Prove $S_n = a + ar + ar^2 + \cdots + ar^{n-1} = a\left(\frac{1-r^n}{1-r}\right)$

Recall

FV = Future Value of annuity

$$S = R\left(\frac{(1+i)^n - 1}{i}\right)$$

$$A = P(1+i)^n$$

PV = Present Value of an annuity

= Loan amount, given interest rate, number of payments, and amount of payments

= Amount you can afford to borrow.

To find this, we want the future value of the one lump sum (loan amt) to be the same at the end of payments as if they just stuck that money in a bank at the interest rate they're charging you.

$$A = P(1+i)^n = S = R\left(\frac{(1+i)^n - 1}{i}\right)$$

A: Future Value of Lump Sum

P: Loan Amt.

$i = \frac{r}{m}$

$n = mt$

S: Future Value of payments.

R: Payment(s)

Solve for P

$$P(1+i)^n = R\left(\frac{(1+i)^n - 1}{i}\right)$$

$$P = \frac{1}{(1+i)^n} \cdot \frac{R((1+i)^n - 1)}{i}$$

$ab = ba$

$$= \frac{(1+i)^{-n}}{1} \cdot \frac{R((1+i)^n - 1)}{i}$$

$a(b+c) = ab + ac$

$$= \frac{R(1+i)^{-n}((1+i)^n - 1)}{i}$$

$a^{-n}a^n = a^0 = 1$

$$= \frac{R\left((1+i)^{-n}(1+i)^n - (1+i)^{-n}\right)}{i}$$

$$\boxed{P = \frac{R(1-(1+i)^{-n})}{i}}$$

R, r, m, t

How much can you afford to borrow?

Set $A = S$, solve for P

$$P(1+i)^n = R\left(\frac{(1+i)^n - 1}{i}\right)$$

Sinking Fund.

You Know how much you need in the future, so you want to know how much to set aside each month to meet that goal.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Solve for R:

$$\frac{Si}{((1+i)^n - 1)} = R$$

Sinking Fund formula.

Parents want to save for their worthless, lazy kid's college. So starting at one month old, they pay into an account for 18 years. How much each month to have $100,000 in the bank at the end? $r = .07,\ t = 18,\ n = 12$

$$R = \frac{(100000)\left(\frac{.07}{12}\right)}{\left(\left(1+\frac{.07}{12}\right)^{12(18)} - 1\right)}$$

20% for correct numbers in final answer.

$\$232.17$?

```
100000*.07/12/((
1+.07/12)^(12*18
)-1)
      232.1688374
```

Suppose you want to know what your monthly payments will be on a LOAN?

You know P, r, m, t

RECALL $A = S$ for P gives present value

$$P(1+i)^n = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Solve the "Loan Equation" for R.

$$R\left[\frac{(1+i)^n - 1}{i}\right] = P(1+i)^n$$

$$R = P(1+i)^n\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= \frac{P}{(1+i)^{-n}}\left[\frac{i}{(1+i)^n - 1}\right]$$

$$= \frac{Pi}{(1+i)^{-n}(1+i)^n - (1+i)^{-n}}$$

$$R = \frac{Pi}{1 - (1+i)^{-n}}$$

$\frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$

You borrowed $150,000 at 4% annual rate on a 30-yr loan.

What's your payment?

$P = 150000,\ r = .04,\ m = 12,\ t = 30$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{(150000)\left(\frac{.04}{12}\right)}{\left(1-\left(1+\frac{.04}{12}\right)^{-(12)(30)}\right)}$$

```
1+.07/12)^(12*18
)-1)
     232.1688374
150000*.04/12/(1
-(1+.04/12)^(-12
*30))
     716.1229432
```

DO THE FREAKIN' 5.3, 5.5, OK?

GRAPH THE SYSTEM

DON'T DO LIKE THE BOOK. Instead, always scratch out the BAD STUFF.

$3x-2y \le 12$

$5x+4y \le 20$

$x \ge 0$

$y \ge 0$

$3x-2y \le 12$

x	y
0	-6
4	0

$0 \le 12$?

Yes

(0,0) GOOD

Tanzan say.

$5x+4y \le 20$

x	y
0	5
4	0

$0 \le 20$?

Yes. (0,0) good

Ten Bears like.