§5.2 #s 29, 33

Under-determined

Dependent: 3 vars, 2 eq'ns.

29 $x + 2y - 3z = -17$

$3x - 2y - z = -3$

$-3R1 \quad -3x - 6y + 9z = 51$

$R2 \quad 3x - 2y - z = -3$

$-3R1+R2 \quad 0 \; -8y + 8z = 48$

$$\begin{bmatrix} 1 & 2 & -3 & -17 \\ 0 & -8 & 8 & 48 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -3 & -17 \\ 0 & 1 & -1 & -6 \end{bmatrix}$$

New System:

$x + 2y - 3z = -17$

$-8y + 8z = 48$

This is far enough to parse the solution.
makes sense to factor out the -8 in R2.

$R1 \quad x + 2y - 3z = -17$

$-\frac{1}{8}R2 \quad 1y - 1z = -6$

Can't eliminate any more.
We can get rid of 'y' in 1st eq'n. That's Reduced Row-Echelon Form. But why bother?

$$\begin{array}{c} -2R2+R1 \\ R2 \end{array} \begin{bmatrix} 1 & 0 & -1 & -5 \\ 0 & 1 & -1 & -6 \end{bmatrix}$$

won't use. But look for "Row Reduce" in wolframalpha.org.

$y - z = -6$

$\boxed{y = z - 6}$

$x + 2y - 3z = -17$

$x + 2(z-6) - 3z = -17$

$x + 2z - 12 - 3z = -17$

$x - z - 12 = -17$

$x - z = -5$

$\boxed{x = z - 5}$

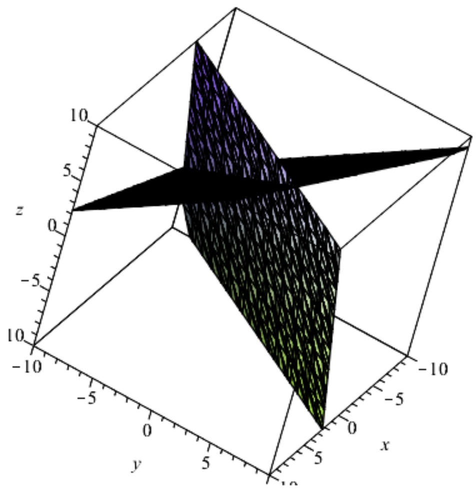
z is FREE

$\{(x, y, z) \mid x = z-5,\ y = z-6,\ z \in \mathbb{R}\}$

$\{(x, y, z) \mid y = x-1,\ z = x+5,\ x \in \mathbb{R}\}$

$x = z - 5$

$y = (z-5) - 1 = z - 6$



A picture for #29 A dependent system with infinitely-many solutions. Two planes intersecting in a line.

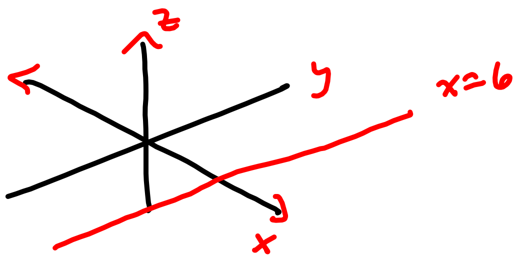
§5.2 #33

$$2x - y - z = 7$$
$$y + z = 5$$

$$\Rightarrow y = -z + 5$$

$$2x - y - z = 7$$
$$2x - (-z+5) - z = 7$$
$$2x + z - 5 - z = 7$$
$$2x - 5 = 7$$
$$2x = 12$$
$$\boxed{x = 6}$$

$$\{(x, y, z) \mid x = 6,\ y = -z+5,\ z \in \mathbb{R}\}$$



$$\begin{matrix} ax+by+cz=d \\ ex+fy+gz=h \\ ix+jy+kz=l \end{matrix} \longleftrightarrow \left[\begin{array}{ccc|c} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{array}\right]$$

Matrix

System 2:

$$\left[\begin{array}{ccc|c} a & b & c & d \\ 0 & 1 & * & * \\ 0 & 2 & * & * \end{array}\right]$$

System 3:

$$\left[\begin{array}{ccc|c} a & b & c & d \\ 0 & * & * & * \\ 0 & 0 & * & * \end{array}\right]$$

Gives you z.
Send z to eq'n 2
Gives you y.
Send y & z to eq'n 1
Gives you x.
Done. (Interpretation)

Back-Substitute

$2x+2y+4z=15$

$3x+4y+z=12$

$5x+2y+5z=21$

$x+y+2z=7.5$

$1.2=\frac{12}{10}=\frac{6}{5}$

$1.5=\frac{15}{10}=\frac{3}{2}$

$2.4=\frac{24}{10}=\frac{12}{5}$

2R1 $2x+2y+4z=15$

$3x+4y+z=12$

$5x+2y+5z=21$

$$\begin{array}{ll} -3R1 & -6x-6y-12z=-45 \\ 2R2 & 6x+8y+2z=24 \\ \hline -3R1+2R2 & 2y-10z=-21 \end{array}$$

$$\begin{array}{ll} -5R1 & -10x-10y-20z=-75 \\ 2R3 & 10x+4y+10z=42 \\ \hline -5R1+2R3 & -6y-10z=-33 \end{array}$$

System 2:

$2x+2y+4z=15$

$2y-10z=-21$

$-6y-10z=-33$

$$\begin{array}{ll} 3R2 & 6y-30z=-63 \\ R3 & -6y-10z=-33 \\ \hline 3R2+R3 & -40z=-96 \end{array}$$

$z=\frac{-96}{-40}=\frac{48}{20}=\frac{24}{10}=\boxed{\frac{12}{5}=z}$

Final System

$2x+2y+4z=15$

$2y-10z=-21$

$\boxed{z=\frac{12}{5}}$

$2y-\cancel{10}^{2}\left(\frac{12}{\cancel{5}}\right)=-21$

$2y-24=-21$

$2y=3$

$\boxed{y=\frac{3}{2}}$

Finally

$2x+2\left(\frac{3}{2}\right)+4\left(\frac{12}{5}\right)=15$

$2x+3+\frac{48}{5}=15$

$2x+\frac{15+48}{5}=15$

$2x+\frac{63}{5}=15$

$2x=\frac{75-63}{5}$

$2x=\frac{12}{5}$

$x=\frac{12}{5}\cdot\frac{1}{2}=\boxed{\frac{6}{5}=x}$

The following matrices are in reduced row-echelon form. Interpret as systems in x, y, and z.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -11 \end{array}\right]$$

$x = 2$
$y = 5$
$z = -11$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

$0 = 2$?!

Inconsistent

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Tommy
"The Who"

Give the general solution

$x - 5z = -1$
$y + 2z = 2$

$x = 5z - 1$
$y = -2z + 2$
$z =$ Anything!