

121 WP #3 (TEST 3 TAKE-HOME) Spring 2018

$$f(x) = 2x^5 - 19x^4 + 166x^3 - 457x^2 + 482x - 174$$

①  $2x^5$

② Descartes'

$$2x^5 - 19x^4 + 166x^3 - 457x^2 + 482x - 174$$

$\underbrace{\hspace{1.5cm}}_1 \quad \underbrace{\hspace{1.5cm}}_2 \quad \underbrace{\hspace{1.5cm}}_3 \quad \underbrace{\hspace{1.5cm}}_4 \quad \underbrace{\hspace{1.5cm}}_5$

5, 3 or 1 positive roots

$$f(-x) = -2x^5 - 19x^4 - 166x^3 - 457x^2 - 482x - 174$$

0 negative zeros

③ Rational zeros

$$\frac{p}{q} \leq \frac{174}{2}$$

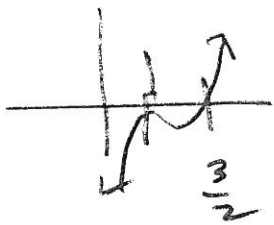
$$\begin{array}{r}
 2 \overline{) 174} \\
 3 \overline{) 87} \\
 \underline{87} \\
 0
 \end{array}$$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 29, \pm 58, \pm 87, \pm 174$   
(12)

$\pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{3}{2}, \pm \frac{6}{2}, \pm \frac{29}{2}, \pm \frac{58}{2}, \pm \frac{87}{2}, \pm \frac{174}{2}$

$\pm \frac{58}{2}, \pm \frac{87}{2}$

(4)



is what it looks like

$$\begin{array}{r}
 \underline{1} \mid 2 \quad -19 \quad 166 \quad -457 \quad 482 \quad -174 \\
 \phantom{\underline{1} \mid} \phantom{2} \phantom{-19} \phantom{166} \phantom{-457} \phantom{482} \phantom{-174} \\
 \phantom{\underline{1} \mid} \phantom{2} \phantom{-19} 2 \phantom{166} \phantom{-457} 149 \phantom{482} \phantom{-174} \\
 \hline
 \underline{1} \mid 2 \quad -17 \quad 149 \quad -308 \quad 174 \quad 0 \\
 \phantom{\underline{1} \mid} \phantom{2} \phantom{-17} \phantom{149} \phantom{-308} \phantom{174} \phantom{0} \\
 \phantom{\underline{1} \mid} \phantom{2} \phantom{-17} 3 \phantom{149} \phantom{-308} 134 \phantom{174} \phantom{0} \\
 \hline
 \underline{2/3} \mid 2 \quad -15 \quad 134 \quad -174 \quad 0 \\
 \phantom{\underline{2/3} \mid} \phantom{2} \phantom{-15} \phantom{134} \phantom{-174} \phantom{0} \\
 \phantom{\underline{2/3} \mid} \phantom{2} \phantom{-15} 3 \phantom{134} \phantom{-174} 174 \\
 \hline
 2 \quad -12 \quad 116 \quad 0
 \end{array}$$

$$\hookrightarrow 2x^2 - 12x + 116 = 0$$

$$x^2 - 6x + 58 = 0$$

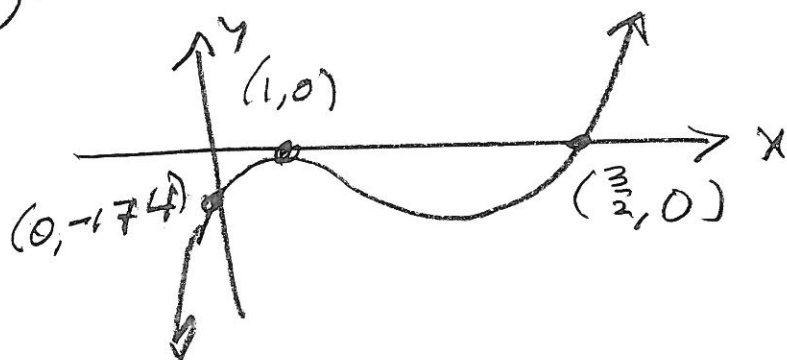
$b^2 - 4ac = (-6)^2 - 4(1)(58)$  is clearly negative,

so no real solns.

$$x = 1, m = 2; \quad x = \frac{3}{2}, m = 1$$

$$f(x) = (x-1)^2 \left(x - \frac{3}{2}\right) (2x^2 - 12x + 116)$$

6



7  $2x^2 - 12x + 116 = 0$

$\Rightarrow x^2 - 6x + 58 = 0$

$a=1, b=-6, c=58$

$b^2 - 4ac = (-6)^2 - 4(1)(58)$

$= 36 - 232 = -196$

$$\begin{array}{r} 2 \overline{) 196} \\ 2 \overline{) 98} \\ 7 \overline{) 49} \\ 7 \end{array}$$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{6 \pm \sqrt{-196}}{2(1)} = \frac{6 \pm 14i}{2} = 3 \pm 7i = x$

$f(x) = 2(x-1)^2(x-\frac{3}{2})(x-(3+7i))(x-(3-7i))$

$x^2 - 6x + 58 = 0$

$x-3 = \pm\sqrt{-49} = \pm 7i$

$x^2 - 6x + 3^2 = -58 + 9$

$x = 3 \pm 7i$

$(x-3)^2 = -49$

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UP 3

$$\textcircled{8} R(x) = \frac{2x^2x - 15}{x^2 - 4x - 21} = \frac{(2x+5)(x-3)}{(x-7)(x+3)}$$

$$D: R \setminus \{-3, 7\}$$

$$V.A.: x = -3, x = 7$$

$$x\text{-int: } \left(-\frac{5}{2}, 0\right) \\ (3, 0)$$

$$y\text{-int: } \frac{15}{21} = \frac{5}{7} \\ \left(0, \frac{5}{7}\right)$$

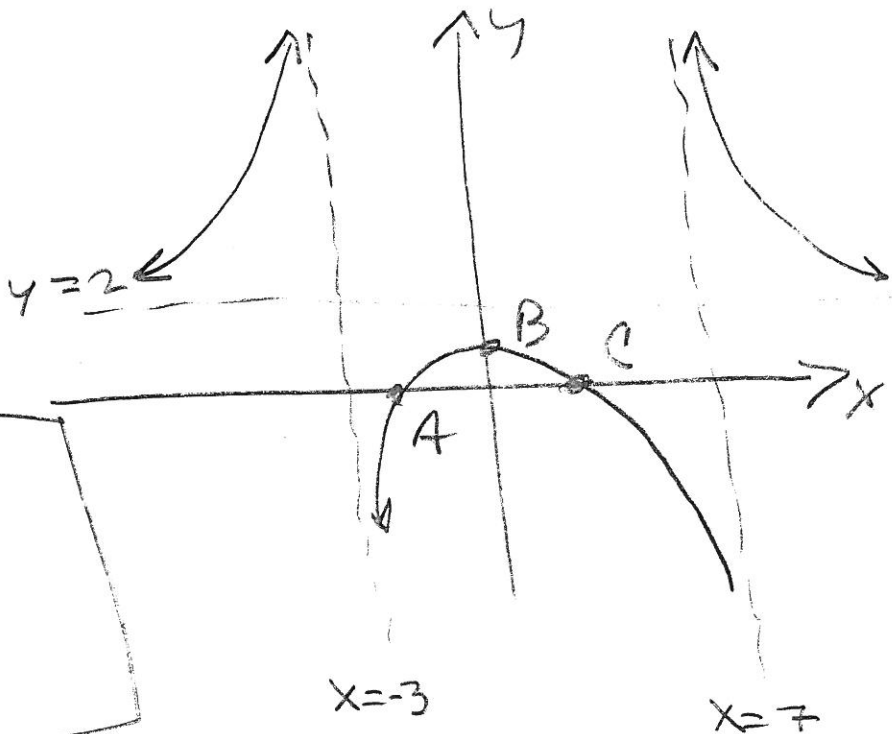
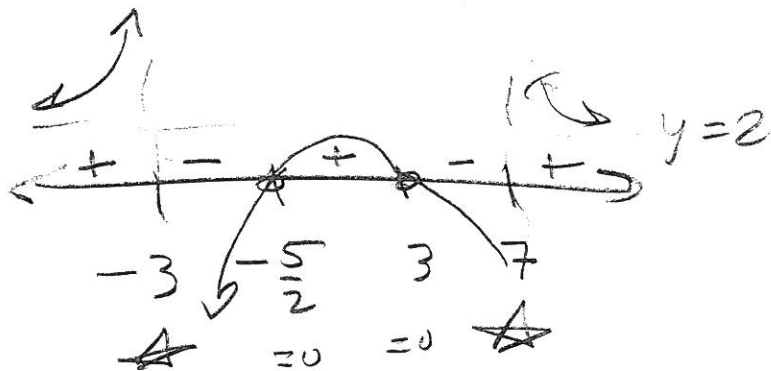
$$E.B.: \frac{2x^2}{x^2} = 2$$

$$y = 2 \text{ H.A.}$$

$$A = \left(-\frac{5}{2}, 0\right)$$

$$B = \left(0, \frac{5}{7}\right)$$

$$C = (3, 0)$$



Bonus

$$R(x) \stackrel{\text{SFT}}{=} 2 = \text{H.A.}$$

$$\text{LCD} = (x+3)(x-7)$$

$$\frac{2x^2 - x - 15}{(x-7)(x+3)} = \frac{2}{1} \cdot \frac{x^2 - 4x - 21}{x^2 - 4x - 21}$$

$$\frac{2x^2 - x - 15}{\text{LCD}} = \frac{2x^2 - 8x - 42}{\text{LCD}}$$

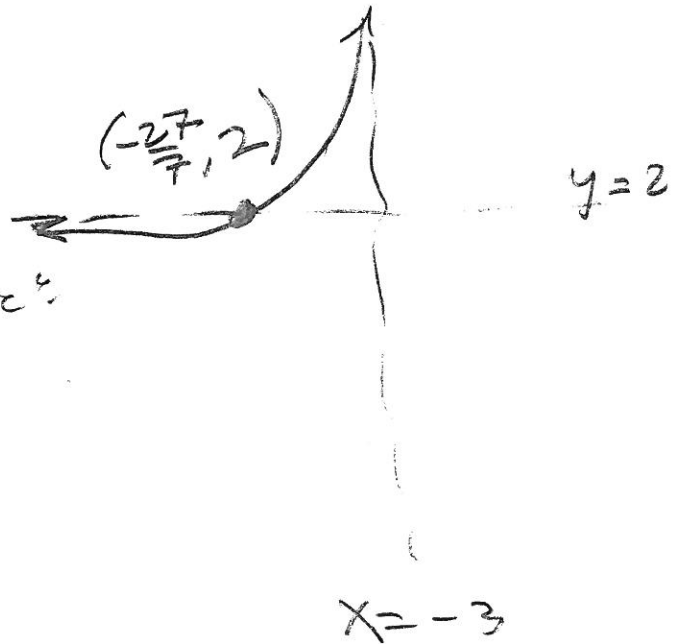
$$2x^2 - x - 15 = 2x^2 - 8x - 42$$

$$-x - 15 = -8x - 42$$

$$7x = -27$$

$$x = \frac{-27}{7}$$

P.S:



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$$\textcircled{9} \quad Q(x) = \frac{2x^3 - 23x^2 - 4x + 165}{x^3 - 15x^2 + 23x + 231}$$

Look for the hole!

$$Q(x) = R(x) \left( \frac{x-c}{x-c} \right) \quad \text{Find } x-c.$$

$$x^3 - 15x^2 + 23x + 231 = (x-7)(x+3)(x-c)$$

$$\begin{array}{r|rrrr} 7 & 1 & -15 & 23 & 231 \\ & & 7 & -56 & -231 \\ \hline & 1 & -8 & -33 & 0 \\ -3 & 1 & -8 & -33 & 0 \\ & & 3 & 33 & \end{array}$$

$$\textcircled{1 \quad -11} \rightarrow x-c = x-11$$

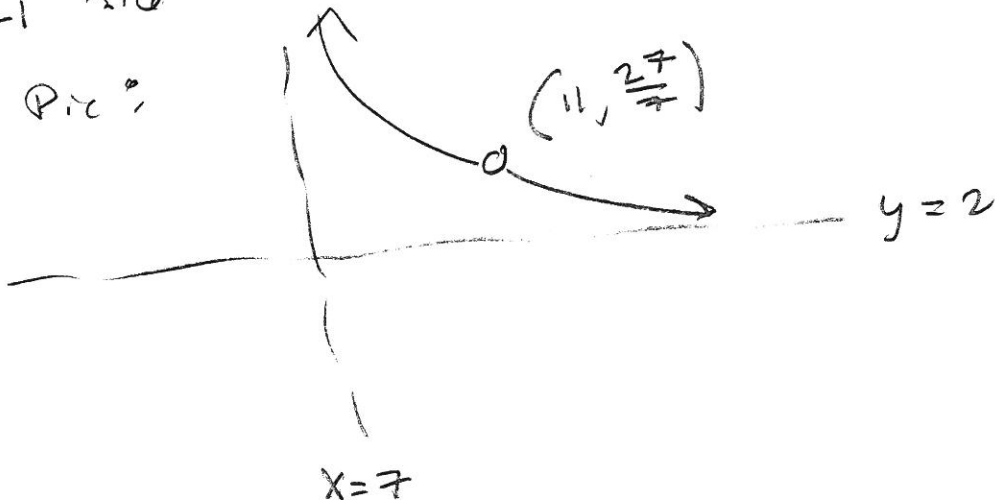
So the hole is  $x=11$

$$R(11) = \frac{2(11)^2 - 11 - 15}{(11)^2 - 4(11) - 21} = \frac{216}{56} = \frac{27}{7} \approx 3.85714$$

$$\begin{array}{r|rrr} 11 & 2 & -1 & -15 \\ & & 22 & 231 \\ \hline & 2 & 21 & 216 \end{array}$$

$$\begin{array}{r|rrr} 11 & 1 & -4 & -21 \\ & & 11 & 77 \\ \hline & & 7 & 56 \end{array} = \frac{27}{7}$$

Pic:



$$(10) \quad T(x) = \frac{x^3 - 13x^2 + 55x - 91}{x^2 - x - 2} = \frac{(x-7)(x^2 - 6x + 13)}{(x-2)(x+1)}$$

Factor Numerator:

FROM QUADRATIC

$$\begin{array}{r|rrrr} \rightarrow 7 & 1 & -13 & 55 & -91 \\ & & 7 & -42 & 91 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$$x^2 - 6x + 13 = 0$$

$$x^2 - 6x = -13$$

$$x^2 - 6x + 3^2 = -13 + 9$$

$$(x-3)^2 = -4$$

$$x = 3 \pm i\sqrt{4} = 3 \pm 2i$$

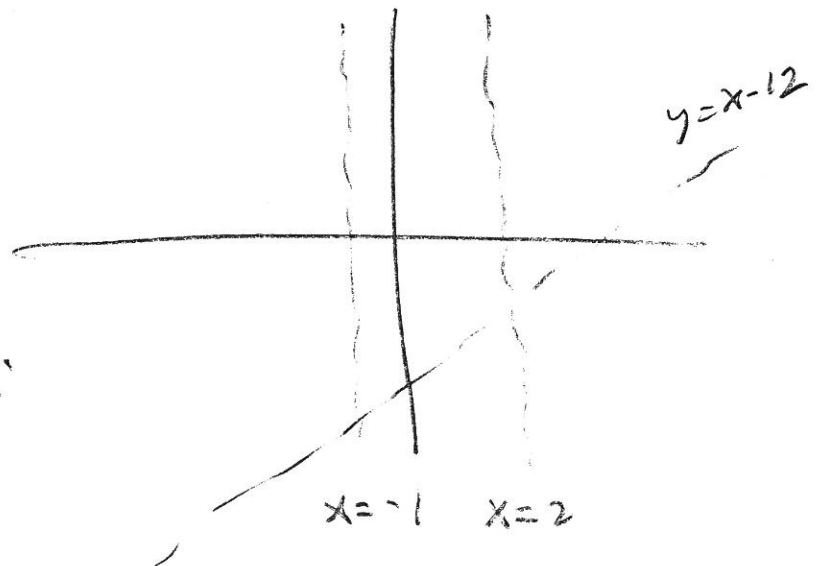
$$D: \mathbb{R} \setminus \{-1, 2\}$$

$$\boxed{\text{V.A.: } x = -1, x = 2}$$

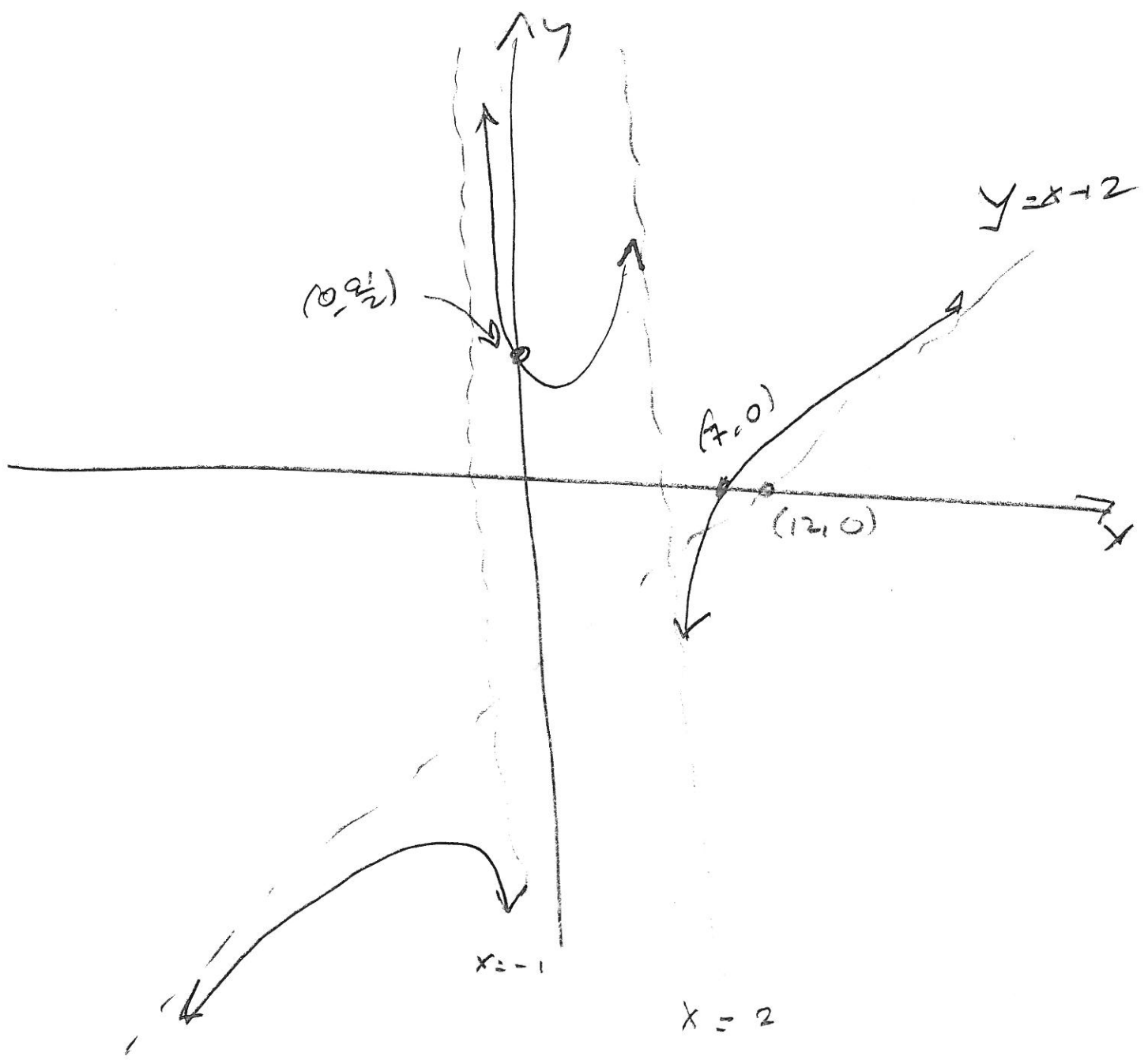
$$\begin{array}{r} x^2 - x - 2 \overline{) x^3 - 13x^2 + 55x - 91} \\ \underline{-(x^3 - x^2 - 2x)} \phantom{- 91} \\ -12x^2 + 57x - 91 \phantom{- 91} \end{array}$$

$$\boxed{y = x - 12 \text{ is O.A}}$$

$$\boxed{x\text{-int: } (7, 0)}$$

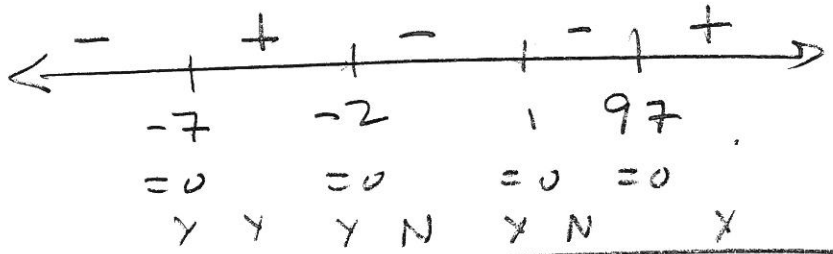


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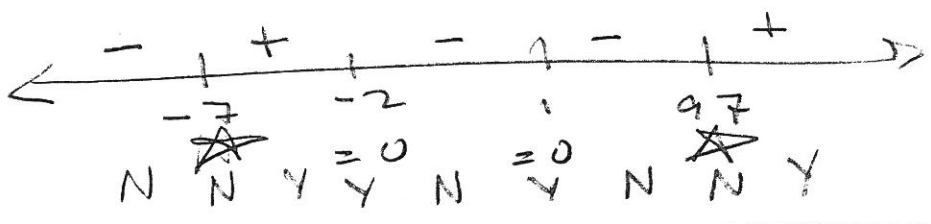
(1)  $D(w) = \text{Need } (x+2)(x-1)^2(x+7)(x-97) \geq 0$



$D(w) = [-7, -2] \cup \{1\} \cup [97, \infty)$

(2)  $D(k)$  is same as  $D(w)$  except for

$x \neq 97$  &  $x \neq -7 \rightarrow$



$> 0$  &  $x \neq -7$  &  $x \neq 97$

$D(k) = (-7, -2] \cup \{1\} \cup (97, \infty)$