121 WP #3 (TEST 3 TAKE-HOME) Spring, 2018

$f(x) = 2x^5 - 19x^4 + 166x^3 - 457x^2 + 482x - 174$

(1) $2x^5$ ...

(2) Descartes:

$$\underbrace{2x^5 - 19x^4}_{1}\ \underbrace{+166x^3}_{2}\ \underbrace{-457x^2}_{3}\ \underbrace{+482x}_{4}\ \underbrace{-174}_{5}$$

$\boxed{5, 3 \text{ or } 1 \text{ positive roots}}$

$f(-x) = -2x^5 - 19x^4 - 166x^3 - 457x^2 - 482x - 174$

$\boxed{0 \text{ negative zeros}}$

(3) Rational zeros

$\frac{p}{q}: \frac{174}{2}$

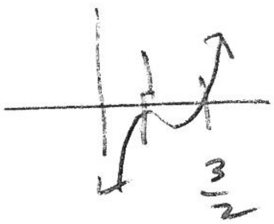
$2\,\underline{|174}$
$3\,\underline{|87}$
$\,|29$

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 29, \pm 58, \pm 87, \pm 174$ (12)

$\pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \pm \frac{3}{2}, \cancel{\pm \frac{6}{2}}, \pm \frac{29}{2},$

$\cancel{\pm \frac{58}{2}}, \pm \frac{87}{2}$

121 WP 3

(4) 

is what it looks like

1	2	−19	166	−457	482	−174
		2	−17	149	−308	174
1	2	−17	149	−308	174	0
		2	−15	134	−174	
$\frac{3}{2}$	2	−15	134	−174	0	
		3	−18	174		
	2	−12	116	0		

$2x^2-12x+116=0$

$x^2-6x+58=0$

$b^2-4ac=(-6)^2-4(1)(58)$ is clearly negative,

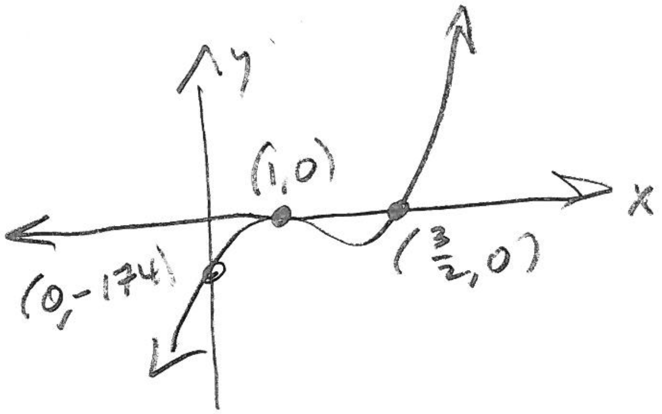
so no real solns.

$\boxed{x=1, m=2;\ x=\frac{3}{2}, m=1}$

(5) $\boxed{f(x)=(x-1)^2\left(x-\frac{3}{2}\right)(2x^2-12x+116)}$

121 WP #3

6



7

$2x^2-12x+116=0$

$x^2-6x+58=0$

$b^2-4ac = 36-232 = -196$

$\sqrt{-196} = 2\cdot 2i\sqrt{11} = 4i\sqrt{11}$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{6 \pm 4i\sqrt{11}}{2} = \boxed{3 \pm 2i\sqrt{11} = x}$$

$x=$

$2 \,|\, 196$
$2 \,|\, 88$
$2 \,|\, 44$
$2 \,|\, 22$
11

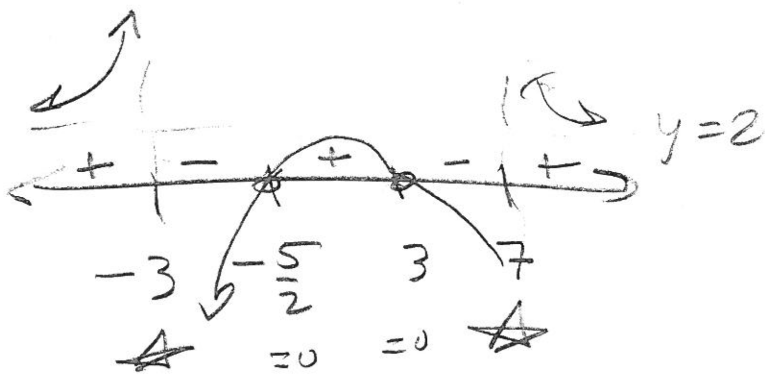
$$f(x) = \boxed{2(x-1)^2\left(x-\tfrac{3}{2}\right)\left(x-(3+2i\sqrt{11})\right)\left(x-(3-2i\sqrt{11})\right)}$$

121 WP 3

(8) $R(x) = \dfrac{2x^2 x - 15}{x^2 - 4x - 21} = \dfrac{(2x+5)(x-3)}{(x-7)(x+3)}$

$D: \mathbb{R} \setminus \{-3, 7\}$

$\boxed{V.A.: x = -3,\ x = 7}$

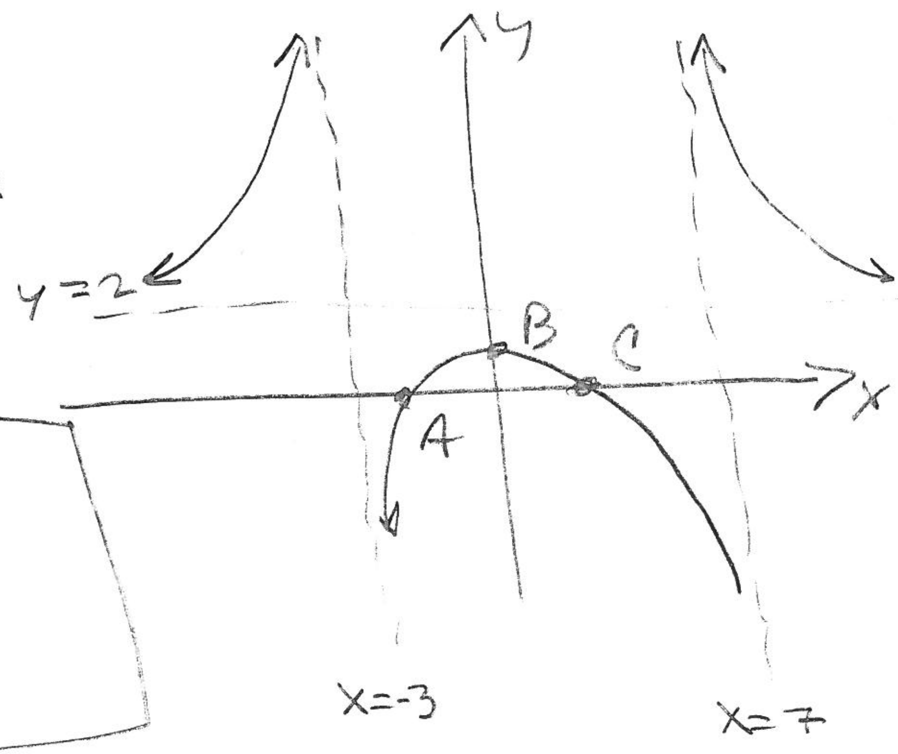


$\boxed{x\text{-int}: \left(-\frac{5}{2}, 0\right) \quad (3, 0)}$

$\boxed{y\text{-int}: \frac{15}{21} = \frac{5}{7} \quad \left(0, \frac{5}{7}\right)}$

E.B.: $\dfrac{2x^2}{x^2} = 2$

$\boxed{y = 2 \quad H.A.}$



$A = \left(-\frac{5}{2}, 0\right)$

$B = \left(0, \frac{5}{7}\right)$

$C = (3, 0)$

121 WP3

$\boxed{\text{Bonus}}$ $R(x) \overset{SET}{=} 2 = H.A.$ $LCD = (x+3)(x-7)$

$$\frac{2x^2-x-15}{(x-7)(x+3)} = \frac{2}{1}\cdot\frac{x^2-4x-21}{x^2-4x-21}$$

$$\frac{2x^2-x-15}{LCD} = \frac{2x^2-8x-42}{LCD}$$

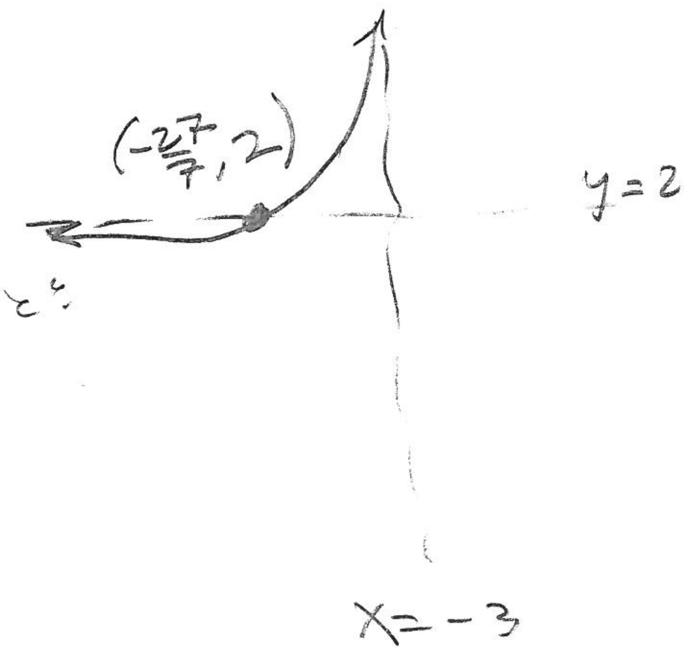
$$2x^2-x-15 = 2x^2-8x-42$$

$$-x-15 = -8x-42$$

$$7x = -27$$

$$x = \frac{-27}{7}$$

Pic:



121 WP3

(9) $Q(x) = \dfrac{2x^3 - 23x^2 - 4x + 165}{x^3 - 15x^2 + 23x + 231}$

Look for the hole!

$Q(x) = R(x)\left(\dfrac{x-c}{x-c}\right)$ Find $x-c$.

$x^3 - 15x^2 + 23x + 231 = (x-7)(x+3)(x-c)$

$$
\begin{array}{r|rrrr}
7 & 1 & -15 & 23 & 231 \\
 & & 7 & -56 & -231 \\
\hline
-3 & 1 & -8 & -33 & 0 \\
 & & -3 & 33 & \\
\hline
 & 1 & -11 & &
\end{array}
$$

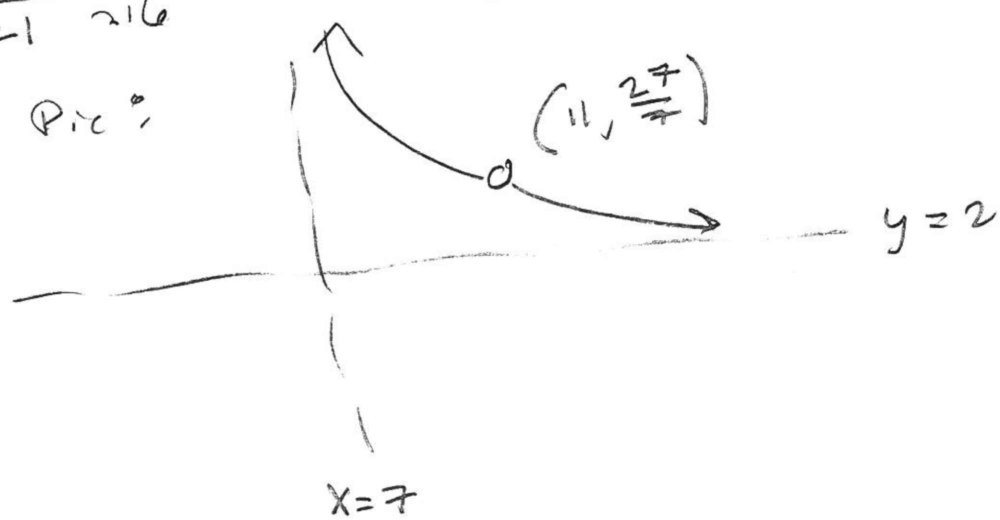
$\rightarrow x - c = x - 11$

So the hole is @ $x = 11$

$R(11) = \dfrac{2(11)^2 - 11 - 15}{(11)^2 - 4(11) - 21} = \dfrac{216}{56} = \dfrac{27}{7} \approx 3.85714$

$$
\begin{array}{r|rrr}
11 & 2 & -1 & -15 \\
 & & 22 & 231 \\
\hline
 & 2 & 21 & 216
\end{array}
\qquad
\begin{array}{r|rrr}
11 & 1 & -4 & -21 \\
 & & 11 & 77 \\
\hline
 & 1 & 7 & 56
\end{array}
= \frac{27}{7}
$$

Pic:



121 WP 3

(10) $T(x) = \dfrac{x^3 - 13x^2 + 55x - 91}{x^2 - x - 2} = \dfrac{(x-7)(x^2-6x+13)}{(x-2)(x+1)}$

Factor Numerator:

$$\begin{array}{r|rrrr} 7 & 1 & -13 & 55 & -91 \\ & & 7 & -42 & 91 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

From Grapher

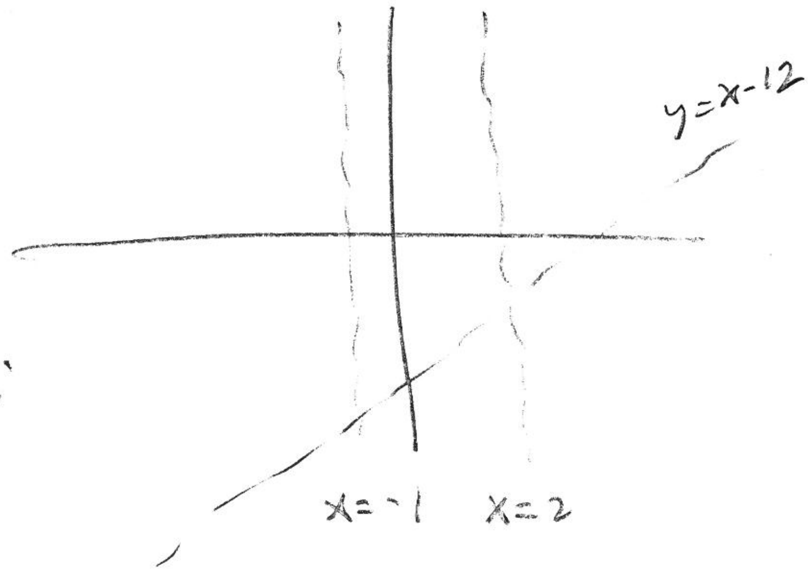
$x^2 - 6x + 13 = 0$

$x^2 - 6x = -13$

$x^2 - 6x + 3^2 = -13 + 9$

$(x-3)^2 = -4$

$x = 3 \pm i\sqrt{4} = 3 \pm 2i$



D: $\mathbb{R} \setminus \{-1, 2\}$

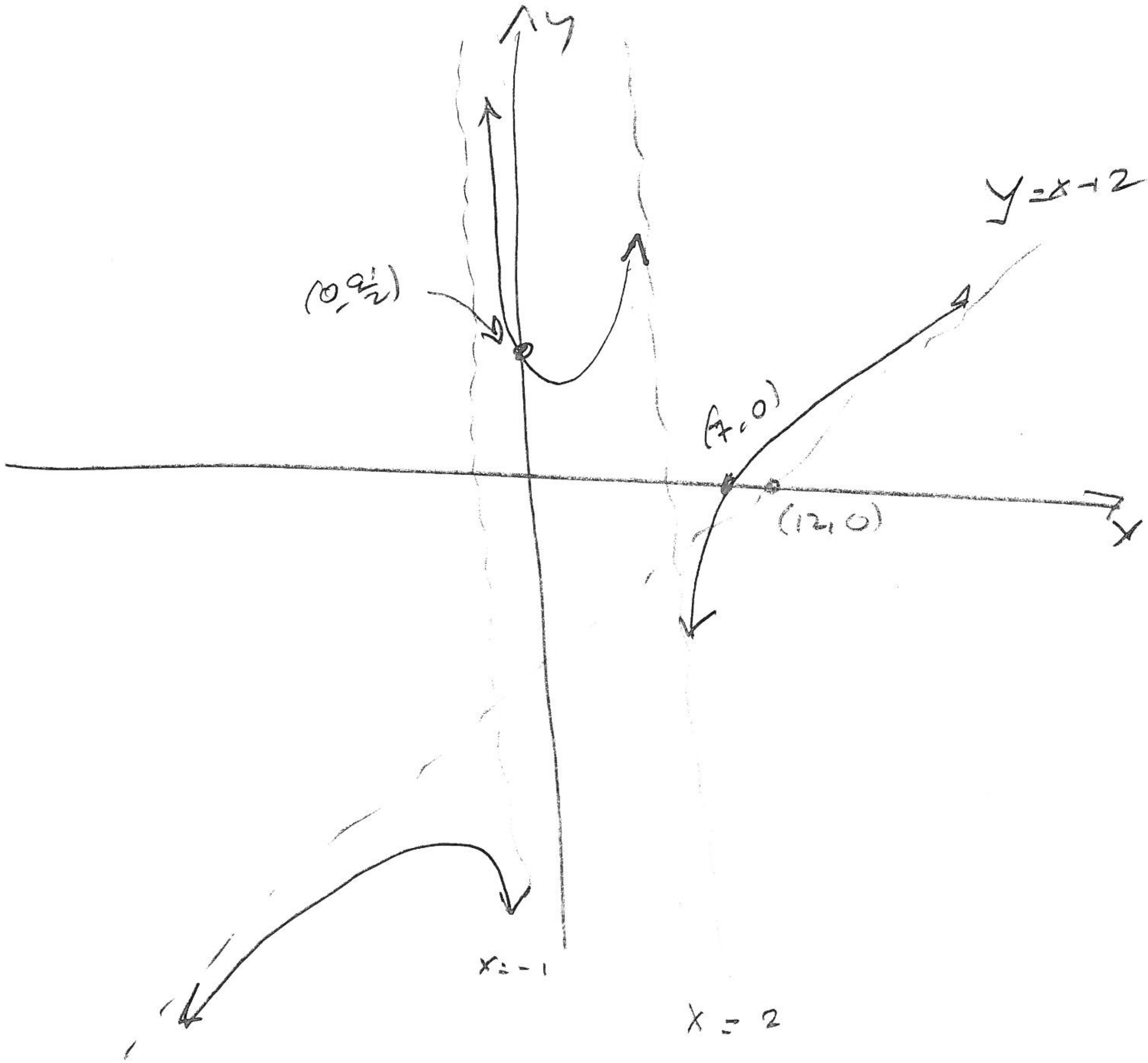
$\boxed{\text{V.A.: } x = -1, x = 2}$

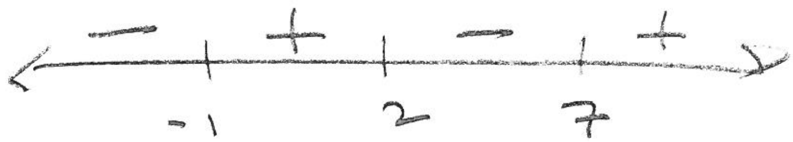
$$\begin{array}{r|l} & x - 12 \\ x^2 - x - 2 & x^3 - 13x^2 + 55x - 91 \\ & -(x^3 - x^2 - 2x) \\ \hline & -12x^2 \ldots \end{array}$$

$\boxed{y = x - 12 \text{, is O.A}}$

$\boxed{x\text{-int: } (7, 0)}$

121 WP 3





121 WP3

(11) $\mathcal{D}(w)$: Need $(x+2)(x-1)^2(x+7)(x-97) \geq 0$

$-\quad +\quad -\quad -\quad +$

$-7\quad -2\quad 1\quad 97$

$=0\quad =0\quad =0\quad =0$

Y Y Y N Y N Y

$$\boxed{\mathcal{D}(w) = [-7,-2] \cup \{1\} \cup [97,\infty)}$$

(12) $\mathcal{D}(k)$ is same as $\mathcal{D}(w)$ except for $x \neq 97$ & $x \neq -7$ $\longrightarrow$ >0 & $x \neq -7$ & $x \neq 97$

$-\quad +\quad -\quad -\quad +$

$-7\quad -2\quad 1\quad 97$

$=0\quad =0$

N N Y Y N Y N N Y

$\longrightarrow$

$$\boxed{\mathcal{D}(k) = (-7,-2] \cup \{1\} \cup (97,\infty)}$$