5.3 # 29

$$\frac{3}{x} - \frac{1}{y} = \frac{13}{10}$$

$$\frac{1}{x} + \frac{2}{y} = \frac{9}{10}$$

R1 $\frac{3}{x} - \frac{1}{y} = \frac{13}{10}$

-3R2 $-\frac{3}{x} - \frac{6}{y} = -\frac{27}{10}$

R1-3R2 $-\frac{7}{y} = -\frac{7}{5}$

$-\frac{1}{y} - \frac{6}{y} = \frac{-1-6}{y} = -\frac{7}{y}$

$\frac{17-27}{10} = -\frac{14}{10} = -\frac{7}{5}$

LCD = 5y

$-\frac{7}{y} \cdot \frac{5}{5} = -\frac{7}{5} \cdot \frac{y}{y}$

$\frac{-35}{LCD} = \frac{-7y}{LCD}$

$-35 = -7y$

$-7y = -35$

$y = \frac{-35}{-7} = 5 = y$

y = 5 →

$\frac{3}{x} - \frac{1}{y} = \frac{13}{10}$ →

LCD = 2.5x

$\frac{3}{x} - \frac{1}{5} = \frac{13}{10}$

$\frac{3}{x} \cdot \frac{10}{10} - \frac{1}{5} \cdot \frac{2x}{2x} = \frac{13}{10} \cdot \frac{x}{x}$

$\frac{30-2x}{LCD} = \frac{13x}{LCD}$

$30 - 2x = 13x$

$30 = 15x$

$15x = 30$

$x = \frac{30}{15} = 2 = x$

$(x, y) = (2, 5)$

$(x, y) \in \{(2, 5)\}$ } either is OK

$\{x \mid x \neq 2, 3\} = \mathbb{R} \setminus \{2, 3\}$

$= (-\infty, 2) \cup (2, 3) \cup (3, \infty)$

5.3 #27

$x+y=-4$

$xy=1 \Rightarrow y=\frac{1}{x}$

$\Rightarrow x+y=-4$ is

LCD $=x$

$x+\frac{1}{x}=-4$

$x\cdot\frac{x}{x}+\frac{1}{x}=-4\cdot\frac{x}{x}$

$\frac{x^2+1}{x}=\frac{-4x}{x}$

$x^2+1=-4x$

$x^2+4x+1=0$

$x^2+4x+2^2=-1+4$

$(x+2)^2=3$

$x+2=\pm\sqrt{3}$

$x=-2\pm\sqrt{3}$

$x=-2+\sqrt{3}$

$x+y=-4$

$-2+\sqrt{3}+y=-4$

$y=-4+2\mp\sqrt{3}$

$y=-2\mp\sqrt{3}$

$x=-2-\sqrt{3}$

$x+y=-4$

$-2-\sqrt{3}+y=-4$

$y=+2+\sqrt{3}-4$

$y=-2+\sqrt{3}$

$(x,y)\in\{(-2+\sqrt{3},-2-\sqrt{3}),(-2-\sqrt{3},-2+\sqrt{3})\}$

5.3 # 31

$x^2+xy-y^2=-5$

$x+y=1 \Rightarrow y=1-x$

$x^2+xy-y^2=-5$

$x^2+x(1-x)-(1-x)^2=-5$

$x^2+x-x^2-(x^2-2x+1)=-5$

$x^2+x-x^2-x^2+2x-1=-5$

$-2x^2+3x-1=-5$

$-x^2+3x+4=0$

$x^2-3x-4=0$

$x^2-4x+1x-4=0$

$x(x-4)+1(x-4)$

$(x-4)(x+1)=0$

$x \in \{-1,4\}$, provisionally.

Write Much
Think Little

$(1-x)^2=(x-1)^2$

$(1-x)^2=((-1)(x-1))^2$

$=(-1)^2(x-1)^2$

$=(x-1)^2$

$(x-1)^2=x^2-2x+1$

$y=1-x$

$x=-1$: $y=1-(-1)=1+1=2$

$y=2$

$(-1,2)$

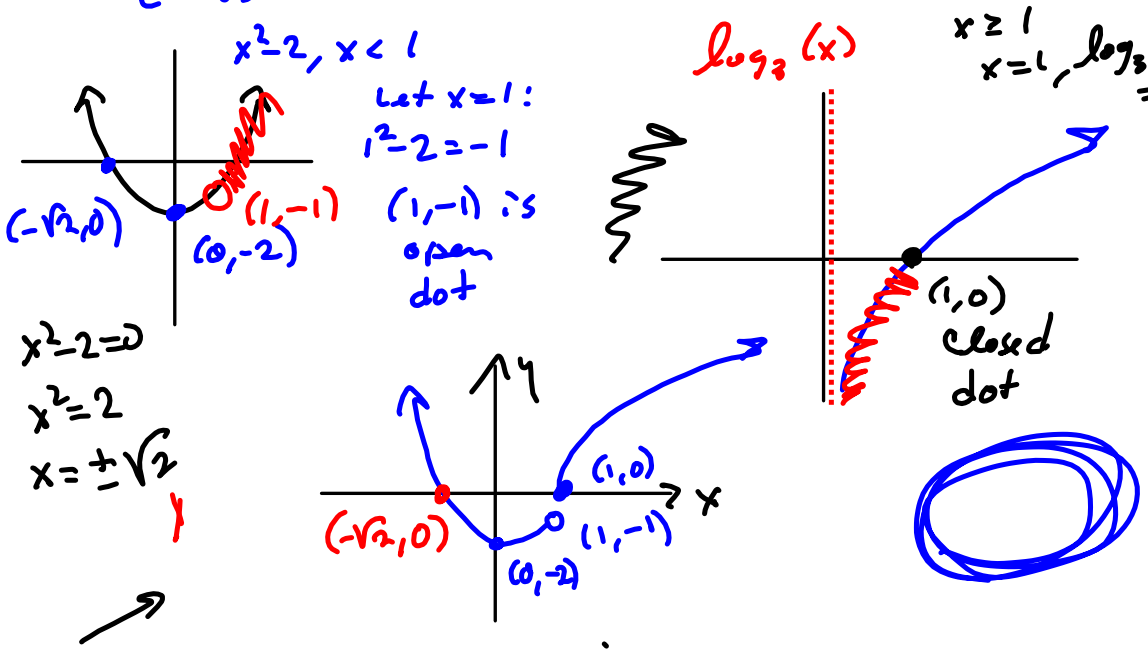
$x=4$: $y=1-4=-3=y$

$(4,-3)$

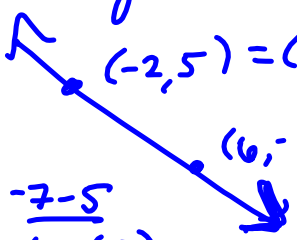
$(x,y) \in \{(-1,2),(4,-3)\}$

Graph the piecewise-defined function

$$f(x)=\begin{cases} x^2-2 & \text{if } x<1 \\ \log_3(x) & \text{if } x\geq 1 \end{cases}$$



Build an equation of the line



$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-7-5}{6-(-2)}$$

$$=\frac{-12}{8}=-\frac{3}{2}$$

$y=mx+b$

$y=-\frac{3}{2}x+b$

$5=-\frac{3}{2}(-2)+b$

$5=3+b$

$2=b$

$y=-\frac{3}{2}x+2$

$y=m(x-x_1)+y_1$

$y=-\frac{3}{2}(x-(-2))+5$

$y=-\frac{3}{2}(x+2)+5$ STOP!

$=-\frac{3}{2}x-3+5$

$y=-\frac{3}{2}x+2$

3. (5 pts) $12x^2 = 9$ (Give your answer in simplified radical form.)
4. (5 pts) $9x^2 - 24x + 23 = 0$ (Give your answer in simplified radical form.)

#s 5 – 7. Compute the discriminant for the following equations. Tell me what it says about the solutions of the equations, *without solving the equations*. How many distinct solutions, how many real zeros. If you can predict rational solutions, that's worth some extra points.

5. (5 pts) $9x^2 - 24x + 23 = 0$ $D = b^2 - 4ac = (-24)^2 - 4(9)(23) = -252 < 0$ ⇒ 2 nonreal soln's
6. (5 pts) $12x^2 = 9$ ⇒ $12x^2 - 9 = 0$, $a = 12$, $b = 0$, $c = -9$
7. (5 pts) $36x^2 - 60x + 25$ $b^2 - 4ac = 0^2 - 4(12)(-9) = 432 > 0$ 2 real solns

Solve by factoring: You can use a "cheat," so long as you show understanding of the connection between solutions and factors. $(x+7)(x-5) = 0 \Rightarrow x \in \{5, -7\}$

8. (10 pts) $x^2 + 2x - 35 = 0$
9. (5 pts) $60x^2 + 61x - 56 = 0$

$x = \frac{-b \pm \sqrt{D}}{2a}$

10 / 36 / 360 — 360 / 72 / 432

(3) $12x^2 = 9$

$x^2 = \frac{9}{12} = \frac{3}{4}$

$x^2 = \frac{3}{4}$

$x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2} = x$

#7 $b^2 - 4ac = (-60)^2 - 4(36)(25)$

$3600 - 3600 = 0$

⇒ 1 real (repeated) soln.

(4) $b^2 - 4ac = (-24)^2 - 4(9)(23)$

$= 576 - 828 = -252$

$\sqrt{-252} = 2 \cdot 3\sqrt{7}\,i = 6\sqrt{7}\,i$

$x = \frac{24 \pm 6\sqrt{7}\,i}{2(9)} = \frac{6(4 \pm \sqrt{7}\,i)}{2 \cdot 3 \cdot 3}$

$= \frac{4 \pm \sqrt{7}\,i}{3}$

BAD $\sqrt{7}i$

OK $\sqrt{7}\,i$ $= i\sqrt{7}$ OK

252 = 2 · 2 · 3 · 3 · 7

24 / 24 / 96 — 48 / 576

36 / 23 / 108 / 720 / 828

$60x^2+61x-56=0$

Cheat:

$b^2-4ac=(61)^2-4(60)(-56)$

$=3721+13440=17161=131^2$

missed the sign.

61×61: $61 \times 1 = 61$, $61 \times 60 = 3660$, $61+3660=3721$

$x=\frac{-61\pm131}{2(60)}=\frac{-61\pm131}{120}$

$\frac{7}{12}=\frac{70}{120}$

$\frac{-192}{120}=\frac{-96}{60}=\frac{-48}{30}=\frac{-16}{10}=\frac{-8}{5}$

56×240: $56 \times 40 = 2240$, $56 \times 200 = 11200$, $2240+11200=13440$

So $x=\frac{7}{12}, \frac{-8}{5}$

$60x^2+61x-56=60\left(x-\frac{7}{12}\right)\left(x+\frac{8}{5}\right)$

$=12\cdot5\left(x-\frac{7}{12}\right)\left(x+\frac{8}{5}\right)$

$=(12x-7)(5x+8)=0$

$12x-7=0$ OR $5x+8=0$

$12x=7$ $5x=-8$

$x=\frac{7}{12}$ OR $x=-\frac{8}{5}$

$\left(x+\frac{3+2\sqrt{3}i}{2}\right)\left(x-\frac{3-2\sqrt{3}i}{2}\right)$