10$

$= \left(\frac{3+9i}{3}\right)^2 + \cancel{\ldots} \quad \cancel{=\left(\frac{3(1+3i)}{3}\right)^2} \text{ +em}$

$= (1+3i)^2 - 2(1+3i) + 10$

$= 1^2 + 2(3i) + (3i)^2 - 2 - 6i + 10$

$= 1+6i-9-2-6i+10 = 0$ ✓

$\cancel{(1-3i)^2-2(1-3i)+10}$

$5-9i$ works, because $5+9i$ worked, plus I know the Conjugate Pairs Theorem! (Chapter 3)