Recall Math of Finance.

Simple Interest : A = future value
P = present value (principal)
r = annual interest rate
t = time, in years

$$A = P + \underbrace{Prt}_{I} = P(1+rt)$$

I = Interest Earned.

Compound Interest

We collect interest on P every period.

m = # of periods per year

n = # of periods total $= mt$

$i = \frac{r}{m}$ = interest rate per period.

Period	A
0	P
1	$P + Pi = P(1+i)$
2	$P(1+i) + P(1+i)i = \Delta + \Delta i = \Delta(1+i)$ $= P(1+i)(1+i) = P(1+i)^2$
3	$P(1+i)^3$
	$\vdots$
n	$P(1+i)^n = \boxed{P\left(1+\frac{r}{m}\right)^{mt} = A}$

Future Value of savings

Trustafarians GET annuities.

Bankers BUY annuities.

They're called LOANS.

Simple Ordinary Annuity Certain

Payments are made at the end of each period.

Fixed # of periods $(n = mt)$

Same pmt every period (month).

S = Future Value of the Annuity

R = Periodic Payment.

1^{st} payment goes in the bank 'til the end.

R = Present Value of the 1 payment $= P$

$A = P(1+i)^{n-1} = R(1+i)^{n-1}$

2^{nd} Pmt: $R(1+i)^{n-2}$

$\vdots$

Next-to-last pmt: $R(1+i)^1$

Last pmt: R

Future Value of the annuity

$$S' = \underbrace{R + R(1+i)^1 + R(1+i)^2 + \cdots + R(1+i)^{n-1}}_{\text{Geometric Growth.}}$$

Let $R = a$ $\qquad x^1(ax^{n-1}) = ax^{n-1+1} = ax^n$

$(1+i) = x$

Then $S' = a + ax + ax^2 + ax^3 + \cdots + ax^{n-1}$

$- \; xS' = \quad ax + ax^2 + ax^3 + \cdots + ax^{n-1} + ax^n$

$S' - xS' = a - ax^n$

$S'(1-x) = a(1-x^n)$

Geometric Sum Formula

$$\boxed{S' = \frac{a(1-x^n)}{1-x} = a + ax + ax^2 + \cdots + ax^{n-1}}$$

So the future value of the annuity is

$$\frac{R(1-(1+i)^n)}{1-(1+i)} = R\,\frac{(1-(1+i)^n)}{-i} = R\left(\frac{-1+(1+i)^n}{i}\right)$$

$1-1-i = -i$

$$S = R\left[\frac{(1+i)^n-1}{i}\right]$$

$$= R\left[\frac{(1+\frac{r}{m})^{mt}-1}{\frac{r}{m}}\right]$$

$$S - xS = \Delta - x\Delta = (1-x)\Delta$$

$$= (1-x)S$$

S 4.4 II is what we're doin' here.

\$200/month 10 yrs @ 5% annual interest rate, compounded monthly.

$R = 200$

$r = .05$

$m = 12$

$t = 10$

$i = \frac{r}{m} = \frac{.05}{12}$

$n = mt = 12 \cdot 10 = 120$

$$S = R\left(\frac{(1+i)^n - 1}{i}\right) = 200\left[\frac{\left(1+\frac{.05}{12}\right)^{120} - 1}{\frac{.05}{12}}\right]$$

```
200*((1+.05/12)^
(10*12)-1)/(.05/
12)
     31056.45589
200*120
     24000.00000
```

Section 8.3 does geometric series, but I THINK you have enough to do the annuity problems in Section 4.4 II.

Next time, come with questions on the weird annuities they set up, like payments made at the BEGINNING of the period, which changes things slightly.

End of Period:

$$R + R(1+i) + R(1+i)^2 + \dots + R(1+i)^{n-1}$$

Beginning of Period

$$R(1+i) + R(1+i)^2 + \dots + R(1+i)^n$$

$= (1+i)$ [end of period version]

Factor out the extra $1+i$ is all.

$$= (1+i)\left[R\left(\frac{(1+i)^n - 1}{i}\right)\right]$$

Previous Problem, But Beginning of the period:

$$(1+i) = \left(1 + \frac{.05}{12}\right)\left[\$31,056.455\ 89\right]$$

```
200*((1+.05/12)^
(10*12)-1)/(.05/
12)
         31056.45589
Ans*(1+.05/1)
         32609.27868
```

$\approx \boxed{\$32,609.28}$

will save time on 4.4 II, but we're only testing over

Simple ordinary annuity certain