12.1 §1.5 I #s 1-14 ALL, [redacted]

See Special Instructions #s 5-14, §1.5 I, [redacted]

" " #s 51-56, §1.5 II

#s 5-14 Special Instructions

Solve each quadratic equation in three ways:

(A) Quadratic Formula

(B) Completing the square

(C) Factoring *

* You may use (A) or (B) as a "cheat" for (C)

(5) $x^2 - x - 20 = 0$

(A) $a = 1, b = -1, c = -20$

$b^2 - 4ac = (-1)^2 - 4(1)(-20)$

$= 1 + 80$

$= 81$

$\rightarrow \sqrt{b^2 - 4ac} = \sqrt{81} = 9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{1 \pm 9}{2(1)} = \begin{cases} \frac{1+9}{2} = \frac{10}{2} = 5 \\ \frac{1-9}{2} = -\frac{8}{2} = -4 \end{cases}$

$x \in \{-4, 5\}$

(B) $x^2 - x = 20$

$x^2 - x + \left(\frac{1}{2}\right)^2 = 20 + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{81}{4}$

$x - \frac{1}{2} = \pm\sqrt{\frac{81}{4}} = \pm\frac{9}{2}$

$x = \frac{1}{2} \pm \frac{9}{2} \begin{cases} \nearrow \frac{10}{2} \\ \searrow -\frac{8}{2} \end{cases}$

$x \in \{-4, 5\}$

(C) $(x-5)(x+4) = 0$

$x - 5 = 0$ OR $x + 4 = 0$

$x = 5$ OR $x = -4$

$x \in \{-4, 5\}$

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(6) $x^2+2x-8=0$

(A) $a=1, b=2, c=-8$

$b^2-4ac = 2^2-4(1)(-8)$

$= 4+32 = 36$

$\sqrt{b^2-4ac} = \sqrt{36} = 6$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-2 \pm 6}{2(1)}$ → $\frac{-2+6}{2} = 2$, $\frac{-2-6}{2} = -4$

$x \in \{-4, 2\}$

(B) $x^2+2x-8=0$

$x^2+2x = 8$

$x^2+2x+1^2 = 8+1$

$(x+1)^2 = 9$

$x+1 = \pm 3$

$x = -1 \pm 3$ → 2, −4

$x \in \{-4, 2\}$

(C) $(x+4)(x-2)=0$

$x \in \{-4, 2\}$

(7) (A) $x^2+3x = -2$

$x^2+3x+2=0$

$a=1, b=3, c=2$

$b^2-4ac = 3^2-4(1)(2)$

$= 9-8 = 1$

$\sqrt{b^2-4ac} = \sqrt{1} = 1$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-3 \pm 1}{2}$ → $\frac{-3+1}{2} = -\frac{2}{2}$, $\frac{-3-1}{2} = -\frac{4}{2}$

$x \in \{-2, -1\}$

(B) $x^2+3x = -2$

$x^2+3x+\left(\frac{3}{2}\right)^2 = -2+\frac{9}{4}$

$\left(x+\frac{3}{2}\right)^2 = -\frac{8}{4}+\frac{9}{4} = \frac{1}{4}$

$x+\frac{3}{2} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$x = -\frac{3}{2} \pm \frac{1}{2}$

$x \in \{-2, -1\}$

(C) $(x+2)(x+1)=0$

$x \in \{-1, -2\}$

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(8) $x^2-4x=12$

$x^2-4x-12=0$

(A) $a=1, b=-4, c=-12$

$b^2-4ac=(-4)^2-4(1)(-12)$

$=16+48$

$=64$

$\sqrt{b^2-4ac}=\sqrt{64}=8$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-4)\pm 8}{2(1)}=\frac{4\pm 8}{2}$

$x\in\{-2,6\}$

(B) $x^2-4x=12$

$x^2-4x+2^2=12+4$

$(x-2)^2=16$

$x-2=\pm\sqrt{16}=\pm 4$

$x=2\pm 4$

$x\in\{-2,6\}$

(C) $x^2-4x-12=0$

$(x-6)(x+2)=0$

$x\in\{-2,6\}$

(9) $2x^2-5x-3=0$

(A) $a=2, b=-5, c=-3$

$b^2-4ac=(-5)^2-4(2)(-3)$

$=25+24=49$

$\sqrt{b^2-4ac}=\sqrt{49}=7$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-5)\pm 7}{2(2)}=\frac{5\pm 7}{4}$ → $\frac{12}{4}$, $-\frac{2}{4}$

$x\in\{-\frac{1}{2},3\}$

(B) $2x^2-5x-3=0$

$x^2-\frac{5}{2}x-\frac{3}{2}=0$

$x^2-\frac{5}{2}x+\quad=\frac{3}{2}$

$x^2-\frac{5}{2}x+\left(\frac{5}{4}\right)^2=\frac{3}{2}+\frac{25}{16}$

$\left(x-\frac{5}{4}\right)^2=\frac{24+25}{16}=\frac{49}{16}$

$x-\frac{5}{4}=\pm\sqrt{\frac{49}{16}}=\pm\frac{7}{4}$

$x=\frac{5}{4}\pm\frac{7}{4}$ → $\frac{5+7}{4}=\frac{12}{4}$, $\frac{5-7}{4}=-\frac{2}{4}$

$x\in\{-\frac{1}{2},3\}$

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(9) (C) $2x^2 - 5x - 3 = 0$

$a = 2, b = -5, c = -3$

Want factors of ac whose sum is -5

$(2)(-3) = -6 = (-6)(1)$

$-6 + 1 = -5$ ✓

$2x^2 - 6x + 1x - 3 = 0$

$2x(x-3) + 1(x-3) = 0$

$(x-3)(2x+1) = 0$

$x - 3 = 0$ OR $2x + 1 = 0$

$x = 3$ OR $2x = -1$

$x = -\frac{1}{2}$

$x \in \{-\frac{1}{2}, 3\}$

CHEAT: (A) & (B) tell us that $x = -\frac{1}{2}$ & $x = 3$ are zeros, so

$2x^2 - 5x - 3 = 2(x - (-\frac{1}{2}))(x - 3)$

$= 2(x + \frac{1}{2})(x - 3)$ Bring the "2" inside

$= (2x + 1)(x - 3)$ & voila! It looks like we factored it!

Stick the leading coefficient out front

FACTORED using Quadratic Formula!

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(4) $2x^2-5x+2=0$

(A) $a=2, b=-5, c=2$

$b^2-4ac=(-5)^2-4(2)(2)$

$=25-16=9$

$\sqrt{b^2-4ac}=\sqrt{9}=3$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{5\pm3}{2(2)}$ → $\frac{5+3}{4}=\frac{8}{4}$, $\frac{5-3}{4}=\frac{2}{4}$

$x\in\{\frac{1}{2},2\}$

(B) $2x^2-5x+2=0$

$x^2-\frac{5}{2}x+1=0$

$x^2-\frac{5}{2}x=-1$

$x^2-\frac{5}{2}x+\left(\frac{5}{4}\right)^2=-1+\frac{25}{16}$

$\left(x-\frac{5}{4}\right)^2=\frac{-16+25}{16}=\frac{9}{16}$

$x-\frac{5}{4}=\pm\sqrt{\frac{9}{16}}=\pm\frac{3}{4}$

$x=\frac{5}{4}\pm\frac{3}{4}$ → $\frac{8}{4}$, $\frac{2}{4}$

$x\in\{\frac{1}{2},2\}$

(C) $2x^2-5x+2=0$

$2(x-\frac{1}{2})(x-2)=0$

$(2x-1)(x-2)=0$ → shows you can factor!

$x\in\{\frac{1}{2},2\}$

121 §1.5 $a=6, b=-7, c=2$

(A) M1 $6x^2-7x+2=0$

$b^2-4ac=(-7)^2-4(6)(2)$

$=49-48=1$

$\sqrt{b^2-4ac}=\sqrt{1}=1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{7\pm1}{2(6)}=\begin{cases}\frac{7+1}{12}=\frac{8}{12}\\ \frac{7-1}{12}=\frac{6}{12}\end{cases}$

$x\in\{\frac{1}{2},\frac{2}{3}\}$

(B) $6x^2-7x=-2$

$x^2-\frac{7}{6}x=-\frac{1}{3}$

$x^2-\frac{7}{6}x+\left(\frac{7}{12}\right)^2=-\frac{1}{3}+\frac{49}{144}$

$\left(x-\frac{7}{12}\right)^2=\frac{-48+49}{144}=\frac{1}{144}$

$x-\frac{7}{12}=\pm\sqrt{\frac{1}{144}}=\pm\frac{1}{12}$

$x=\frac{7}{12}\pm\frac{1}{12}\begin{cases}\nearrow\frac{8}{12}\\ \searrow\frac{6}{12}\end{cases}$

$x\in\{\frac{1}{2},\frac{2}{3}\}$

(C) M1 Factors of 12 whose sum is -7

$(-4)(-3)=+12$

$-4-3=-7$ ✓

$6x^2-4x-3x+2=0$

$2x(3x-2)-1(3x-2)=0$

$(3x-2)(2x-1)=0$

$x\in\{\frac{1}{2},\frac{2}{3}\}$

M2 Use previous work in (A) & (B):

$6(x-\frac{1}{2})(x-\frac{2}{3})$

$=2\cdot3(x-\frac{1}{2})(x-\frac{2}{3})$

$=2(x-\frac{1}{2})(3)(x-\frac{2}{3})$

$=(2x-1)(3x-2)=0$

$\Rightarrow x\in\{\frac{1}{2},\frac{2}{3}\}$

FACTORED BY FACTOR THEOREM & A LITTLE MANIPULATION!

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(12) $12x^2 - 17x + 6 = 0$

$a = 12, b = -17, c = 6$

$b^2 - 4ac = (-17)^2 - 4(12)(6)$

$= 289 - 288 = 1$

$\sqrt{b^2 - 4ac} = \sqrt{1} = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{17 \pm 1}{2(12)}$ → $\frac{18}{24} = \frac{3}{4}$, ↓ $\frac{16}{24} = \frac{2}{3}$

$x \in \{\frac{2}{3}, \frac{3}{4}\}$

$(10+7)(10+7)$

$= 100 + 140 + 49$

$48 \cdot 6 = 288$

$24 \cdot 24 = 96 \cdot 48$

(B) $12x^2 - 17x = -6$

$x^2 - \frac{17}{12}x = -\frac{1}{2}$

$x^2 - \frac{17}{12}x + \left(\frac{17}{24}\right)^2 = -\frac{1}{2} + \frac{289}{576}$

$\left(x - \frac{17}{24}\right)^2 = \frac{-288 + 289}{576} = \frac{1}{576}$

$x - \frac{17}{24} = \pm\sqrt{\frac{1}{576}} = \pm\frac{1}{24}$

$x = \frac{17 \pm 1}{24}$ → $\frac{18}{24}$, ↓ $\frac{16}{24}$

$x \in \{\frac{2}{3}, \frac{3}{4}\}$

FACTOR By Parts A & B,

we have $12x^2 - 17x + 6$

$= 12\left(x - \frac{2}{3}\right)\left(x - \frac{3}{4}\right)$

$= 3 \cdot 4\left(x - \frac{2}{3}\right)\left(x - \frac{3}{4}\right)$

$= 3\left(x - \frac{2}{3}\right)\left(4\left(x - \frac{3}{4}\right)\right)$ → See?

$= (3x - 2)(4x - 3) = 0$ →

$x \in \{\frac{2}{3}, \frac{3}{4}\}$

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(13) $(x-3)(x+4)=30$

(A) $x^2+x-12=30$

$x^2+x-42=0$

$a=1, b=1, c=-42$

$b^2-4ac = 1^2-4(1)(-42)$

$=1+168=169$

$\sqrt{b^2-4ac} = \sqrt{169} = 13$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-1\pm 13}{2}$ → $\frac{-1+13}{2}=\frac{12}{2}=6$, $\frac{-1-13}{2}=\frac{-14}{2}=-7$

$x\in\{-7,6\}$

(B)

$x^2+x = 12$

$x^2+x+\left(\frac{1}{2}\right)^2 = 12+\frac{1}{4}$

$\left(x+\frac{1}{2}\right)^2 = \frac{48}{4}+\frac{1}{4} = \frac{49}{4}$

$x+\frac{1}{2} = \pm\sqrt{\frac{49}{4}} = \pm\frac{7}{2}$

$x = -\frac{1}{2}\pm\frac{7}{2}$ → 6

(B) $x^2+x-12=30$

$x^2+x = 42$

$x^2+x+\left(\frac{1}{2}\right)^2 = 42+\frac{1}{4}$

$\left(x+\frac{1}{2}\right)^2 = \frac{168}{4}+\frac{1}{4} = \frac{169}{4}$

$x+\frac{1}{2} = \pm\sqrt{\frac{169}{4}} = \pm\frac{13}{2}$

$x=\frac{-1\pm 13}{2}$ → $\frac{12}{2}$, $-\frac{14}{2}$

$x\in\{-7,6\}$

(C) By "Cheat",

x^2+x-42

$=(x+7)(x-6)=0$

$x\in\{-7,6\}$

ALTERNATE = FACTORS of -42 whose sum is $+1$

$(-7)(6)=-42$

$-7+6=-1$

$x^2-7x+6x-42$

$x(x-7)+6(x-7)$

$=(x-7)(x+6)=0$

$x\in\{-7,6\}$

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(14) $(w-1)(w-2)=6$

(A) $(x-1)(x-2)=6$

$x^2-3x+2=6$

$x^2-3x-4=0$

$a=1, b=-3, c=-4$

$b^2-4ac=(-3)^2-4(1)(-4)$

$=9+16=25$

$\sqrt{b^2-4ac}=\sqrt{25}=5$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{3\pm5}{2}$ → $\frac{8}{2}$, $-\frac{2}{2}$

$x\in\{-1,4\}$

(B) $(x-1)(x-2)=6$

$x^2-3x+2=6$

$x^2-3x=4$

$x^2-3x+\left(\frac{3}{2}\right)^2=4+\frac{9}{4}$

$\left(x-\frac{3}{2}\right)^2=\frac{16+9}{4}=\frac{25}{4}$

$x-\frac{3}{2}=\pm\sqrt{\frac{25}{4}}=\pm\frac{5}{2}$

$x=\frac{3}{2}\pm\frac{5}{2}$ → $\frac{8}{2}$, $-\frac{2}{2}$

$x\in\{-1,4\}$

(C) Using "cheat" ↓

$x=-1$ is zero $x-(-1)=x+1$ is factor.

$x=4$ is zero $x-4$ is factor

$x^2-3x-4=(x+1)(x-4)$ SET $=0$

$x\in\{-1,4\}$