Combinations & Permutations.

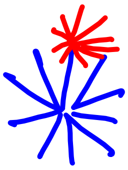
Counting

Multiplication Principle (for independent events)

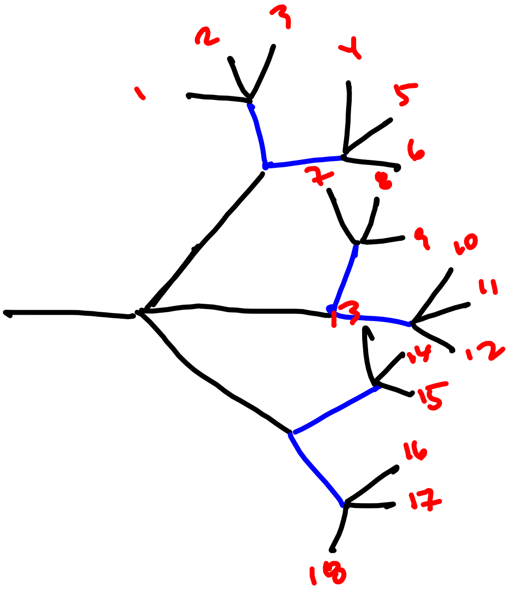
How many phone #s in any given area code?

$\underline{10}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10}$

No restrictions
(911, 555, 411)
Aside from these
'0' for "operator"



John has 3 pairs of pants, 2 shirts, 3 pairs of shoes
How many different outfits can he put together?



$\underline{3}\cdot\underline{2}\cdot\underline{3}$

5 people $P_1, P_2, \ldots, P_5$

How many ways can you choose 3 of them for 3 jobs: President, Treasurer, Clown

$\underline{5} \cdot \underline{4} \cdot \underline{3}$

Pres Treas Clown

Select and arrange, without replacement.

Permutation

$3! = 3$ factorial $= 3 \cdot 2 \cdot 1 = 6$

$0! = 1$ by convention / definition.

5 people, choose 3 for specific jobs

$= 5 \cdot 4 \cdot 3 = P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2 \cdot 1}}{\cancel{2 \cdot 1}}$

4 people, choose 1 to be pathfinder & the other to be sacrificial lamb.

$$P(4,2) = \frac{4!}{(4-2)!} = \frac{4!}{2!} = 4 \cdot 3 = 12$$

a, b, c, d are our peeps.

ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc,

12 ways.

Notice $5!$ = The # of ways to arrange 5 people.

$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

$n!$ = # of arrangements on n people.

How many 2-man teams can be chosen from 4 people? Nobody's better than anyone else.

ab, ac, ad, ~~ba~~, bc, bd, ~~ca~~, ~~cb~~, cd, ~~da~~, ~~db~~, ~~dc~~,

6 ways.

$P(4,2) = 12$

$C(4,2)$ = combos on 4 people taken 2 at a time = The # of 2-person subsets of $\{a, b, c, d\}$ = "4 choose 2"

$\{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}$

$P(4,2)$ = "4 choose 2 and arrange the 2."

$$C(n,k) = \frac{n!}{k!(n-k)!}$$

$$C(4,2) = \frac{4!}{2!2!}$$

$$P(4,2) = \frac{4!}{2!}$$

$$P(n,k) = \frac{n!}{(n-k)!}$$

Combos = Perms divided by the # of ways to arrange the chosen.

How many poker hands are there?

52 cards, Choose 5

$$C(52,5) = \frac{52!}{5!(52-5)!} = \frac{52!}{5!\,47!}$$

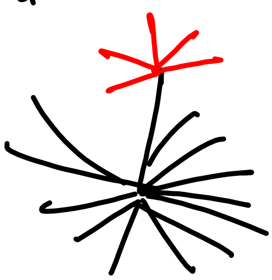
$$= \frac{52\cdot 51\cdot 50\cdot 49\cdot 48\cdot \cancel{47!}}{5!\,\cancel{47!}} = \frac{52\cdot 51\cdot \cancel{50}^{10}\cdot 49\cdot \cancel{48}^{2}}{\cancel{5\cdot 4\cdot 3\cdot 2}}$$

$$= 52\cdot 51\cdot 10\cdot 49\cdot 2 = 2,598,960$$

How many ways to get 3 of a kind?

$$\underline{C(13,1)}\cdot \underline{C(4,3)}\cdot \underline{C(48,2)}$$

pick a #



Full house is included in this, because I didn't account for the last 2 being a pair.

How many ways to get 2 pair?

$C(13,1)\cdot C(4,2)\cdot C(12,1)\cdot C(4,2)\cdot C(44,1)$

$C(13,2)$

$$C(13,2) = \frac{13!}{11!\,2!} = \frac{13\cdot 12}{2}$$

$$\frac{C(13,1)\,C(12,1)}{2} = 13\cdot 12$$

I'm off by a factor of 2 on this reasoning

TEST QUESTION:

10 people. Choose 3 ....

(a) and arrange them $P(10,3)$

(b) .. don't arrange them. $C(10,3)$

NOTATION

$${}_nC_k = C(n,k) = \binom{n}{k}$$

is called the binomial coefficient

$a+b$, $3x+2y$, $x-7$ are binomials.

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$1 = \binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$

$n=4 \quad \binom{4}{0}a^4b^0 + \binom{4}{1}a^3b^1 + \binom{4}{2}a^2b^2 + \binom{4}{3}a^1b^3 + \binom{4}{4}a^0b^4$

$$= \sum_{k=0}^{4} \binom{4}{k} a^{4-k} b^k$$

In general $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

In practice, Pascal's Triangle.

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(3x-2y)^5$$

$$= 1(3x)^5(-2y)^0 + 5(3x)^4(-2y)^1 + 10(3x)^3(-2y)^2$$
$$+ 10(3x)^2(-2y)^3 + 5(3x)^1(-2y)^4 + 1(3x)^0(-2y)^5$$

$$= 243x^5 + 5(8x^4)(-2y) + 10(27x^3)(4y^2)$$
$$+ 10(9x^2)(-8y^3) + 5(3x)(16y^4) + (-32y^5)$$

$$= 243x^5 - 80x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$$